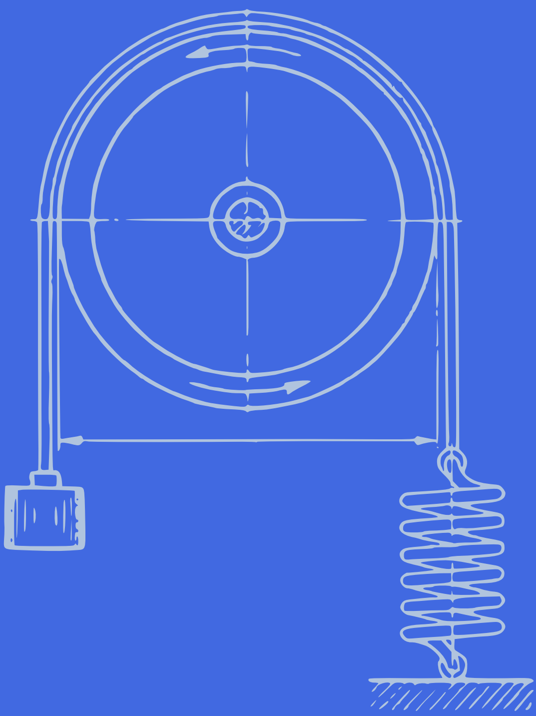
I. V. MESHCHERSKY

COLLECTION OF PROBLEMS IN THEORETICAL MECHANICS



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COLLECTION OF PROBLEMS IN THEORETICAL MECHANICS

Edited
by Professor *A. I LURIE*,
Dr. Tech. Sc.

The Supplement is written
by V I. KUZNETSOV,
Cand. Tech. Sc.

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CONTENTS

Editor's note 5
Preface . 7

Part I. **Statics of Rigid Bodies**

I. Coplanar Force System
1. Collinear Forces 9
2. Concurrent Forces 10
3. Parallel Forces and Couples 19
4. Arbitrary Coplanar Force Systems 26
5. Graphical Statics. 40

II. Statics in Space
6. Concurrent Forces . . . 43
7. Reduction of a System of Forces to Its Simplest Possible Form 47
8. Equilibrium of an Arbitrary System of Forces 48
9. Centre of Gravity 56

Part II. **Kinematics**

III. Motion of a Particle
10. Equation of Motion and Path of a Particle 59
11. Velocity of a Particle . 61
12. Acceleration of a Particle 63

IV. Simplest Motions of a Rigid Body
13. Rotation of a Rigid Body about a Fixed Axis 68
14. Conversion of Simplest Motions of a Rigid Body 70

V. Composition and Resolution of Motions of a Particle
15. Equations of Motion and Path of the Resultant Motion of a Particle 74
16. Composition of Velocities of a Particle 76
17. Composition of Accelerations of a Particle Undergoing Translatory Motion of Transport 79
18. Composition of Accelerations of a Particle Performing Rotational Motion of Transport about a Fixed Axis 82

VI. A Rigid Body Motion in a Plane
19. Equations of Motion of a Body and Its Particles in a Plane . 87
20. Velocity of a Point of a Body Which Performs Motions in a Plane. Instantaneous Centres of Velocities 89
21. Space and Body Centrodes 96
22. Accelerations of a Point on a Body Which Performs Motions in a Plane. Instantaneous Centres of Accelerations 99
23. Composition of Motions of a Body in a Plane 102

VII. Motion of a Rigid Body about a Fixed Point
24. Rotation of a Rigid Body about a Fixed Point . 105
25. Composition of Rotations of a Rigid Body about Intersecting Axes 107

PREFACE

This book is an abridged translation of the latest (28th) Russian edition of Meshchersky's *Collection of Problems in Theoretical Mechanics,* published in the USSR in 1962.

This *Collection* was compiled by a large group of Soviet professors and highly experienced instructors of the Leningrad Politechnical Institute, named after M. I. Kalinin, among whom are S. A. Sorokov (statics), N. N. Naugolnaya and A. S. Kelson (kinematics), A. S. Kelson (dynamics of a particle), M. I. Baty (dynamics of a system), G. J. Djanelidze (analytical statics, dynamics of bodies having variable masses, stability of motion).

Professor A. I. Lurie, Dr. Tech. Sc. a prominent Soviet scientist, is the general editor of this book.

The first *Collection of Problems*, edited by I. V. Meshchersky, was published in 1914. With every new edition the book has been revised and supplemented taking into consideration the developments in science and engineering for the past period.

The present edition embraces all basic principles of theoretical mechanics usually taught during the first two years of studies in higher and secondary technical schools.

The material in this book is presented consistently, i. e., proceeding from the particular to the general. The same method is applied to the order of paragraphs and the arrangement of the text. Accordingly, the initial section of the book deals with fairly easy problems on basic concepts and principles of statics of rigid bodies, while the last section of the book embraces rather complicated problems on principles of stability of motion.

It should be noted that most of the problems, chosen for this collection, are not only an illustration of the theoretical material, but are well in accord with the materials which serve as a bridge between theoretical mechanics and intermediate sciences.

The primary objective of the book was to present to the reader problems of practical value by giving the examples in such fields as the operation of machines and mechanisms, hydrodynamics, resistance of materials, and other branches of science and engineering. All this has made the book widely popular among the Soviet students of technical schools.

The book *Collection of Problems in Theoretical Mechanics* is at present one of the basic text-books for Soviet students of theoretical mechanics.

The material collected in this book will aid students in the practical application of laws and methods of theoretical mechanics in their engineering practice.

The present translation of the *Collection of Problems* is supplemented with solutions of certain typical problems for each paragraph to help the student to apply to specific situations the principles and theorems that he has learned.

The book is well illustrated with drawings and diagrams.

Part I
STATICS OF RIGID BODIES

I. COPLANAR FORCE SYSTEM

1. Collinear Forces

1. Two weights of 10 kgf and 5 kgf, respectively, are suspended from a string at different points. The heavier weight is suspended lower than the lighter one. Find the tensions in the string.

Ans. 10 kgf and 15 kgf.

2. A uniform vertical cylindrical column with the height $h=5$ m and weight $Q=3000$ kgf is mounted on a solid foundation. It carries a load $P=4000$ kgf. Determine the pressure that the column exerts on the foundation and the compressive forces in the sections located at distances $l_1=l_2=0.5$ m from the top and bottom ends of the column.

Ans. $N=7000$ kgf; $N_1=4300$ kgf; $N_2=6700$ kgf.

3. The weight of a man standing at the bottom of a pit is 64 kgf. By means of a rope running over a fixed pulley the man lifts a load of 48 kgf.

(1) Determine the pressure that the man exerts on the bottom of the pit.
(2) Calculate the maximum weight hanging on the rope that the man can hold up.

Ans. (1) 16 kgf; (2) 64 kgf.

4. A train moves at constant speed along a horizontal straight track. The weight of the train, excluding the weight of the locomotive, is 180,000 kgf. What is the tractive force exerted by the locomotive, if the resistance to motion is 0.005 of the train pressure on the rails?

Ans. 900 kgf.

5. Define an average magnitude of the force transmitted by a piston rod of a steam engine with two cylinders located in tandem (Fig. 1). The diameters of pistons are: $D_1=320$ mm; $D_2=600$ mm; the diameters of the piston rod are: $d_1=60$ mm;

$d_3=100$ mm. The mean vapour pressure is $p_1=9.5$ kgf/cm^2; $p_2=2.5$ kgf/cm^2; $p_3=0.1$ kgf/cm^2.

Ans. 12,100 kgf.

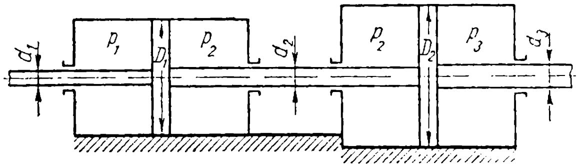


Fig. 1

2. Concurrent Forces

6. Concurrent forces of 1, 3, 5, 7, 9 and 11 kgf applied at the centre of a rectilinear hexagon act towards the vertices. Determine the magnitude and direction of the resultant and the equilibrant.

Ans. 12 kgf; the direction of the equilibrant is opposite to the direction of the given 9-kgf force.

7. Resolve a force of 8 kgf into two components, 5 kgf each. Is it possible to resolve the same force of 8 kgf into two components of 10 kgf each; 15 kgf each; 20 kgf each, or even two forces of 100 kgf each?

Ans. The answer is positive if the directions of resolutions are not given.

8. A force $Q=250$ kgf acts in the direction of a rafter inclined at an angle $\alpha=45°$ to the horizontal (Fig. 2). Compute the magnitudes of the force S which acts in the direction of the horizontal joining beam, and the force N which acts on the wall in the vertical direction.

Ans. $S=N=177$ kgf.

9. The rings A, B and C of three spring balances are tightly fixed on the horizontal board. Three strings are tied up to the hooks of

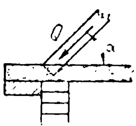


Fig. 2

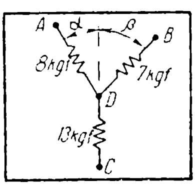


Fig. 3

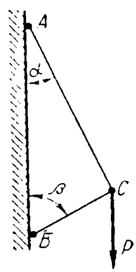


Fig. 4

the balances, stretched and then tied into one knot D. The balances show readings of 8, 7 and 13 kgf.

Define the angles α and β, formed by the strings, as represented in Fig. 3.

Ans. $\alpha=27.8°$; $\beta=32.2°$.

10. The rods AC and BC are hinged with each other and with the vertical wall (Fig. 4). The vertical force $P=1000$ kgf acts on a hinge pin C. Define the reactions of these rods on the hinge pin C, if the angles formed by the rods and the wall are: $\alpha=30°$ and $\beta=60°$.

Ans. 866 kgf; 500 kgf.

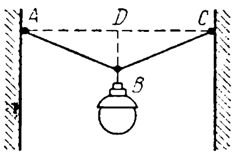


Fig. 5

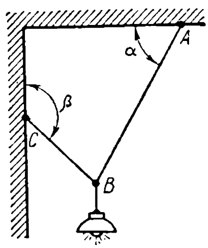


Fig. 6

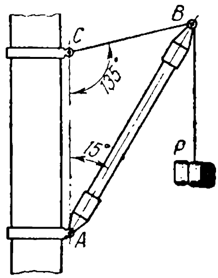


Fig. 7

11. A street lamp hangs at point B in the middle of the wire cable ABC. The ends of this wire cable are fastened to the hooks A and C located on the same level (Fig. 5). Determine the tensions T_1 and T_2 in the cable parts AB and BC, if the weight of the street lamp is 15 kgf and the total length of the cable ABC is 20 m. The sag BD at the point of suspension from the horizontal is 0.1 m. Neglect the weight of the cable.

Ans. $T_1=T_2=750$ kgf.

12. A 2-kgf electric lamp is suspended from the ceiling by a cord AB and then it is pulled towards a wall by a string BC (Fig. 6.). Determine the tensions T_A in the cord and T_C in the string, provided that the angles α and β are 60° and 135°, respectively. Neglect the weight of the cord and the string.

Ans. $T_A=1.46$ kgf; $T_C=1.04$ kgf.

13. A derrick crane consists of a boom AB hinged to a tower at A and a chain CB (Fig. 7). A weight $P=200$ kgf is suspended from the end B of the boom; the angles $BAC=15°$, $ACB=135°$ Determine the tension T in the chain CB, and the thrust Q in the boom AB.

Ans. $T=104$ kgf; $Q=283$ kgf.

14. A 25-kgf weight is held in equilibrium by two strings which run over two pulleys. Counterbalances are attached to free ends of the strings. One of the counterbalances weighs 20 kgf; the sine of the angle formed by the string and the vertical is 0.6. Neglecting friction of the pulleys, determine the weight p of the other counterbalance, and the angle α, formed by the second string with the vertical. The weight of the strings may be neglected.

Ans. $p=15$ kgf; $\sin\alpha=0.8$.

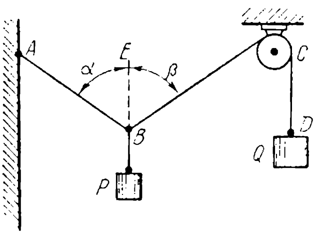


Fig. 8

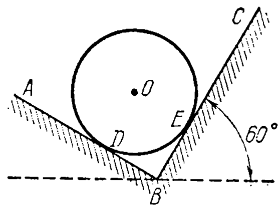


Fig. 9

15. A string AB is fixed with one end at point A. A weight P and a string BCD are fastened to the other end of the string at point B. From here the string BCD runs over a pulley, and at the end D of the string a weight $Q=10$ kgf is attached (Fig. 8).

Neglecting the friction in the pulley, determine the tension T in the string AB and the weight P, if the angles between the strings and the vertical BE are: $\alpha=45°$ and $\beta=60°$ The system is in equilibrium.

Ans. $T=12.2$ kgf; $P=13.7$ kgf.

16. A 6-kgf homogeneous ball O lies between two mutually perpendicular smooth planes AB and BC. Determine the pressure of the ball against each plane, assuming that the plane BC is inclined at 60° to the horizontal, as shown in Fig. 9.

Ans. $N_D=5.2$ kgf; $N_E=3$ kgf.

17. A homogeneous ball O suspended from a string AC rests against a smooth vertical wall AB (Fig. 10). The angle BAC between the string and the wall is α, and the weight of the ball is P. Determine the tension T in the string and the pressure Q of the ball against the wall.

Ans. $T=\frac{P}{\cos\alpha}$; $Q=P\tan\alpha$.

18. A small ball B of weight P is suspended by a thread AB from a fixed point A (Fig. 11). It rests on the surface of a smooth sphere of radius r. The distance $AC=d$. The length of the thread

$AB=l$. AO is a vertical straight line. Calculate the tension T in the thread and the reaction Q of the sphere. The radius of the ball is to be neglected.

Ans. $T=P\frac{l}{d+r}$; $Q=P\frac{r}{d+r}$.

19. A 10-kgf homogeneous ball is held in equilibrium by two strings AB and CD, lying in the same vertical plane, and forming

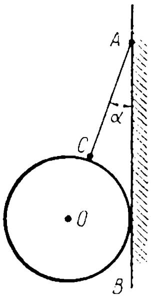


Fig. 10

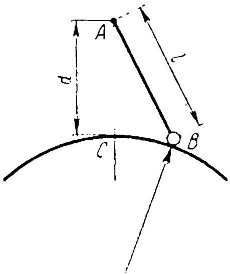


Fig. 11

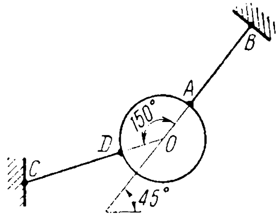


Fig. 12

an angle of 150° (Fig. 12) The string AB is inclined at 45° to the horizontal. Determine the tensions in the strings.

Ans. $T_B=19.3$ kgf; $T_C=14.1$ kgf.

20. A homogeneous roller of 2000-kgf weight has a radius of 60 cm. Determine the horizontal force P which is necessary to apply to pull the roller over a stone step 8 cm high, as shown in Fig. 13.

Ans. $P=1150$ kgf.

21. A 16-kgf uniform rod AB 1.2 m long is suspended from a point C by two strings AC and CB each 1 m long (Fig. 14). Determine the tensions in the strings.

Ans. The tension in each string is 10 kgf.

22. A uniform rod AB is hinged at A to a vertical wall and is held at an angle of 60° to the vertical by a string BC, forming an angle of 30° with the rod (Fig. 15).

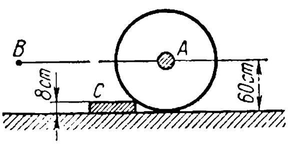


Fig. 13

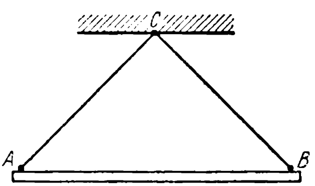


Fig. 14

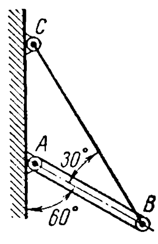


Fig. 15

Determine the magnitude and direction of the reaction R of the hinge, if the weight of the rod is 2 kgf.

Ans. $R=1$ kgf; angle $(R, AC)=60°$

23. The beam AB is hinged to the support A. The end B is supported by rollers. A force $P=2000$ kgf acts in the centre point of the beam at an angle of 45° to its axis. Determine the reactions of the supports for cases *a* and *b*. Use dimensions shown in Fig. 16. The weight of the beam should be neglected.

Ans. (a) $R_A=1580$ kgf; $R_B=710$ kgf;
(b) $R_A=2240$ kgf; $R_B=1000$ kgf.

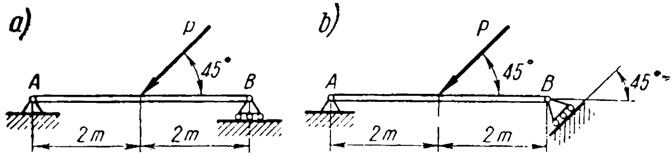


Fig. 16

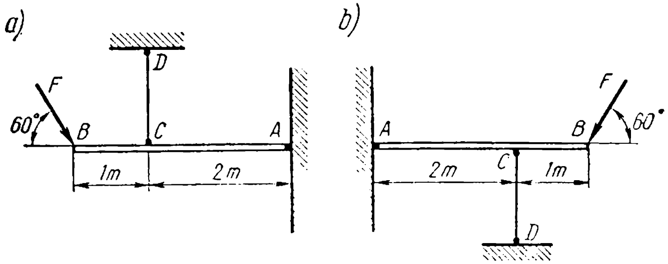


Fig. 17

24. Beams AB are held in horizontal position by vertical rods CD, as shown in Fig. 17. Two equal forces F of 3000 kgf each are applied at both ends of the beams, and act on them at an angle of 60° with the horizontal. Referring to the dimensions in the figure, determine the tensions S in the rods CD, and the pressures Q exerted by the beams on the wall, assuming that the joints A, C and D are hinged. The weight of the beams and rods may be neglected.

Ans. (a) $S=3900$ kgf; $Q=1980$ kgf;
(b) $S=3900$ kgf; $Q=1980$ kgf.

25. Determine reactions of the supports R_A and R_D, caused by the horizontal force P, which is applied to the frame at B, as shown in Fig. 18. Neglect the weight of the frame.

Ans. $R_A=P\frac{\sqrt{5}}{2}$; $R_D=\frac{P}{2}$.

26. A steam engine (Fig. 19) has a piston with the area of 0.1 m². The connecting rod AB is 2 m and the crank BC is 0.4 m long, the front pressure of the steam in the cylinder is $p_0=6$ kgf/cm² and the back pressure is $p_1=1$ kgf/cm².

Determine the force T which acts on the crank, and the pressure N of the sliding block A on the guides when the angle ABC is 90°. The effect of friction between the sliding block and the guides may be neglected.

Ans. $T=5100$ kgf; $N=1000$ kgf.

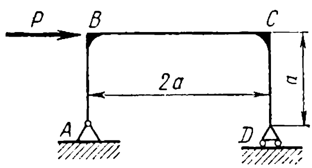


Fig. 18

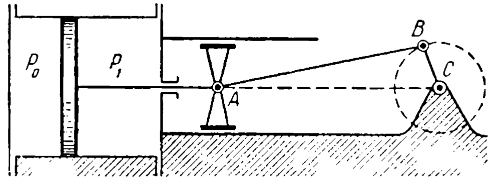


Fig. 19

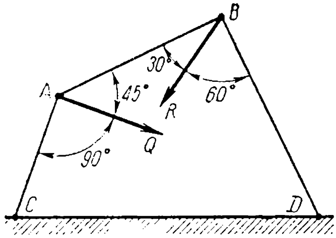


Fig. 20

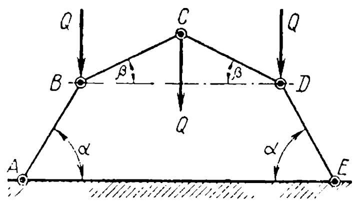


Fig. 21

27. Fig. 20 represents an $ABCD$ system of links, one side of which CD is fixed. A force $Q=10$ kgf acts at a hinge A at an angle $BAQ=45°$ Determine the force R which, acting at the hinge B at an angle $ABR=30°$, keeps the system in equilibrium. Angles $CAQ=90°$ and $DBR=60°$

Ans. $R=16.3$ kgf.

28. Fig. 21 represents a system consisting of four rods of equal length. The ends A and E are fixed pivots. The joints B, C and D are acted on by identical vertical forces Q. At equilibrium the angle of inclination of the extreme rods with the horizontal is $\alpha=60°$ Determine the angle of inclination of the middle rods to the horizontal.

Ans. $\beta=30°$.

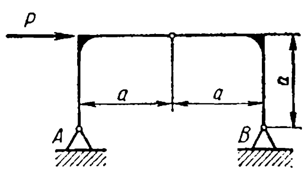


Fig. 22

29. Find the reactions of the supports A and B when a horizontal force P is applied to a three-hinged arch (Fig. 22). Neglect the weight.

Ans. $R_A = R_B = P\frac{\sqrt{2}}{2}$.

30. A system consists of three three-hinged arches (dimensions are given in Fig. 23). Express the reactions of the supports A, B, C and D in terms of the applied horizontal force P

Ans. $R_A = P\frac{\sqrt{2}}{2}$; $R_B = P$; $R_C = P$; $R_D = P\frac{\sqrt{2}}{2}$.

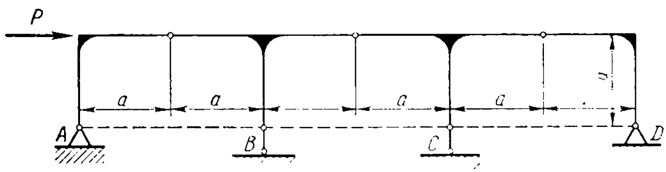


Fig. 23

31. A derrick crane (Fig. 24) consists of the fixed tower AC and the movable truss BC which is hinged at C and is supported by a cable AB. A weight $Q = 40{,}000$ kgf is held by a chain which runs over the pulley at B and from there it goes to the winch along the straight line BC. The length $AC = BC$.

Determine (as functions of the angle $ACB = \varphi$) the tension T in the cable AB, and the force P which compresses the truss along the straight line BC. Neglect the weight of the truss and the friction on the pulley.

Ans. $T = 80{,}000 \sin\frac{\varphi}{2}$ kgf; $P = 80{,}000$ kgf, independently of the angle φ.

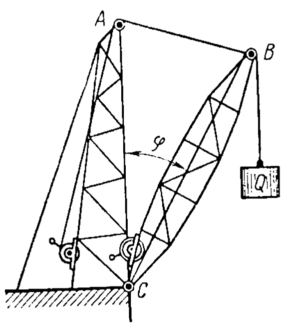


Fig. 24

32. A pulley C with the weight $P = 18$ kgf can slide along a flexible 5-m long cable ACB (Fig. 25) The ends of the cable are

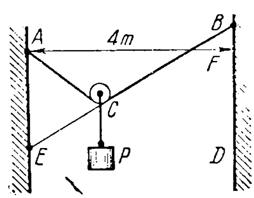


Fig. 25

fastened to the walls; the distance between the walls is 4 m. Find the tension in the cable when the pulley and the weight are in equilibrium. Neglect the weight of the cable and the friction on the pulley.

Hint. The tensions in the parts AC and CB are equal, and their magnitude can be determined by similarity between the force triangle and the isosceles triangle, one side of which is a straight line BCE and its base is the vertical BD.

Ans. 15 kgf, independently of the height BF

33. Two small balls A and B weighing 0.1 kgf and 0.2 kgf, respectively, rest on a smooth circular cylinder with horizontal axis and radius $OA=0.1$ m. The balls are connected by a thread $AB=0.2$ m long (Fig. 26).

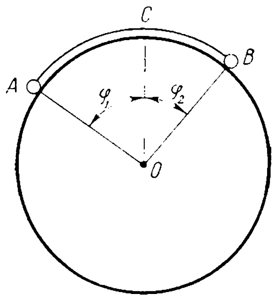


Fig. 26

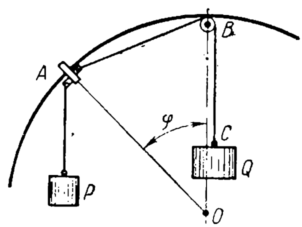


Fig. 27

Determine the angles φ_1 and φ_2 formed by the radii OA and OB and the vertical straight line OC when the system is in equilibrium. Determine the pressures N_1 and N_2 exerted by the balls on the cylinder at points A and B. Neglect the sizes of the balls.

Ans. $\varphi_1=2-\varphi_2$; $\tan\varphi_2=\frac{\sin 2}{2+\cos 2}$; $\varphi_1=84°45'$; $\varphi_2=29°50'$.

$N_1=0.1\cos\varphi_1$ kgf$=0.0092$ kgf;
$N_2=0.2\cos\varphi_2$ kgf$=0.173$ kgf.

34. A smooth ring A can slide without friction on the fixed wire bent into a circular arch in a vertical plane (Fig. 27). A weight P is suspended from the ring; a cord ABC, also attached to it, runs over the fixed pulley B. The latter is suspended from the highest point of the wire. (Neglect the size of the block.) A weight Q is attached at the point C. Determine the central angle φ, subtended by the arch AB, for equilibrium, and find the position when equilibrium is possible. The weight of the ring and friction may be neglected.

Ans. $\sin\frac{\varphi_1}{2}=\frac{Q}{2P}$; $\varphi_2=\pi$. The first state of equilibrium is possible when $Q<2P$, the second at any Q and P.

35. A point M is attracted to three fixed centres $M_1(x_1, y_1)$, $M_2(x_2, y_2)$ and $M_3(x_3, y_3)$ by forces proportional to distances $F_1=k_1r_1$, $F_2=k_2r_2$, $F_3=k_3r_3$, where $r_1=MM_1$, $r_2=MM_2$, $r_3=MM_3$ and k_1, k_2, k_3 are coefficients of proportionality. Determine the coordinates x, y of the point M when in equilibrium.

Ans. $x=\frac{k_1x_1+k_2x_2+k_3x_3}{k_1+k_2+k_3}$; $\quad y=\frac{k_1y_1+k_2y_2+k_3y_3}{k_1+k_2+k_3}$.

36. A 5-kgf uniform rectangular plate is suspended in such a way, that it can easily rotate about a horizontal axis along one of the sides of the plate. The wind, blowing with uniform velocity, keeps the plate inclined at an angle of 18° to the vertical. Determine the wind pressure on the plate perpendicular to its plane.

Ans. 5 sin 18°=1.55 kgf.

37. A water-tower has a cylindrical tank 6 m high and 4 m in diameter, as shown in Fig. 28. It is mounted on four symmetrically located legs inclined to the horizontal. The bottom of the tank is 17 m above the ground level of the supports. The weight of the tower is 8000 kgf. The wind pressure is calculated on the basis of the projected area of the tank on the plane perpendicular to the direction of the wind. Assuming that the specific wind pressure is 125 kgf/m², determine the required distance AB between the supports of the legs so that the structure will not topple over.

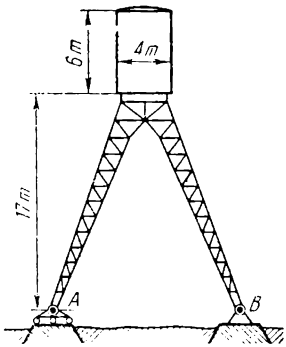


Fig. 28

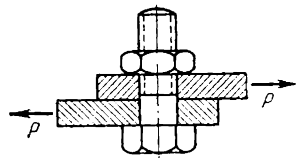


Fig. 29

Ans. $AB \geqslant 15$ m.

38. What force should be applied in order to give a start to a 50-kgf carriage along the horizontal plane of a lathe bed? Lubrication is poor, so the coefficient of static friction is only 0.15.

Ans. 7.5 kgf.

39. Determine the required tightening force for a bolt, holding two steel strips together (Fig. 29), when a disruptive force $P=2000$ kgf acts on the steel strips. The bolt has a clearance and it should not be under shearing stress. The coefficient of static friction between the strips is 0.2.

Hint. The bolt should not be under shearing stress; therefore it has to be tightened by such a force that the friction developed between the two strips should prevent them from slipping. The force which acts along the axis of the bolt is the one that is required.

Ans. 10,000 kgf.

40. A rough surface is placed at such an angle of inclination with the horizontal that a heavy body on the surface descends with constant velocity it was given initially. What is the coefficient of friction f?

Ans. $f=\tan\alpha$.

41. A wedge A with an angle of inclination $\tan\alpha=0.05$ is pressed into a recess BB_1 by a force $Q=6000$ kgf (Fig. 30). If the coefficient of friction f is 0.1, determine the normal force N on the face of the wedge, and the force P required to pull it out.

Ans. $N=20{,}000$ kgf; $P=2000$ kgf.

42. A box of the weight P rests on a rough horizontal surface whose coefficient of friction is f (Fig. 31). Determine at what angle β the minimum force Q must be applied to move the box, and the value of this force.

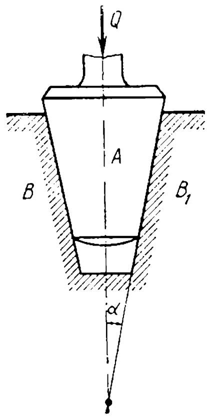


Fig. 30

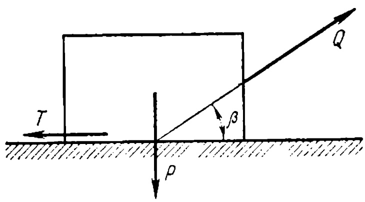


Fig. 31

Ans. $\beta=\arctan f$; $Q_{\min}=\dfrac{fP}{\sqrt{1+f^2}}$.

3. Parallel Forces and Couples

43. Determine the vertical reactions of the supports of the horizontal beam with span l, if a weight P is applied to the beam at a distance x from the first support.

Ans. $R_1=P\dfrac{l-x}{l}$; $R_2=P\dfrac{x}{l}$.

44. Two weights $C=200$ kgf and $D=100$ kgf are located on a horizontal beam supported at A and B (Fig. 32) The distance between the supports is 4 m. The weights are placed in such a way that the reaction of the support A is twice the reaction of the support B. The distance CD between the weights is 1 m. Neglecting the weight of the beam, find the distance x between the weight C and the support A.

Ans. $x=1$ m.

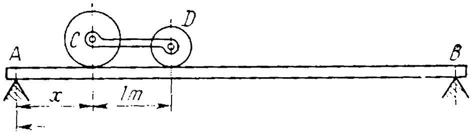


Fig. 32

45. Find the pressures exerted by a bridge crane AB on the rails as a function of the position of the carriage C on which a winch is mounted (Fig. 33). The position of the carriage should be determined by the distance between its centre and the left rail as a fraction of the total length of the bridge. The weight of the crane is $P=6000$ kgf, and $P_1=4000$ kgf is the weight of the crane carriage with the lifting load.

Ans. $F_A=1000\ (7-4n)$ kgf; $F_B=1000\ (3+4n)$ kgf,

where $n=\dfrac{AC}{AB}$.

46. A horizontal rod AB of weight 0.1 kgf can rotate about a fixed axle of a hinge A (Fig. 34). The end B is pulled vertically upwards by a rope running over a pulley and attached to a 0.15-kgf weight P. A weight $Q=0.5$ kgf is suspended to a point 20 cm from the end B. What is the length x of the rod AB for the system to be in equilibrium?

Ans. $x=25$ cm.

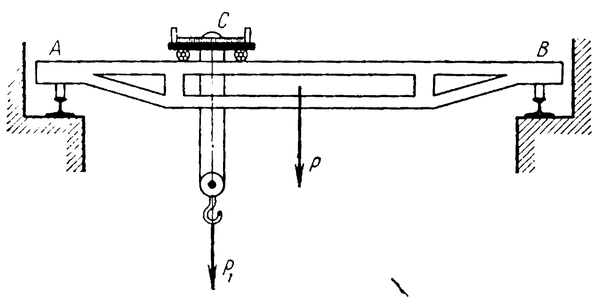


Fig. 33

47. Four weights are suspended at equal distances from a uniform bar 3 m long and weighing 6 kgf. The first weight at the left is 2 kgf, and each successive weight is 1 kgf heavier than the previous one so that the extreme weights are at the ends. At what distance x from the left end should the bar be suspended so that it remains horizontal?

Ans. $x = 1.75$ m.

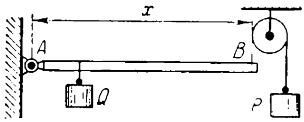


Fig. 34

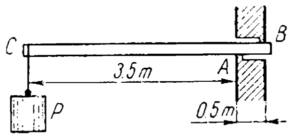


Fig. 35

48. A uniform beam hinged to a wall has a support 160 cm from the wall. The beam is 400 cm long and weighs 320 kgf. Two weights of 160 kgf and 240 kgf are applied to the beam at the distances of 120 cm and 180 cm, respectively, from the wall. Determine the reactions of the supports.

Ans. 790 kgf upwards; 70 kgf downwards.

49. A uniform horizontal beam 4 m long and weighing 500 kgf is placed with one end in a wall of 0.5 m thick so that it rests against points A and B (Fig. 35).

Calculate the reactions of the supports at the points A and B, if a load $P = 4000$ kgf is attached to the free end of the beam.

Ans. $R_A = 34{,}000$ kgf upwards; $R_B = 29{,}500$ kgf downwards.

50. A horizontal double-cantilever beam is acted on by a couple (P, P), as shown in Fig. 36. The left side of the beam carries a uniformly distributed load p, while a vertical load Q acts at a point D on the right side of the beam. $P = 1000$ kgf, $Q = 2000$ kgf, $p = 2000$ kgf/m, $a = 0.8$ m. Determine the reactions of the supports.

Ans. $R_A = 1500$ kgf; $R_B = 2100$ kgf.

51. The rails of a jenny are mounted on the beam AB 10 m long (Fig. 37). The weight of the crane is 5000 kgf and its centre

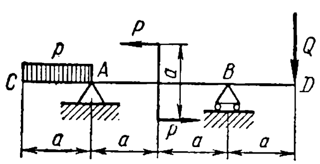


Fig. 36

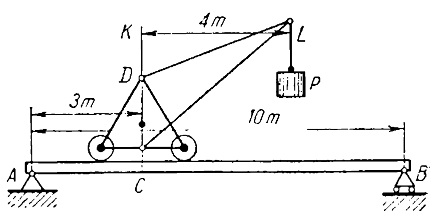


Fig. 37

of gravity is on the axis CD. The weight of a lifting load P is 1000 kgf. The weight of the beam AB is 3000 kgf, the sweep of the crane KL is 4 m; the distance $AC=3$ m. Find the reactions of the supports A and B when the brace DL and the beam AB are in the same vertical plane.

Ans. $R_A=5300$ kgf; $R_B=3700$ kgf.

52. Several identical uniform slabs, each $2l$ long, are stacked together in such a way that each slab overhangs the one below it, as shown in Fig. 38. What is the maximum overhang for each slab when all the slabs are in equilibrium?

Hint. When solving this problem the weight of each slab should be added successively beginning from the topmost one.

Ans. $l, \frac{1}{2}l, \frac{1}{3}l, \frac{1}{4}l, \frac{1}{5}l,$

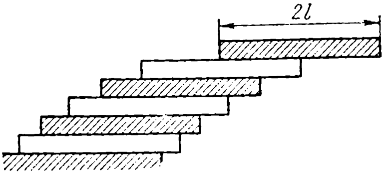


Fig. 38

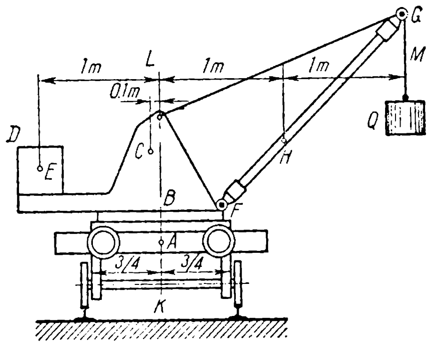


Fig. 39

53. A railway crane rests on rails of 1.5 m gauge. The travelling crab of the crane weighs 3000 kgf, and its centre of gravity is at a point A located on the centre line KL, as shown in Fig. 39. A crane winch B weighs 1000 kgf, and its centre of gravity is at a point C 0.1 m from KL. The weight of a counterweight D is 2000 kgf, and its centre of gravity is at a point E 1 m from the centre line KL. The boom FG weighs 500 kgf, and its centre of gravity is at a point H 1 m from KL. $LM=2$ m. Determine the maximum weight Q under which the crane will not tip over.

Ans. $Q=5180$ kgf.

54. A crane for charging an open-hearth furnace has a winch A, running on wheels along rails mounted on the beams of a movable bridge B (Fig. 40). An inverted column D carrying a ladle C is attached to the bottom of the winch. What must be the weight P of the winch and column to keep the winch from tipping, if a load $Q=1500$ kgf is placed into the ladle at 5-m distance from the vertical axis OA? The weight of the winch is assumed to

be directed along the axis OA. The distance between the axle of every wheel and the axis OA is 1 m.

Ans. $P \geqslant 6000$ kgf.

55. A turning crane is mounted on a stone foundation (Fig. 41). Its weight $Q=2500$ kgf acts at A at the distance $AB=0.8$ m from the axis of the crane. The sweep of the crane $CD=4$ m. The foundation has a square bed with sides $EF=2$ m. The specific weight of the masonry is 2 gf/cm^3. Calculate the minimum depth of the foundation, if the crane is designed to lift the weights up to 3000 kgf. The foundation should be calculated by considering the tipping moment about a rib F.

Ans. 1.1 m.

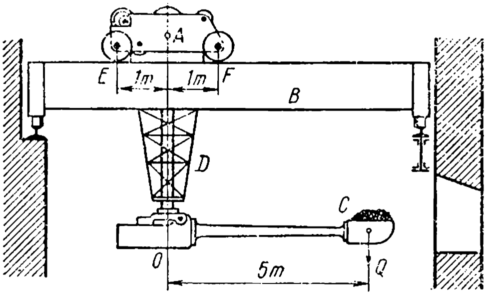


Fig. 40

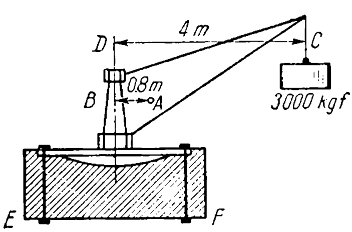


Fig. 41

56. Two uniform rods AB and BC with equal cross-sections are connected with their ends at an angle of 60° thus forming a cranking lever ABC, as shown in Fig. 42. AB is one half BC. The lever is suspended by a thread AD from the end A. Determine the angle α of inclination formed by the rod BC and the horizontal when in equilibrium. The sizes of the cross-sections may be neglected.

Ans. $\tan\alpha=\frac{\sqrt{3}}{5}$ $\quad \alpha=19°05'$.

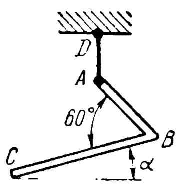


Fig. 42

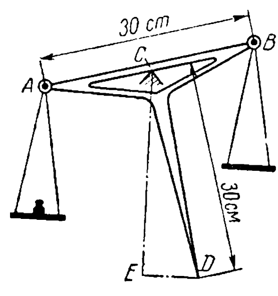


Fig. 43

57. A balance beam AB is 30 cm long and weighs 0.3 kgf (Fig. 43). The length of the pointer CD is 30 cm. An overload of 0.01-gf weight in one of the pans causes a deflection of the pointer from its vertical position by a distance $DE=3$ mm. Calculate the distance from the centre of gravity of the balance beam to the prism edge C when the system is in equilibrium.

Ans. 0.05 cm.

58. A hoist bridge AB is lifted by two beams CD each 8 m long and weighing 400 kgf (Fig. 44). The beams are located

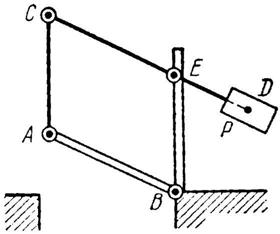


Fig. 44

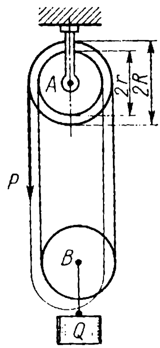


Fig. 45

on each side of the hoist bridge. The length of the hoist bridge is $AB=CE=5$ m and its weight, 3000 kgf, is assumed to act at the centre of AB. The length of the chain AC equals BE. Calculate the value of the counterweight P balancing the bridge.

Ans. $P=1383$ kgf.

59. A main part of a differential chain block consists of two pulleys A fixed together (Fig. 45) Their axle is attached to a fixed hook. The pulleys form two sprockets for an endless chain making two loops. A movable pulley is mounted onto one of the loops and it carries a load Q, while a force P is applied to the part of the free loop hanging down from the larger pulley. The radii of the pulleys A are R and r; $r<R$. Find the ratio between the force P and the magnitude of the weight Q, and also determine this force, assuming that $Q=500$ kgf, $R=25$ cm, and $r=24$ cm. Neglect friction.

Ans. $P=\frac{1}{2}Q\left(1-\frac{r}{R}\right)=10$ kgf.

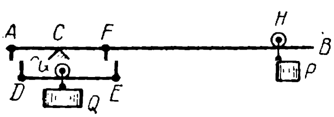


Fig. 46

60. A differential lever consists of a rod AB, supported by a fixed fulcrum at C, and a traverse DE, which is hinged by links AD and EF to the lever AB (Fig. 46). A weight $Q=1000$ kgf is suspended from a

traverse at G by means of a prism. The distance between the verticals passing through the points C and G is 1 mm. Compute the weight P which should be suspended to the lever AB at H ($CH = 1$ m) to counter-balance the weight Q. Friction is to be neglected.

Ans. $P = 1$ kgf.

61. A cantilever bridge consists of three parts: AC, CD and DF (Fig. 47). Each extreme part rests on two supports $AC = DF = 70$ m, $CD = 20$ m, $AB = EF = 50$ m.

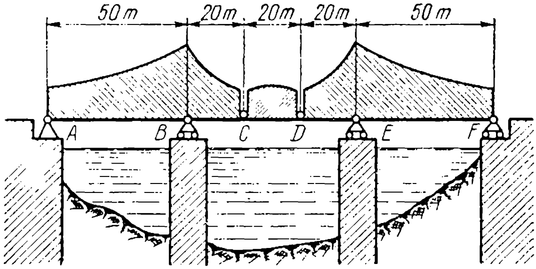


Fig. 47

The linear load of the bridge is 6000 kgf/m. Calculate the pressure of this load on the supports A and B.

Ans. $N_A = 102{,}000$ kgf; $N_B = 378{,}000$ kgf.

62. A cantilever bridge consists of a main truss AB and two lateral trusses AC and BD (Fig. 48). The weight per metre of the truss AB is 1500 kgf and of the trusses AC and BD is 1000 kgf. $AC = BD = 20$ m; $AE = FB = 15$ m; $EF = 50$ m.

Determine the reactions of all supports at a particular instant when the whole of right span FD is loaded with a train of 3000-kgf weight per metre.

Ans. $R_C = 10{,}000$ kgf; $R_D = 40{,}000$ kgf;
$R_E = 54{,}250$ kgf; $R_F = 160{,}750$ kgf.

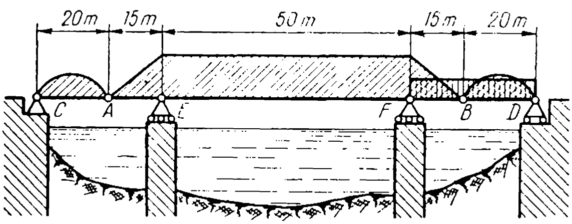


Fig. 48

63. One end A of a split horizontal beam ACB is fixed into a wall, and the other end B rests on a roller support. A hinge is at a point C. A jenny bearing a weight $P=1000$ kgf is mounted on the beam. The weight of the jenny is $Q=5000$ kgf, and its sweep $KL=4$ m. The centre of gravity of the jenny acts along the vertical CD. The dimensions are shown in Fig. 49. Neglecting the weight of the beam, determine the reactions of the supports at A and B, provided that the jenny and the beam AB are in the same vertical plane.

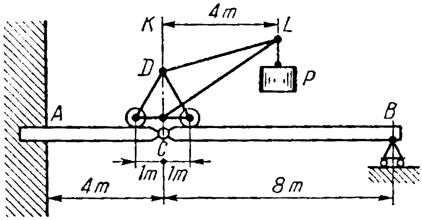


Fig. 49

Ans. $R_A=5375$ kgf; $R_B=625$ kgf; $M_A=20{,}500$ kgfm.

4. Arbitrary Coplanar Force Systems

64. A horizontal crane beam of the length l is hinged at one end, and at the other end B it is suspended from the wall by a brace rod BC forming an angle of inclination α with the horizontal (Fig. 50). A load P moves along the beam, and its position is defined by the distance x to the hinge A. Express the tension T in the rod BC as a function of the load position. Neglect the weight of the beam.

Ans. $T=\dfrac{Px}{l\sin\alpha}$.

65. A homogeneous ball of weight Q and radius a as well as a weight P are suspended by cords to a point O as shown in Fig. 51. The distance $OM=b$. When the system is in equilibrium, what angle does the straight line OM form with the vertical?

Ans. $\sin\varphi=\dfrac{a}{b}\,\dfrac{P}{P+Q}$.

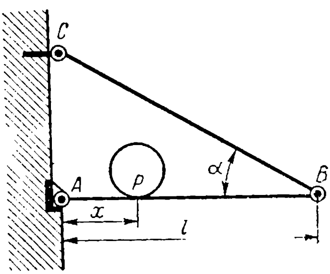


Fig. 50

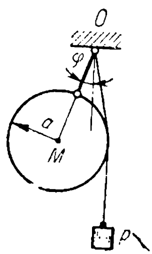


Fig. 51

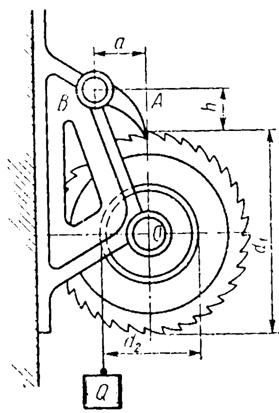


Fig. 52

66. A winch is equipped with a ratchet wheel of diameter d_1 and a pawl A (Fig. 52). A drum of diameter d_2, rigidly tied to the wheel, is wound with a rope holding the weight $Q=50$ kgf; $d_1=$ $=420$ mm; $d_2=240$ mm; $h=50$ mm: $a=120$ mm. Find the force R acting on the axis B of the pawl. Neglect the weight of the pawl.

Ans. $R=Q\frac{d_2}{d_1}\frac{\sqrt{a^2+h^2}}{a}=31$ kgf.

67. A 4-m long uniform beam weighing 60 kgf rests with one end on a smooth floor and with an intermediate point B leaning against a pole 3 m high forming an angle of 30° with the vertical, as shown in Fig. 53. In this position the beam is held by the rope AC stretched along the floor. Neglecting friction, determine the tension in the rope and the reactions of the pole R_B and the floor R_C.

Ans. $T=15$ kgf; $R_B=17.3$ kgf; $R_C=51.3$ kgf.

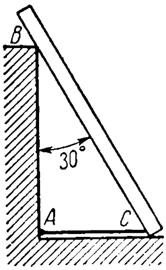


Fig. 53

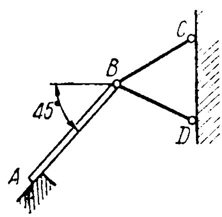


Fig. 54

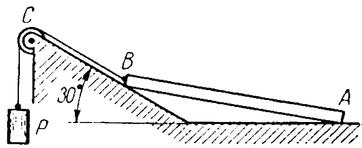


Fig. 55

68. A uniform, slab AB weighing $P=100$ kgf rests freely at a point A, and is held by two rods BC and BD at an angle of 45° to the horizontal, as shown in Fig. 54. BCD is an equilateral triangle. The points C and D lie on the vertical. Neglecting the weights of the rods and assuming that the joints at B, C and D are hinged, compute the reaction of the support A and the tensions in the rods.

Ans. $R_A=35.4$ kgf; $S_C=89.5$ kgf; $S_D=-60.6$ kgf.

69. A 100-kgf uniform rod AB rests with one of its ends on a smooth horizontal floor and the other end on a smooth surface inclined at an angle of 30° to the horizontal as shown in Fig. 55. The rod is held at the end B by a cord running over a pulley C and carrying a weight P Part of the cord BC is parallel to the inclined surface. Neglecting friction, calculate the weight P and the pressures N_A and N_B, acting on the floor and on the inclined surface.

Ans. $P=25$ kgf; $N_A=50$ kgf; $N_B=43.3$ kgf.

70. Fig. 56 represents a rafter AB. The upper end B rests on a smooth surface and the lower end A leans against a wall. The beam of the rafter is inclined at an angle of $\tan\alpha = 0.5$.

Determine the reactions of the supports at points A and B when a vertical load of 900 kgf is applied at the centre of the beam.

Ans. $X_A = 180$ kgf;
$Y_A = 540$ kgf;
$R_B = 402$ kgf.

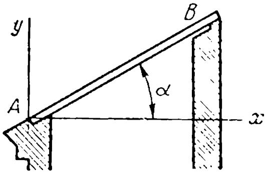


Fig. 56

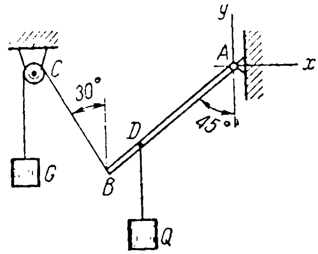


Fig. 57

71. A uniform beam AB weighing $P = 100$ kgf is hinged at A to a wall and is held at an angle of 45° to the vertical by a cable running over a pulley and carrying a weight G, as shown in Fig. 57 Part of the cable BC forms an angle of 30° with the vertical. A weight $Q = 200$ kgf is suspended to the beam at a point D. $BD = {}^1/_4 AB$. Neglecting friction on the block, calculate the magnitude of the weight G and the reaction of the hinge A.

Ans. $G = 146$ kgf; $X_A = 73$ kgf; $Y_A = 173$ kgf.

72. A turning pit crane which is lifting a weight $P = 4000$ kgf has an end thrust bearing A, and at a point B it rests on a smooth cylindrical surface with vertical axis Ay, as shown in Fig. 58. $AB = 2$ m. The sweep of the crane $DE = 5$ m. The 2000-kgf weight of the crane acts along a straight line 2 m from the vertical Ay. Determine the reactions of the supports A and B.

Ans. $X_A = 12{,}000$ kgf; $Y_A = 6000$ kgf;
$X_B = -12{,}000$ kgf.

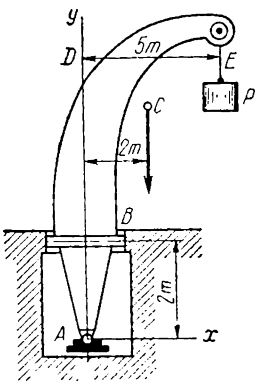


Fig. 58

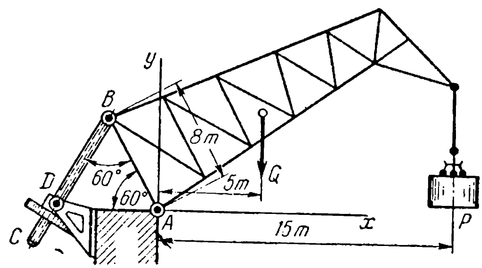


Fig. 59

73. A crane is pivoted at a point A and can be inclined by means of a screw BC which is hinged with the crane truss at B and passes through a nut D, as shown in Fig. 59. $AB=AD=8$ m. When the position of the crane is such that ABD forms an equilateral triangle, the centre of gravity of the truss is 5 m from the vertical passing through the point A. The sweep of the crane

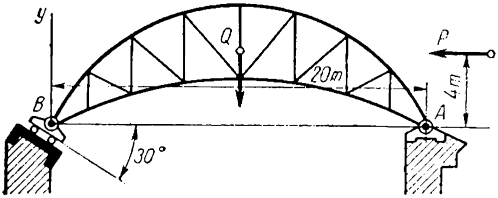


Fig. 60

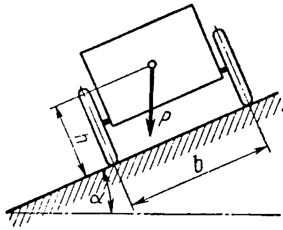


Fig. 61

is 15 m taken from the point A. The hoisting capacity of the crane is $P=20{,}000$ kgf. The weight of the truss is $Q=$ $=12{,}000$ kgf. Determine the reactions of the supports, and the tension T in the screw when the crane is in the position described.

Ans. $X_A=26{,}000$ kgf; $Y_A=77{,}000$ kgf; $T=52{,}000$ kgf.

74. An arch truss has a fixed hinged support A and a roller support B on a smooth plate inclined at 30° to the horizontal (Fig. 60). The span $AB=20$ m. The weight of the truss is $Q=10{,}000$ kgf. The resultant of the wind forces is $P=2000$ kgf. It is directed parallel to AB at a point 4 m above AB. Determine the reactions of the supports.

Ans. $X_A=-1120$ kgf; $Y_A=4600$ kgf; $R_B=6240$ kgf.

75. Usually each rear wheel of a car exerts a normal pressure of 500 kgf on a horizontal road (Fig. 61). What would be the normal pressure of the wheels on a road whose camber angle to the horizontal is $\alpha=5°$? The height of the centre of gravity of the car above the ground is $h=0.8$ m; and the wheel spacing is $b=1.4$ m.

Ans. The pressure equals 548 kgf for the lower wheel.

76. A uniform rod AB of weight P is hinged at one end A to a horizontal floor AD (Fig. 62). The other end B is tied to the wall CD by a cord BC. If the angles $CED=\alpha$, $BAD=\beta$, what is the reaction of the hinge, and the tension in the cord T?

Ans. $X_A=-P\dfrac{\cos\alpha\cos\beta}{2\sin(\alpha-\beta)}$; $\quad Y_A=P\left[1-\dfrac{\sin\alpha\cos\beta}{2\sin(\alpha-\beta)}\right]$;

$$T=P\frac{\cos\beta}{2\sin(\alpha-\beta)}$$

77. A bridge is made of two parts hinged together at A and it is hinged to piers at B and C. The weight of each part of the bridge is 4000 kgf and their centres of gravity are at D and E. A load $P=2000$ kgf is applied to the bridge. The dimensions are shown in Fig. 63.

Determine the value of the pressure at the hinge A and the reactions at points B and C.

Ans. $X_A=\pm 2000$ kgf; $Y_A=\mp 800$ kgf; $X_B=-X_C=2000$ kgf; $Y_B=5200$ kgf; $Y_C=4800$ kgf.

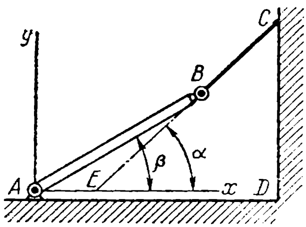


Fig. 62

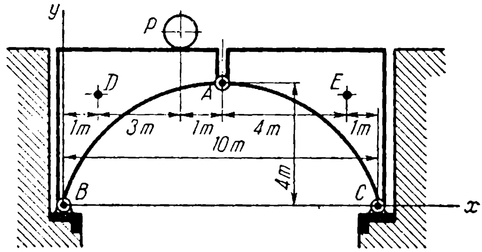


Fig. 63

78. The rafter consists of two beams AC and BC of equal length connected at point C (Fig. 64). The lower ends of the beams are fixed into a horizontal beam AB. The angle of inclination of the roof is $\tan\alpha=0.5$. A 900-kgf load is vertically applied at the centre of each beam. Determine the pressures acting at point C between two beams and at point A acting on the beam at the base, considering that the joints A, B and C are hinged.

Ans. $X_A=-900$ kgf; $Y_A=-900$ kgf;
$X_C=\pm 900$ kgf; $Y_C=0$.

79. A rafter consists of beams AC and CB. A linear vertical load of 100 kgf/m is applied to each beam. A force $P=800$ kgf acts perpendicularly on the beam AC at D. AD $DC=3:2$. The dimensions are shown in Fig. 65. Assuming that the joint C is hinged, determine the pressure acting between the beams at the point C and the reactions at A and B.

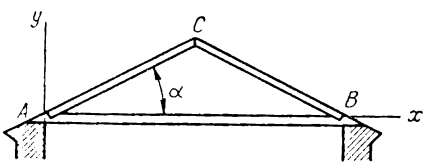


Fig. 64

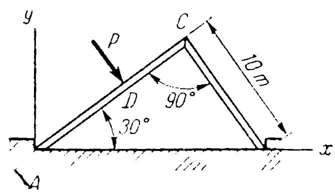


Fig. 65

Ans. $X_A=431$ kgf; $Y_A=1483$ kgf; $X_B=-831$ kgf; $X_C=\pm 831$ kgf; $Y_B=1940$ kgf; $Y_C=\mp 940$ kgf.

80. A portable step-ladder stands on a smooth horizontal surface (Fig. 66). It consists of two parts AC and BC each 3 m long and of weight 12 kgf. Both parts are hinged at C and are held together by a rope EF. $BF=AE=1$ m. The centre of gravity of each part AC and BC is at its centre. A man weighing 72 kgf stands at a point D where $CD=0.6$ m. If the angle $BAC=ABC=45°$, calculate the reactions of the floor and hinge, and the tension T in the rope EF.

Ans. $R_A=40.8$ kgf;
$R_B=55.2$ kgf;
$X_C=\pm 52.2$ kgf;
$Y_C=\pm 28.8$ kgf;
$T=52.2$ kgf.

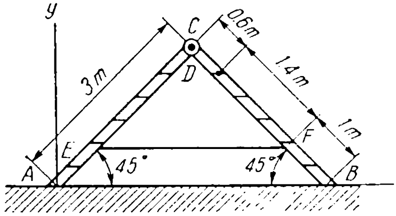


Fig. 66

81. A bridge consists of two horizontal beams hinged together at A and is hinged to its base by four rigid struts *1, 2, 3, 4*, the two outermost ones being vertical, while the middle struts are inclined at an angle $\alpha=60°$ to the horizontal. $BC=6$ m, $AB=8$ m (Fig. 67).

Determine the value of the forces in the struts, and the reaction of the hinge A, assuming that the bridge carries a vertical load $P=15{,}000$ kgf at a distance $a=4$ m from point B.

Ans. $S_1=-6250$ kgf; $S_2=S_3=-5770$ kgf; $S_4=1250$ kgf; $X_A=\pm 2890$ kgf; $Y_A=\mp 3750$ kgf.

82. Fig. 68 shows a three-hinged truss which holds the whole structure of a shop. A bridge crane runs on the rails along the shop. A traverse beam weighs $P=1200$ kgf and it can also move along the rails. The weight of the crane being unloaded is 800 kgf and its centre of gravity lies on the beam one-quarter of the distance from the left rail. Each part of the truss weighs $Q=6000$ kgf, and this weight acts 2 m from the vertical which passes through the respective support A or B. The rails of the crane are 1.8 m from these verticals. The shop is 12 m high and the span of the truss is 16 m. The resultant of the wind pressure equals 1200 kgf and it is directed parallel to AB, its line of action being located 5 m from AB. Determine the reactions at the

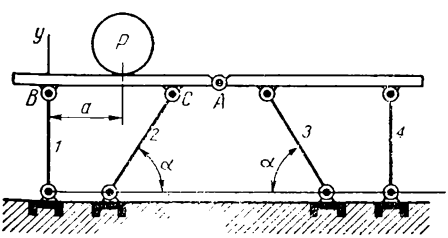


Fig. 67

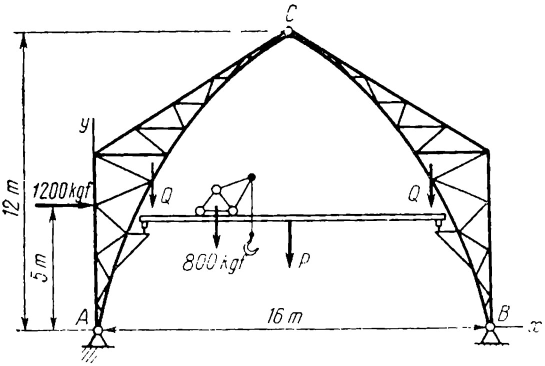


Fig. 68

hinges A and B, and also the pressure at the hinge C.

Ans. $X_A=200$ kgf; $X_B=-1400$ kgf; $X_C=\pm 1400$ kgf; $Y_A=6780$ kgf; $Y_B=7220$ kgf; $Y_C=\mp 420$ kgf.

83. A load $P=25$ kgf is suspended from the end of a horizontal beam AB. The weight of the beam is $Q=10$ kgf and acts at the point E. The beam is hinged to the wall at the point A and propped up by the hinged strut CD. Neglect the weight of the strut.

Dimensions are given in Fig. 69. Determine the reactions of the hinges A and C.

Ans. $X_A=-30$ kgf; $Y_A=-17$ kgf; $R_C=60$ kgf.

84. A suspension clip consists of two beams AB and CD hinged together at the point D and fixed to the ceiling by hinges A and C (Fig. 70) The 60-kgf weight of the beam AB acts at the point E and the 50-kgf weight of the beam CD acts at the point F A vertical force $P=200$ kgf is applied to the beam AB at the point B. $AB=1$ m; $CD=0.8$ m; $AE=0.4$ m; $CF=0.4$ m. AB and CD are inclined at angles $\alpha=60°$ and $\beta=45°$ to the horizontal, respectively. Determine the reactions of the hinges A and C.

Ans. $-X_A=X_C=135$ kgf; $Y_A=150$ kgf; $Y_C=160$ kgf.

85. A bridge truss carries two equal vertical loads $P=10{,}000$ kgf at the joints C and D (Fig. 71). The inclined rods form angles of 45° with the horizontal. Find the forces in the rods *1*, *2*, *3*, *4*, *5* and *6* due to the above loads.

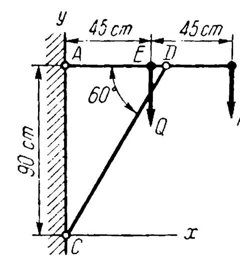


Fig. 69

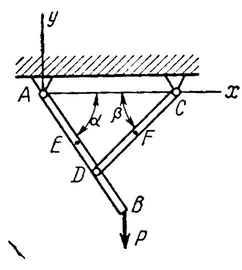


Fig. 70

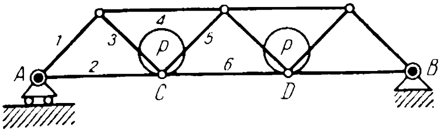


Fig. 71

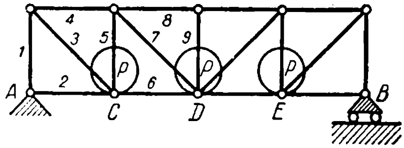


Fig. 72

Ans. $S_1 = -14{,}100$ kgf; $S_2 = +10{,}000$ kgf;
$S_3 = +14{,}100$ kgf; $S_4 = -20{,}000$ kgf;
$S_5 = 0$; $S_6 = +20{,}000$ kgf.

86. The joints C, D and E of a bridge truss carry equal vertical loads $P = 10{,}000$ kgf (Fig. 72). The inclined web members form angles of 45° with the horizontal. Calculate the forces in the web members *1*, 2, *3*, *4*, *5*, *6*, *7*, *8* and *9*, caused by the mentioned loads.

Ans. $S_1 = -15{,}000$ kgf; $S_2 = 0$; $S_3 = +21{,}200$ kgf;
$S_4 = -15{,}000$ kgf; $S_5 = -5000$ kgf;
$S_6 = +15{,}000$ kgf; $S_7 = +7100$ kgf;
$S_8 = -20{,}000$ kgf; $S_9 = 0$.

87. To assemble a bridge a temporary wooden crane is built. It is mounted on wheels which run on rails A and B. A block which lifts loads by means of a chain is suspended from the centre C of the lower chord DE of the crane. The chain lifts a load of $P = 5000$ kgf from a scaffold and at the instant of lifting the angle of the chain is $\alpha = 20°$. To prevent the load from swinging the latter is held horizontally by a rope GH. Assuming that the horizontal component of the tension in the chain is taken entirely by the right rail B, determine the force S_1 in the horizontal rod CF at the instant of lifting the load from the scaffold, and compare this force with that of the force S_2 which would act, when $\alpha = 0$. All dimensions are shown in Fig. 73.

Ans. $S_1 = 10{,}460$ kgf;
$S_2 = 5000$ kgf.

88. Compute the magnitude of the force which compresses an object M on a press (Fig. 74). The force $P = 20$ kgf is directed perpendicularly to the lever OA 1 m long with a fixed axle O. At a certain position of the press the pull rod BC is perpendic-

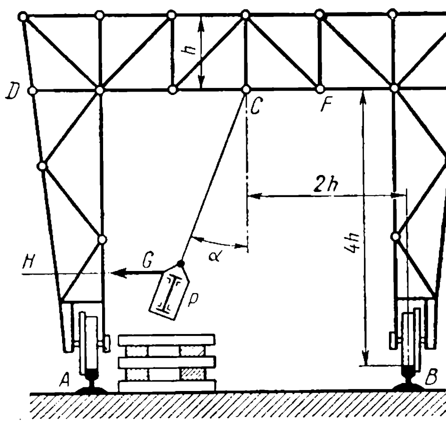


Fig. 73

ular to OB and bisects the angle ECD. The angle $CED=$ $=\arctan 0.2=11°20'$; the length $OB=10$ cm.

Ans. 500 kgf.

89. A chain OO_1 of a self-gripping device is hinged at O with the rods $OC=OD=60$ cm (Fig. 75). The rods are hinged with two equal bell-cranks CAE and DBF which rotate about points A and B on the connecting bar GH. Two special shoes hinged at E and F

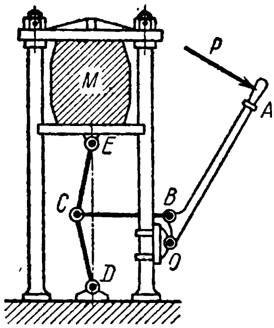


Fig. 74

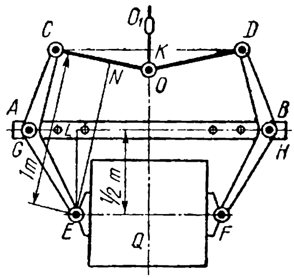


Fig. 75

hold a load $Q=1000$ kgf by means of friction. The distance between the point E and the bar GH is $EL=50$ cm, and between the point E and the rod OC is $EN=1$ m. The height of the triangle COD is $OK=10$ cm. Find the force which stretches the connecting bar GH. Neglect the weight of the device.

Ans. 6000 kgf.

90. A uniform rod AB of length $2l$ and weight P can rotate about a horizontal axis at its end A. This rod is supported by another uniform rod CD with the same length $2l$, which can rotate about the horizontal axis passing through its centre E (Fig. 76). The points A and E are on the same

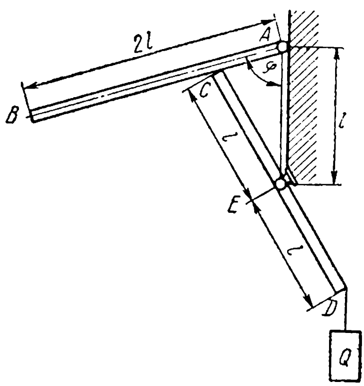


Fig. 76

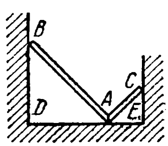


Fig. 77

vertical and the distance $AE=l$. A weight $Q=2P$ is suspended from the end D.

Determine the magnitude of the angle φ formed by the rod AB and the vertical when the system is in equilibrium. Neglect friction.

Ans. $\varphi=\arccos\frac{1}{8}=82°50'$

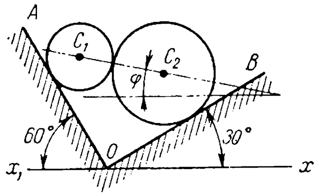


Fig. 78

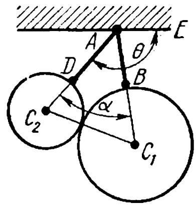


Fig. 79

91. Two uniform rods AB and AC rest at the point A on a smooth horizontal floor being connected together by their smooth vertical surfaces (Fig. 77). The ends B and C rest against smooth vertical walls. $AB=a$; $AC=b$; the weight of the rod AB equals P_1 and of the rod AC equals P_2. Derive a formula for the distance DE between the walls when the rods are in equilibrium and inclined to each other at an angle of 90°

Ans. $DE=\frac{a\sqrt{P_2}+b\sqrt{P_1}}{\sqrt{P_1+P_2}}$.

92. Two smooth homogeneous cylinders which touch each other rest between two smooth inclined planes OA and OB (Fig. 78). One cylinder with centre at C_1 weighs $P_1=10$ kgf and the other one with centre at C_2 weighs $P_2=30$ kgf. Angle $AOx_1=60°$, $BOx=30°$ Determine the angle φ between the straight line C_1C_2 and the horizontal axis xOx_1. Calculate pressures N_1 and N_2 which the cylinders exert on the planes, and the reciprocal pressure N between the cylinders.

Ans. $\varphi=0$; $N_1=20$ kgf; $N_2=34.6$ kgf; $N=17.3$ kgf.

93. Two smooth homogeneous balls C_1 and C_2 with radii R_1 and R_2 weighing P_1 and P_2, respectively, are suspended by cords AB and AD from point A (Fig. 79). $AB=l_1$; $AD=l_2$; $l_1+R_1=l_2+R_2$; the angle $BAD=\alpha$. Derive formulae for the angle θ between the cord AD and the horizontal plane AE, the tensions T_1 and T_2 in the cords and the reciprocal pressure between two balls.

Ans. $\tan\theta = -\frac{P_2+P_1\cos\alpha}{P_1\sin\alpha}$; $T_1 = P_1\frac{\sin\left(\theta-\frac{\alpha}{2}\right)}{\cos\frac{\alpha}{2}}$;

$$T_2 = P_2\frac{\sin\left(\theta-\frac{\alpha}{2}\right)}{\cos\frac{\alpha}{2}}; \quad N = -P_2\frac{\cos\theta}{\cos\frac{\alpha}{2}}.$$

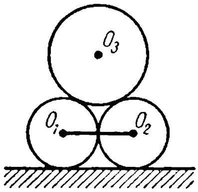


Fig. 80

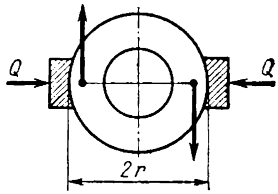


Fig. 81

94. Two round solid identical cylinders each of radius r and weight P, tied together by a thread of length $2r$, rest on a horizontal surface. They hold a third homogeneous cylinder of radius R and weight Q, as shown in Fig. 80. Derive formulae for the tension in the thread, the forces which the cylinders exert on the plane, and the reciprocal pressure between the cylinders. Neglect friction.

Ans. The force exerted by each lower cylinder on the plane is

$$P+\frac{Q}{2}.$$

The reciprocal force between the upper cylinder and each lower cylinder equals

$$\frac{Q(R+r)}{2\sqrt{R^2+2rR}}.$$

The tension in the thread is

$$\frac{Qr}{2\sqrt{R^2+2rR}}.$$

95. The moment of a couple of forces $M=100$ kgfm is applied to a shaft to which a brake wheel of radius $r=25$ cm is wedged (Fig. 81). If the coefficient of static friction between the wheel and shoe is 0.25, calculate the magnitude of the force Q that the brake shoe exerts on the wheel in order to maintain equilibrium.

Ans. $Q=800$ kgf.

96. A cylindrical shaft of weight Q and radius R is rotated by means of a weight P attached to a rope (Fig. 82). The radius of each shaft journal is $r=\frac{R}{2}$ and the coefficient of friction in the bearings is 0.05. Determine the ratio of the weights P and Q when P falls uniformly.

Ans. $\frac{Q}{P}=39$.

97. A bracket loaded by a vertical force $P=600$ kgf is fixed to a wall by two bolts (Fig. 83). Determine the magnitude

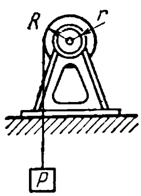


Fig. 82

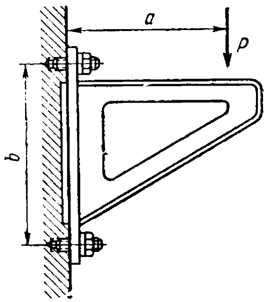


Fig. 83

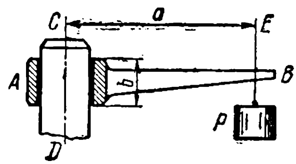


Fig. 84

of the force required to tighten the bolts and fix the wall bracket to the wall.

The coefficient of friction between the bracket and the wall is $f=0.3$. Make the calculation assuming that only the upper bolt is tightened and that the bolts fit with a clearance and must not be under any shearing stress. $\frac{b}{a}>f$.

Hint. The tightening force is the name of the force which acts along the axis of the bolt. The complete tightening of the bolt is made in two steps: the first eliminates the possibility of the wall bracket coming free from the wall and its tipping over the lower bolt, the second one ensures that normal pressure of the upper part of the bracket on the wall which balances the force of friction.

Ans. 2000 kgf.

98. A horizontal rod AB with a hole at end A is set on the vertical round prop CD (Fig. 84). The length of the bushing is $b=2$ cm. A weight P is suspended from a point E of the rod at distance a from the axis of the prop. Neglect the weight of the rod AB. If the coefficient of friction between the rod and the prop is $f=0.1$, calculate the distance a for the position when the rod is in equilibrium under the action of the weight P.

Ans. $a \geqslant 10$ cm.

99. A ladder AB leans against a rough wall and stands on the rough floor, forming an angle of 60° with the floor (Fig. 85). A weight P is applied to the ladder. Neglecting the weight of the ladder, determine graphically the maximum distance BP when the ladder is in equilibrium. The angle of friction for the wall and the floor is 15°

Ans. $BP = \frac{1}{2} AB.$

100. A uniform beam rests on a rough horizontal floor at a point A and is held by a rope at a point B (Fig. 86). The coefficient of friction is f. The beam forms an angle

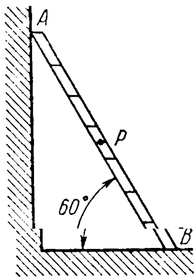


Fig. 85

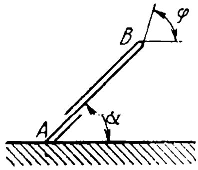


Fig. 86

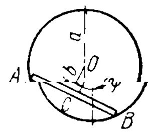


Fig. 87

$\alpha = 45°$ with the floor. Determine the angle φ of inclination of the rope to the horizontal when the beam just starts to slide.

Ans. $\tan\varphi = 2 + \frac{1}{f}$.

101. A uniform rod can slide with its ends A and B along the rough surface of a cylinder of radius a (Fig. 87). The distance OC from the rod to the centre O of the circle is b. The coefficient of friction between the rod and the cylinder is f. Determine the angle φ between the straight line OC and the vertical diameter of the circle, when the rod is in equilibrium.

Ans. $\cot\varphi \geqslant \frac{b^2(1+f^2)}{a^2 f} - f.$

102. A rolling mill consists of two rollers each of diameter $d = 50$ cm rotating in opposite directions, as shown in Fig. 88. The distance between the rollers is $a = 0.5$ cm. If the coefficient of friction between the rollers and hot steel is $f = 0.1$, what is the thickness b of a plate rolled at this mill?

H i n t. For the successful operation of the rolling mill the plate has to be seized by two rotating rollers, i. e., the resultant of all the normal reactions and forces of friction at points A and B applied to the plate should be directed along the horizontal to the right.

Ans. $b \leqslant 0.75$ cm.

103. A homogeneous cylinder is placed between two plates AO and BO hinged together at O (Fig. 89). The axis O_1 of the cylinder is parallel to the axis of the hinge. Both axes are horizontal and lie in the same vertical plane. The plates compress the cylinder under the action of two equal and opposite horizontal forces P applied at points A and B. The weight of the cylinder is Q and its radius is r. The coefficient of friction between the cylinder and the

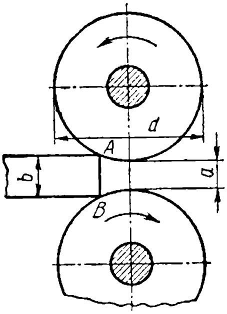


Fig. 88

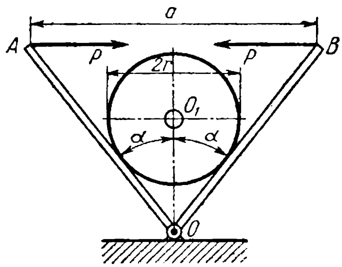


Fig. 89

plates is f. The angle $AOB=2\alpha$. $AB=a$. What conditions must be satisfied by the forces P to keep the cylinder in equilibrium?

Ans. (1) $\tan\alpha>f$; $\frac{r}{a}\frac{Q}{\sin\alpha+f\cos\alpha}\leqslant P\leqslant\frac{r}{a}\frac{Q}{\sin\alpha-f\cos\alpha}$;

(2) $\tan\alpha\leqslant f$; $P\geqslant\frac{r}{a}\frac{Q}{\sin\alpha+f\cos\alpha}$.

104. Two similar objects G and H of equal weight P rest on the edges AB and BC of the prism ABC (Fig. 90). They are connected by a thread which runs over a pulley at B. The coefficient of friction between the objects and the edges of the prism is f. Angles $BAC=$ $=BCA=45°$

Neglecting the friction of the pulley, determine the value of the angle of inclination α between the side AC and the horizontal, when the weight G just starts to slide down.

Ans. $\tan\alpha=f$.

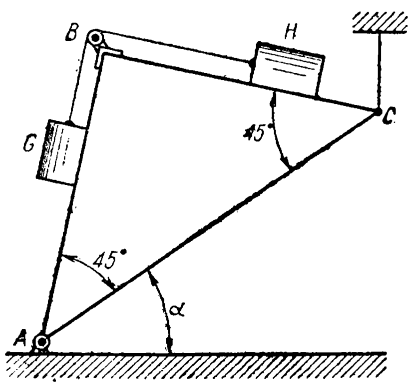


Fig. 90

105. A ball of radius R and the weight Q rests on the horizontal plane. The coefficient of sliding

friction between the ball and the plane is f, and the coefficient of rolling friction is k.

Determine under what conditions the horizontal force P applied at the ball centre causes a uniform rolling.

Ans. $\frac{k}{R} < f; \quad P = Q\frac{k}{R}$

5. Graphical Statics

In the answers to the problems in graphical statics all positive numbers signify strain forces and negative numbers signify compressive forces.

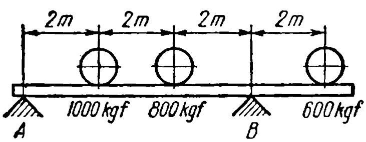


Fig. 91

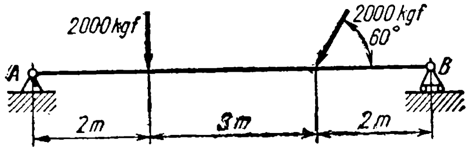


Fig. 92

106. A cantilever beam 8 m long with a span of 6 m carries three loads: 1000 kgf, 800 kgf, and 600 kgf, which are located in the way, shown in Fig. 91. Neglecting the weight of the beam, determine the reactions of the supports using graphical and analytical methods.

Ans. $R_A = 730$ kgf; $R_B = 1670$ kgf.

107. A weightless beam AB is loaded with two forces, as shown in Fig. 92. Determine graphically the reactions of the supports and then verify the results analytically.

Ans. $R_A = 2170$ kgf; $R_B = 1810$ kgf.

108. First determine graphically and then verify analytically the supporting reactions and the forces in the roof-truss members, when under the loads, shown in Fig. 93.

Ans. $R_A = 3400$ kgf;
$R_B = 2600$ kgf.

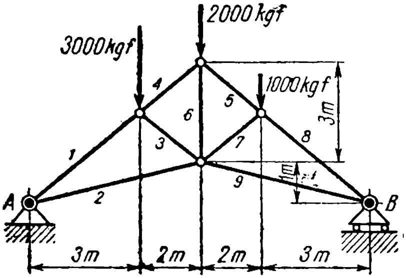


Fig. 93

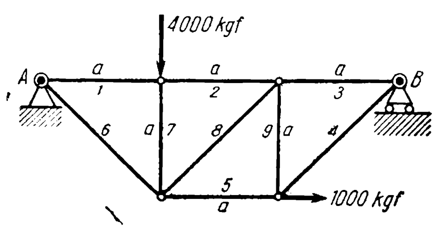


Fig. 94

Member No.	1	2	3	4	5	6	7	8	9
Force, kgf	—7300	+5800	—2440	—4700	—4700	+3900	—810	—5500	+4400

109. Determine the supporting reactions and the forces in the members of a diagonal truss with the loads acting on it, as shown in Fig. 94.

Ans. $X_A = -1000$ kgf; $Y_A = 3000$ kgf; $Y_B = 1000$ kgf.

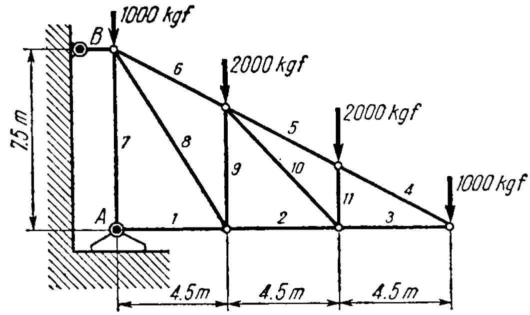


Fig. 95

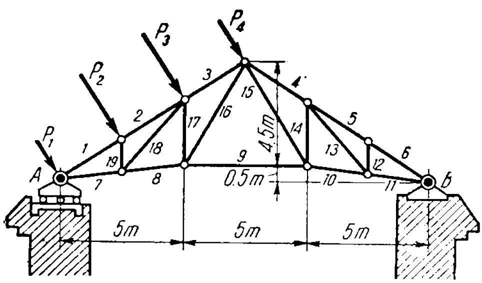


Fig. 96

Member No.	1	·2	3	4	5	6	7	8	9
Force, kgf	— 2000	—2000	—1000	+1410	+2000	+4240	—4000	+1410	—1000

110. Determine the supporting reactions and the forces acting in the members of the suspended truss when loads are applied to it, as shown in Fig. 95.

Ans. $X_A = 5400$ kgf; $Y_A = 6000$ kgf; $X_B = -5400$ kgf.

Member No	1	2	3	4	5	6	7	8	9	10	
Force, kgf	−5400	−3600	−1800	+2060	+2060	+4100	−6000	+3500	−3000	+2700	−2000

111. As a result of wind pressure, forces perpendicular to the roof appear in the joints of a roof truss with equal planes. $P_1=P_4=312.5$ kgf; $P_2=P_3=625$ kgf. Compute the reactions of the supports due to the wind and the forces acting in the truss members. The dimensions are shown in Fig. 96.

Ans. $Y_A=997$ kgf; $X_B=1040$ kgf; $Y_B=563$ kgf;
$S_1=-1525$ kgf; $S_2=-1940$ kgf; $S_3=-1560$ kgf;
$S_4=S_5=S_6=-970$ kgf; $S_7=+1100$ kgf; $S_8=440$ kgf;
$S_9=-215$ kgf; $S_{10}=S_{11}=-230$ kgf; $S_{12}=S_{13}=S_{14}=0$;
$S_{15}=-26$ kgf; $S_{16}=+1340$ kgf; $S_{17}=-1130$ kgf;
$S_{18}=+1050$ kgf; $S_{19}=-750$ kgf.

6. Concurrent Forces

112. Horizontal telegraph wires, forming an angle $DAE=90°$, are suspended to a post AB supported by an inclined strut AC. The tensions in the wires $AD=12$ kgf, $AE=16$ kgf. The joint at point A is hinged (Fig. 97).

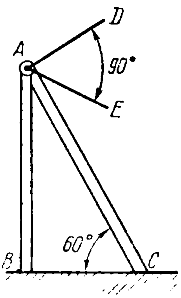


Fig. 97

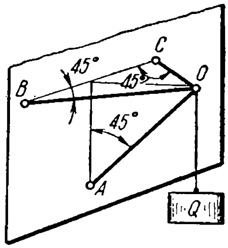


Fig. 98

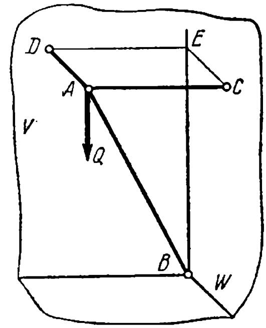


Fig. 99

Calculate the angle α between the planes BAC and BAE when the post does not bend laterally. Determine the force S in the strut, if it is inclined at a 60° angle to the horizontal. Neglect the weights of the post and the strut.

Ans. $\alpha = \arcsin\frac{3}{5} = 36°50'$; $S=-40$ kgf.

113. A weight $Q=100$ kgf is supported by a beam AO, hinged at point A and inclined at a 45° angle to the horizontal, and by two horizontal chains BO and CO of equal length (Fig. 98). $\angle CBO = \angle BCO = 45°$

Determine the force S in the beam, and the tension T in the chains.

Ans. $S=-141$ kgf; $T=71$ kgf.

114. Calculate the forces in a rod AB, fastened at a hinge B and chains AC and AD holding a weight $Q=42$ kgf (Fig. 99),

if $AB=145$ cm; $AC=80$ cm; $AD=60$ cm. The plane of the rectangle $CADE$ is horizontal. The planes V and W are vertical.

Ans. $T_C=32$ kgf; $T_D=24$ kgf; $T_B=-58$ kgf.

115. Determine the tensions in a cable AB and rods AC and AD, which hold a load $Q=180$ kgf (Fig. 100), if $AB=170$ cm, $AC=AD=100$ cm, $CD=120$ cm, $CK=KD$. The plane of the triangle CDA is horizontal. The joints at points A, C and D are hinged.

Ans. 204 kgf; -60 kgf.

116. A weight $Q=1000$ kgf is suspended from a point D, as shown in Fig. 101. The rods are hinged at A, B and D. Determine the reactions of the supports at A, B and C.

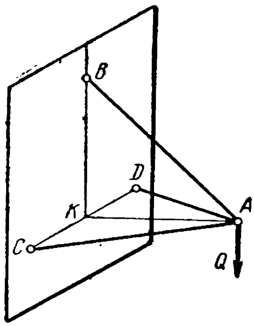


Fig. 100

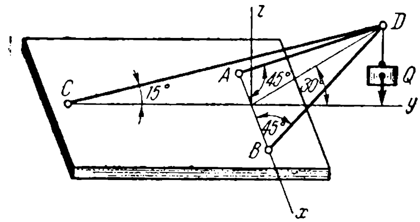


Fig. 101

Ans. $R_A=R_B=2640$ kgf; $R_C=3350$ kgf.

117. A gas-holder of spherical shape of the radius $R=10$ m weighs $Q=200{,}000$ kgf when filled with gas. It rests on three supports A, B and C located in the same horizontal plane, as shown in Fig. 102. The supports form an equilateral triangle with a side $a=10$ m. The support A is a fixed ball hinge, whereas the supports B and C are movable ball rollers whose smooth supporting planes are perpendicular to the radii OB and OC.

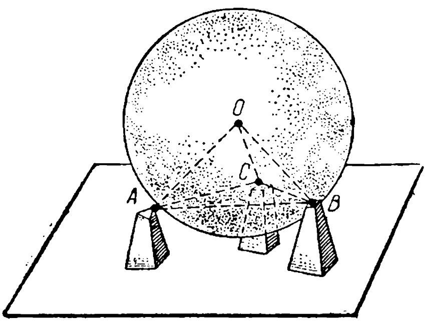


Fig. 102

The wind blows against the sphere in a direction perpendicular to the vertical plane through the line BC in the direction from BC to A with the force $p=120$ kgf per square

metre of the area of projection of the sphere on the mentioned plane. Determine the reactions of the supports.

Ans. $R_A=125{,}000$ kgf; $R_B=R_C=60{,}000$ kgf.
All reactions pass through the centre of the sphere.

118. Determine the force in the vertical pole and the force in the struts of the crane, which form an angle α, as shown in Fig. 103. $AB=BC=AD=AE$. The joints at points A, B, D and E are hinged.

Ans. $S_{BD}=P(\cos\alpha-\sin\alpha)$; $S_{BE}=P(\cos\alpha+\sin\alpha)$;
$S_{AB}=-P\sqrt{2}\cos\alpha$.

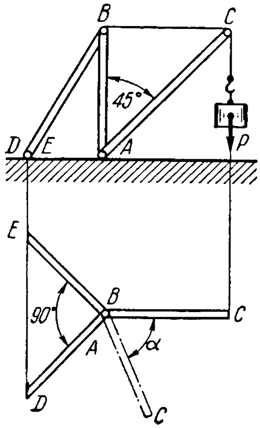


Fig. 103

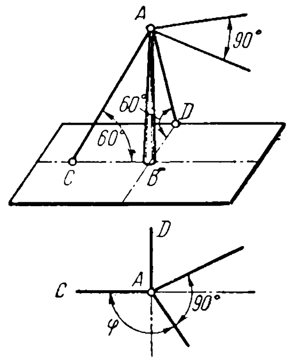


Fig. 104

119. A corner post holding an overhead cable is supported by two guys AC and AD (Fig. 104). The angle CBD equals 90°. Determine the forces in the post and the guys as a function of the angle φ between one of the parts of the cable and the plane CBA. Both parts of the cable are horizontal and perpendicular to each other. The tensions T in these parts are equal.

Ans. $S_{AC}=2T(\sin\varphi-\cos\varphi)$; $S_{AD}=2T(\sin\varphi+\cos\varphi)$;
$S_{AB}=-2\sqrt{3}T\sin\varphi$.
The guys will be simultaneously stretched, when $\frac{\pi}{4}<\varphi<\frac{3\pi}{4}$. One of the guys must be replaced by the beam when $\varphi<\frac{\pi}{4}$ or $\varphi>\frac{3\pi}{4}$.

120. Four symmetrically located guys support a 200-kgf mast AB in vertical position, as shown in Fig. 105. All the angles

between the guys are equal to 60° Determine the pressure exerted by the mast on the earth, assuming that the tension in each guy is 100 kgf.

Ans. 483 kgf.

121. Determine the reaction S in the legs AD, BD, and CD of the tripod forming angles of 60° with the horizontal plane (Fig. 106), when the tripod holds a weight of 3000 kgf. $AB=BC=AC$.

Ans. $S=2300$ kgf.

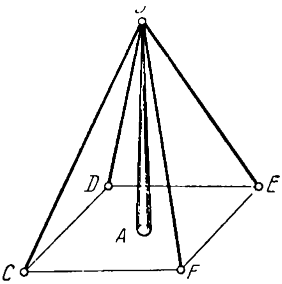


Fig. 105

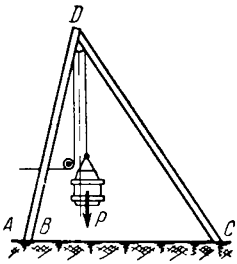


Fig. 106

122. Three homogeneous balls A, B, and C of equal radii rest on a horizontal surface touching each other. They are tied together by a cord wrapped round their equatorial planes. The fourth homogeneous 10-kgf ball O of the same radius rests on the three balls (Fig. 107). Determine the tension in the cord due to the pressure of the upper ball. Neglect the friction between the balls as well as between the balls and the horizontal surface.

Ans. $T=1.36$ kgf.

123. Threads AD, BD and CD, tied together at point D, are fixed at points A, B and C each at an equal distance l from the

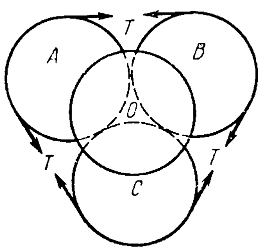


Fig. 107

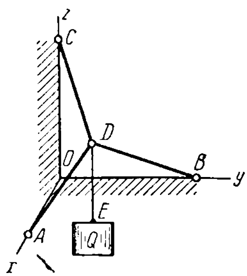


Fig. 108

origin O of a set of cartesian coordinates (Fig. 108). $AD=BD=$ $=CD=L$. The coordinates of the point D are

$$x=y=z=\frac{1}{3}(l-\sqrt{3L^2-2l^2}).$$

A weight Q is suspended from point D. Assuming that $\sqrt{\frac{2}{3}}\,l<$ $<L<l$, deduce the formulae for the tensions in the threads T_A, T_B and T_C.

Ans. $T_A=T_B=\dfrac{l-\sqrt{3L^2-2l^2}}{3l\sqrt{3L^2-2l^2}}\,LQ;\quad T_C=\dfrac{l+2\sqrt{3L^2-2l^2}}{3l\sqrt{3L^2-2l^2}}\,LQ.$

7. Reduction of a System of Forces to Its Simplest Possible Form

124. Four equal forces $P_1=P_2=P_3=P_4=P$ are applied to four vertices A, H, B and D of a cube (Fig. 109). The force P_1 is directed along AC, P_2 along HF, P_3 along BE and P_4 along DG. Reduce this system to one force.

Ans. The resultant force equals $2P$ and is directed along the diagonal DG.

125. Reduce to canonical form a system of forces: $P_1=8$ kgf directed along Oz, and $P_2=12$ kgf directed parallel to Oy, as shown in Fig. 110, where $OA=1.3$ m. First determine the magnitude of the principal vector V of all these forces as well as the value of their principal moment M, acting relative to an arbitrary point on the central screw axis. Compute the angles α, β and γ between the central screw axis and the coordinate axes. Define the coordinates x and y of the point of intersection of that axis with the plane Oxy.

Ans. $V=14.4$ kgf; $M=8.65$ kgfm; $\alpha=90°$; $\beta=\arctan\frac{2}{3}$; $\gamma=\arctan\frac{3}{2}$; $x=0.9$ m; $y=0$.

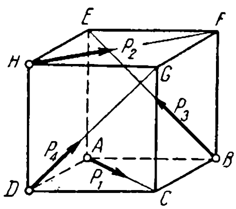


Fig. 109

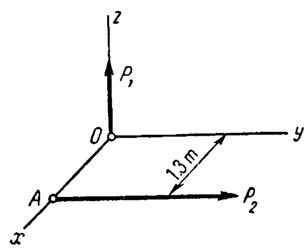


Fig. 110

126. A regular tetrahedron $ABCD$ of equal edge a is acted on by the force F_1 along the edge AB, and the force F_2 along the edge CD (Fig. 111). Determine the coordinates x and y of the point of intersection of the central screw axis with the plane Oxy.

Ans. $x = \dfrac{a\sqrt{3}\left(2F_2^2 - F_1^2\right)}{6\left(F_1^2+F_2^2\right)}$; $y = -\dfrac{a}{2}\,\dfrac{F_1F_2}{F_1^2+F_2^2}$

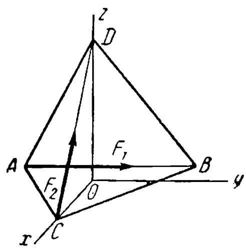


Fig. 111

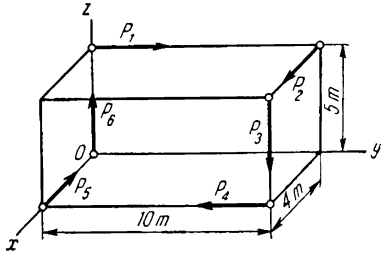


Fig. 112

127. Six forces $P_1=4$ kgf; $P_2=6$ kgf; $P_3=3$ kgf; $P_4=2$ kgf; $P_5=6$ kgf; $P_6=8$ kgf act along the edges of a rectangular parallelepiped 10 m, 4 m, and 5 m long, respectively, as shown in Fig. 112. Reduce this system of forces to canonical form, and determine the coordinates x and y of the point of intersection of the central screw axis with the plane Oxy.

Ans. $V=5.4$ kgf; $M=47.5$ kgfm; $\cos\alpha=0$; $\cos\beta=0.37$; $\cos\gamma=0.93$; $x=11.9$ m; $y=-10$ m.

8. Equilibrium of an Arbitrary System of Forces

128. An electric motor, mounted on the axle of a pair of the tram wheels, exerts a force which tends to turn it in a counterclockwise direction (Fig. 113). The magnitude of the torque moment of the forces (P, P) is 600 kgfm, the radius of the wheels is 60 cm. Assuming that the wheels are standing on horizontal rails, determine the tractive force Q.

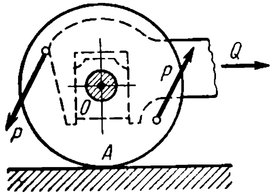


Fig. 113

Hint. First determine the resultant of the force of friction acting between the wheels and the rails, taking the moments of the forces about the axis O. Then all the forces applied to a pair of wheels must be projected on the horizontal plane.

Ans. $Q=1000$ kgf.

129. Circumferences of three disks: A of a radius 15 cm, B of a radius 10 cm, and C of a radius 5 cm, are subjected to three force couples: 10 kgf, 20 kgf and P, respectively (Fig. 114). The axes OA, OB and OC lie in the same plane. AOB is a right angle. Determine the magnitude of the force P. What is the magnitude of the angle $BOC=\alpha$ for equilibrium?

Ans. $P=50$ kgf; $\alpha=\arctan(-0.75)=143°10'$

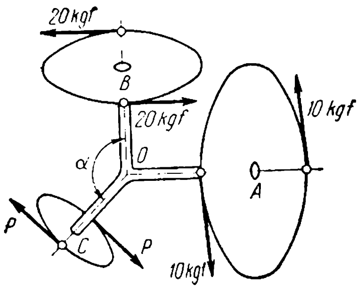


Fig. 114

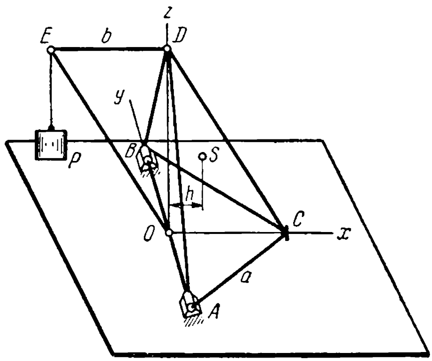


Fig. 115

130. A temporary lifting crane consists of a pyramid with horizontal base, forming an equilateral triangle ABC with vertical edge of an isosceles triangle shape ADB (Fig. 115). A vertical axis of the crane is hinged at O and D. A boom OE, holding a weight $P=1200$ kgf, can rotate about the vertical axis of the crane. The base ABC is fixed to the foundation by bearings A and B, and by the vertical bolt C.

Calculate the reactions of the supports when the boom is in the plane of symmetry of the crane. The weight Q of the crane is 600 kgf. The distance between the centre of gravity S of the crane and the axis OD is $h=1$ m; $a=4$ m; $b=4$ m.

Ans. $Z_A=Z_B=1506$ kgf; $Z_C=-1212$ kgf; $X_A=X_B=0$.

131. A uniform rectangular lamina with sides a and b and weight P rests horizontally on three point supports at the vertices of the rectangle A and B and at some point E (Fig. 116). The force exerted on the supports at the points A and B are $\frac{P}{4}$ and $\frac{P}{5}$, respectively. Compute the force N_E acting on the support at the point E, and the coordinates of this point.

Ans. $N_E=\frac{11}{20}P$; $x=\frac{6}{11}a$; $y=\frac{10}{11}b$.

132. A round weightless lamina rests in a horizontal position with its centre on the spike O (Fig. 117). The weights $P_1=1.5$ kgf, $P_2=1$ kgf and $P_3=2$ kgf are applied to the circumference of the lamina without disturbing the equilibrium. Calculate the values of the angles α and β.

Ans. $\alpha=75°30'$; $\beta=151°$.

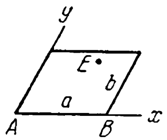


Fig. 116

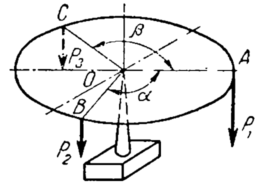


Fig. 117

133. A belt pulley CD of a dynamo has a radius of 10 cm. The dimensions of the shaft AB are shown in Fig. 118. The tension in the upper driving part of the belt is 10 kgf and in the lower driven one is 5 kgf. Determine the torque moment M and the reactions of the bearings A and B in case of uniform rotation. Neglect the weight of the machine parts; (P, P) is a couple caused by the reaction forces.

Ans. $M=50$ kgfcm; $X_A=-18$ kgf; $X_B=3$ kgf; $Z_A=Z_B=0$.

134. A gear C of radius 1 m and a pinion D of radius 10 cm are mounted on a horizontal shaft AB. All other dimensions are shown in Fig. 119. The gear C is acted upon tangentially by a horizontal force $P=10$ kgf and the pinion D is acted upon also tangentially by a vertical force Q. Determine the force Q and the reactions of the bearings A and B for equilibrium.

Ans. $Q=100$ kgf; $X_A=-1$ kgf; $X_B=-9$ kgf;
$Z_A=-90$ kgf; $Z_B=-10$ kgf.

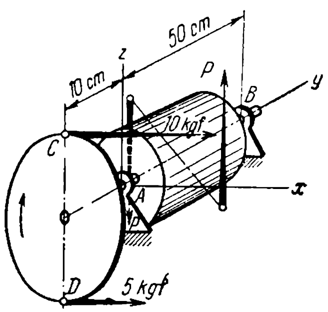


Fig. 118

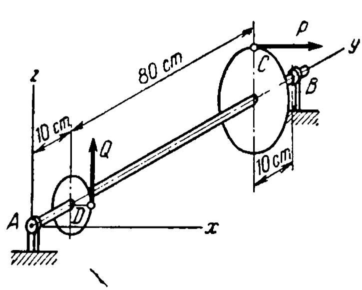


Fig. 119

135. A uniform rectangular lid of weight $P=40$ kgf is held partly opened at an angle of 60° to the horizontal by a counterbalance Q (Fig. 120). Neglecting friction on the pulley D, determine the weight Q, and the reactions of hinges A and B, if the pulley D is mounted on the same vertical as A. $AD=AC$.

Ans. $Q=10.4$ kgf; $X_A=10$ kgf; $X_B=0$; $Z_A=17.3$ kgf; $Z_B=20$ kgf.

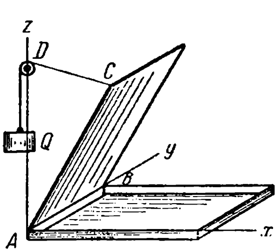


Fig. 120

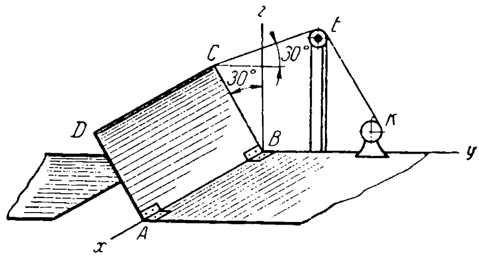


Fig. 121

136. The lifting part $ABCD$ of a railway draw-bridge weighing 1500 kgf is raised by a chain CE running over a pulley E to the winch K. The point E is in the vertical plane CBy. Determine the tension in the chain CE for the position, shown in Fig. 121, and the reactions at points A and B. The centre of gravity of the lifting part coincides with the centre of the rectangle $ABCD$.

Ans. $T=375$ kgf; $Y_A=0$; $Z_A=750$ kgf; $Y_B=-325$ kgf; $Z_B=562.5$ kgf.

137. A square uniform lamina $ABCD$ of side $a=30$ cm and weight $P=5$ kgf is fixed at point A by a ball joint and at a point B by a cylindrical hinge (Fig. 122). The side AB is horizontal. The lamina rests on the spike at the point E. A force $F=10$ kgf acts parallel to AB at the point H. $CE=ED$, $BH=10$ cm. The angle α between the plate and the horizontal surface equals 30° Find the reactions at the points A, B and E.

Ans. X_A 10 kgf; $Y_A=2.35$ kgf;
Z_A -0.11 kgf;
Y_B -3.43 kgf;
Z_B 3.23 kgf;
R_E = 2.17 kgf.

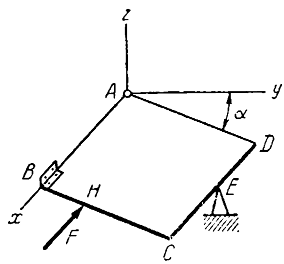


Fig. 122

138. Determine the forces in the 6 supporting rods which hold the square plate BCD when a horizontal force P acts long the side AD. The dimensions are hown in Fig. 123.

Ans. $S_1=P$; $S_2=-P\sqrt{2}$; $S_3=-P$; $S_4=P\sqrt{2}$;
$S_5=P\sqrt{2}$; $S_6=-P$.

139. Calculate the reactions of the bearing B and the thrust bearing A of the crane as well as the tensile force S in the cable, if the crane is pulled off by a horizontal guy-rope running over a pulley and supporting a weight $Q=100$ kgf. The angle of inclination between the cable and the horizontal is 60° The weight of the crane is $G=2000$ kgf. The weight P is 4000 kgf. The dimensions are shown in Fig. 124. Neglect the friction between the rope and the pulley.

Ans. $S=100$ kgf; $X_A=2.4$ kgf;
$Y_A=3395$ kgf;
$Z_A=6087$ kgf;
$X_B=47.6$ kgf;
$Y_B=-3395$ kgf.

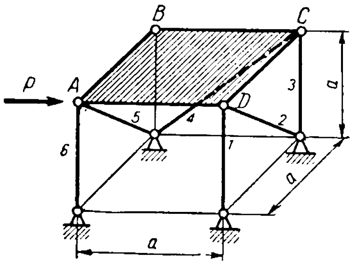


Fig. 123

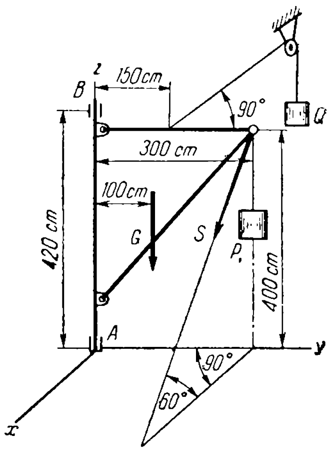


Fig. 124

140. Two horizontal cords AD and BC hold a rod AB with weight of 8 kgf in an inclined position (Fig. 125). At a point A the rod leans against the vertical wall on which the point D is located. At a point B the rod rests on the horizontal floor. The points A and C are in the same vertical. Neglect the forces of friction at A and B.

Calculate whether the rod can remain in equilibrium, and determine the tensions T_A and T_B in the cords as well as the reactions of the supporting planes, if the angles $ABC=BCE=60°$

Ans. $T_A=1.15$ kgf; $T_B=2.3$ kgf;
$R_A=2$ kgf; $R_B=8$ kgf.

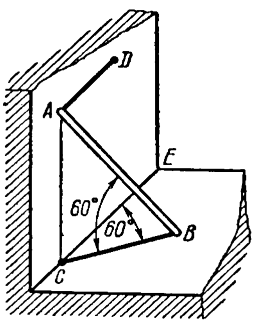


Fig. 125

141. A force couple rotating a water turbine T with a torque moment of 120 kgfm is balanced by pressure acting on the tooth B

of a bevel gear OB and by the reactions of the bearings (Fig. 126). The pressure exerted on the tooth is perpendicular to the radius $OB=0.6$ m, and it forms an angle $\alpha=15°=\arctan 0.268$ with the horizontal. If the total weight of the turbine with the shaft and the wheel is 1200 kgf and it is directed along the axis OC, determine the reactions of the thrust bearing C and the bearing A. $AC=3$ m, $AO=1$ m.

Ans. $X_A=266\frac{2}{3}$ kgf; $X_C=-66\frac{2}{3}$ kgf; $Y_A=-Y_C=10.7$ kgf; $Z_C=1254$ kgf.

142. A motor M uniformly lifts a weight Q by means of an endless chain (Fig. 127). Determine the reactions of the supports A and B and the tension in the chain, if its parts are inclined at an angle of 30° to the horizontal (the axis O_1x_1 is parallel to the axis Ax.) $R=20$ cm, $r=10$ cm and $Q=1000$ kgf. The tension in the driving part of the chain is twice the tension in the driven part, i. e., $T_1=2T_2$.

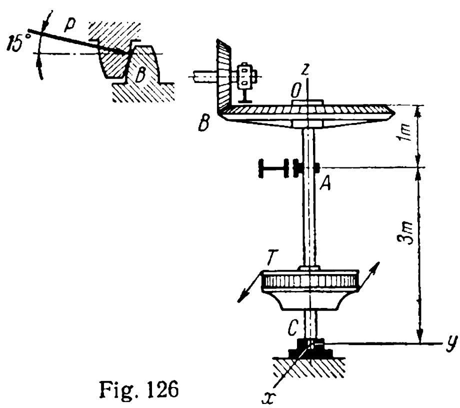


Fig. 126

Ans. $T_1=1000$ kgf; $T_2=500$ kgf; $X_A=-520$ kgf; $Z_A=600$ kgf; $X_B=-780$ kgf; $Z_B=150$ kgf.

143. A horizontal transmission shaft carries two belt pulleys C and D and it can rotate in bearings A and B (Fig. 128). The radii of the pulleys are: $r_C=20$ cm, and $r_D=25$ cm. The distances between the pulleys and the bearings are $a=b=50$ cm, and the distance between pulleys is $c=100$ cm. The tensions in each side of the belt running over the pulley C act in the horizontal direction and their values are t_1 and $T_1=2t_1=500$ kgf, whereas the tensions in each side of the belt running over the pulley D form an angle $\alpha=30°$ with the vertical, and their values are t_2 and $T_2=2t_2$. Determine the tensions T_2 and t_2 for equilibrium, as well as the reactions of the bearings due to the pull of the belts.

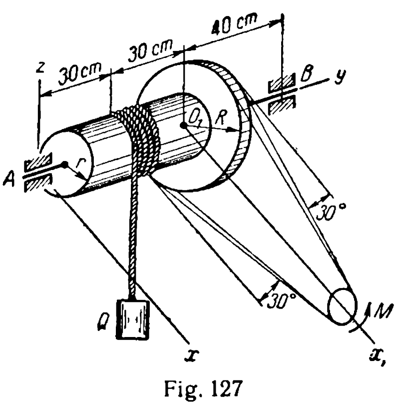


Fig. 127

Ans. $T_2=400$ kgf; $t_2=200$ kgf; $X_A=-637.5$ kgf;
$Z_A=130$ kgf; $X_B=-412.5$ kgf; $Z_B=390$ kgf.

144. The connecting rod of a steam engine exerts a force $P=2000$ kgf at the centre D of the crankshaft pin, and it acts at an angle of 10° to the horizontal, as shown in Fig. 129. The plane

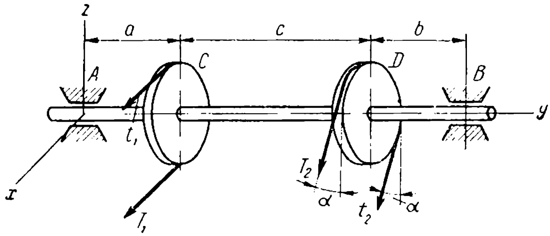


Fig. 128

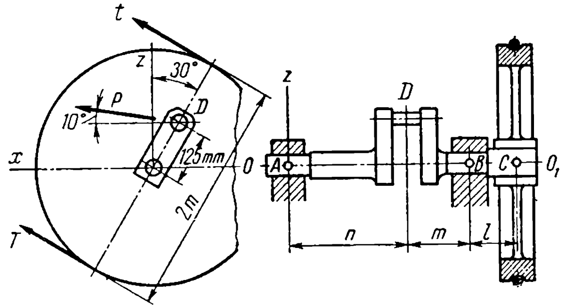


Fig. 129

ODO_1 wich passing through the axis OO_1 of the shaft and the axis of the shaft pin D forms an angle of 30° with the vertical.

The flywheel transmits power by means of a rope with the branches parallel and inclined to the horizontal at an angle of 30°. The force P is balanced by the tensions T and t in the branches of the rope and the reactions of the bearings at A and B. The weight of the flywheel is 1300 kgf and its diameter is $d=2$ m. The sum of tensions in the branches of the rope $T+t$ is 750 kgf and the distance from point D to the axis OO_1 is $r=125$ mm. $l=250$ mm, $m=300$ mm, $n=450$ mm. Determine the reactions of the bearings and the forces t and T

Ans. $X_A=-571$ kgf; $X_B=-2048$ kgf; $T=492$ kgf;
$Z_A=-447$ kgf; $Z_B=1025$ kgf; $t=258$ kgf.

145. To transmit a torque from a shaft to a parallel one two identical auxiliary pulleys are used (Fig. 130). Both pulleys are wedged on a horizontal axle KL. The axle KL rotates freely in a bearing M fixed on a column MN. The latter has a base of

triangular form, which is fixed to the floor by two bolts A and B while at C it rests freely on the floor. A bolt A passes through a round hole at the base, while a bolt B through an oblong hole directed parallel to AB. The axis of the support passes through the centre of the triangle ABC. Determine the reactions at A, B and C, if the distance between KL and the floor is 1 m, and the distance between the centres of the pulleys and the axis of the column is 0.5 m. The tensions in all four sides of the belts are the same and equal 60 kgf. The sides of the right belt are horizontal and the sides of the left belt are inclined at 30° to the horizontal. The weight of the whole system is 300 kgf with the centre of gravity at a point located on the axis of the column. $AB=BC=CA=50$ cm.

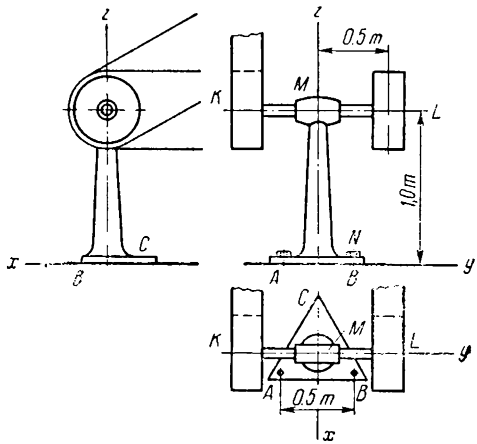


Fig. 130

Ans. $X_A=96$ kgf; $Y_A=0$; $Z_A=-239$ kgf; $X_B=128$ kgf; $Z_B=-119$ kgf; $Z_C=597$ kgf.

146. The suspension of a belt pulley D is fixed to a smooth horizontal ceiling MN by bearings A and C and it rests against the ceiling at point B (Fig. 131). These points are at the vertices of an equilateral triangle ABC of side 30 cm. The location of the centre of the belt pulley D is defined by

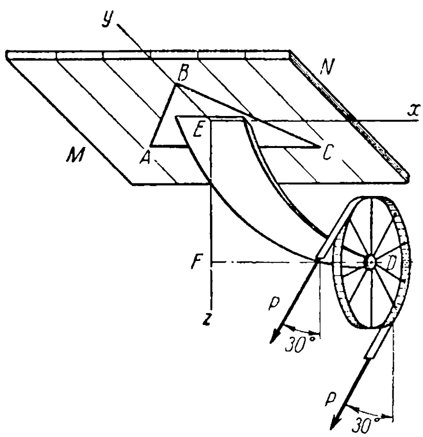


Fig. 131

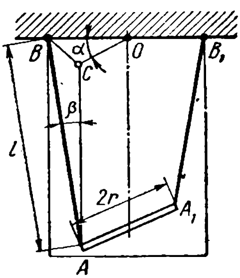


Fig. 132

the vertical $EF=40$ cm dropped from the centre E of the triangle ABC and the horizontal $FD=50$ cm parallel to AC. The plane of the pulley is perpendicular to the straight line FD. The tension P in each branch of the belt is 120 kgf, and they are inclined at 30° to the vertical. Determine the reactions in the supports A, B and C, neglecting the weight of the parts of the system.

Ans. $Y_A=140$ kgf; $Z_A=185$ kgf; $Z_B=115$ kgf; $Y_C=-260$ kgf; $Z_C=-508$ kgf.

147. A uniform rod AA_1 is suspended on two inextensible threads with the length l. The threads are fixed at points B and B_1 (Fig. 132). The length of the rod is $AA_1=BB_1=2r$ and its weight is P. The rod is turned around a vertical axis through an angle α. Derive formulae for the moment M of the couple which should be applied to the rod to keep it in equilibrium, and the tension T in the threads.

Ans. $M=\dfrac{Pr^2\sin\alpha}{\sqrt{l^2-4r^2\sin^2\dfrac{\alpha}{2}}}$; $\quad T=\dfrac{lP}{2\sqrt{l^2-4r^2\sin^2\dfrac{\alpha}{2}}}$.

9. Centre of Gravity

148. Find the position of the centre of gravity C of an area limited by a semicircle AOB of a radius R and two straight lines of equal length AD and DB (Fig. 133). $OD=3R$.

Ans. $OC=\dfrac{3\pi+16}{3\pi+12}R=1.19\,R$.

149. Find the position of the centre of gravity of a homogeneous disk with a round hole, assuming that the radius of the disk is r_1 and that of the hole is r_2 (Fig. 134). The centre of the hole is at $\dfrac{r_1}{2}$ distance from the disk centre.

Ans. $x_C=-\dfrac{r_1r_2^2}{2\left(r_1^2-r_2^2\right)}$.

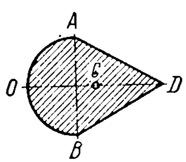


Fig. 133

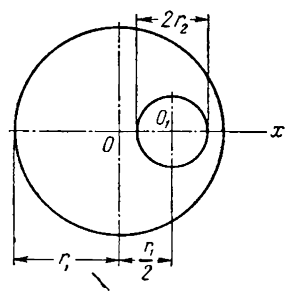


Fig. 134

150. Find the centre of gravity of the section of a beam, whose dimensions are given in Fig. 135.

Ans. $x_C = 9$ cm.

151. Find the coordinates of the centre of gravity of a uniform plate, shown in Fig. 136, if the dimensions are as follows: $AH = 2$ cm, $HG = 1.5$ cm, $AB = 3$ cm; $BC = 10$ cm, $EF = 4$ cm, $ED = 2$ cm.

Ans. $x = 5\frac{10}{13}$ cm; $y = 1\frac{10}{13}$ cm.

152. A square hole $EFGH$ is cut in a homogeneous square board $ABCD$ of side $AB = 2$ m (Fig. 137). The sides of the hole,

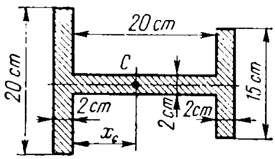


Fig. 135

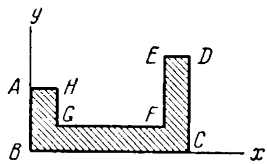


Fig. 136

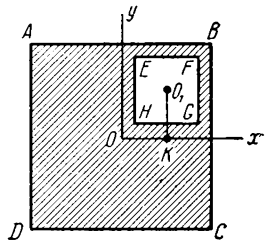


Fig. 137

which are 0.7 m long, are parallel to the sides of the board. Determine the coordinates x and y of the centre of gravity of the remaining part of the board, assuming that $OK = O_1K = 0.5$ m, that O and O_1 are the centres of the squares, and that OK and O_1K are parallel to the sides of the squares.

Ans. $x = y = -0.07$ m.

153. Draw a straight line DE from the vertex D of a homogeneous rectangle $ABCD$ in such a way that when the orthogonal trapezium $ABED$ so formed is suspended at the vertex E, the side $AD = a$ is horizontal (Fig. 138).

Ans. $BE = 0.366\,a$.

154. Determine the coordinates of the centre of gravity of a system of weights located at the vertices of a right-angled parallel-

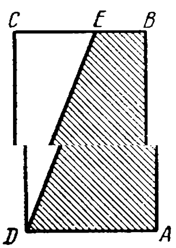


Fig. 138

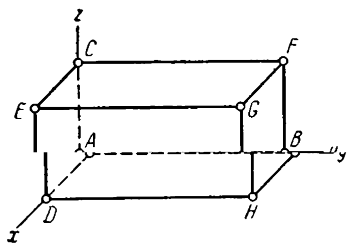


Fig. 139

epiped (Fig. 139). Its edges are: $AB=20$ cm, $AC=10$ cm, $AD=$ $=5$ cm. The weights at the vertices A, B, C, D, E, F, G and H are equal to 1 kgf, 2 kgf, 3 kgf, 4 kgf, 5 kgf, 3 kgf, 4 kgf, and 3 kgf, respectively.

Ans. $x=3.2$ cm, $y=9.6$ cm, $z=6$ cm.

155. Determine the coordinates of the centre of gravity of a flat truss consisting of seven rods with the lengths, shown in Fig. 140, if the weight per metre of the rods is the same.

Ans. $x=1.47$ m, $y=0.94$ m.

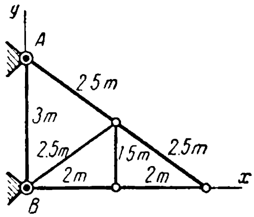


Fig. 140

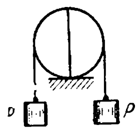


Fig. 141

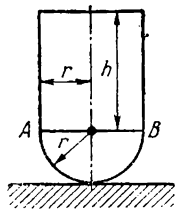


Fig. 142

156. A 30,000-kgf load is transferred at a distance of 60 m from the fore compartment of a ship to the aft one. The displacement of the ship equals 4,500,000 kgf. Calculate the distance by which the centre of gravity of the ship and the load is displaced.

Ans. By the distance of 0.4 m.

157. Two halves of a round homogeneous cylinder are tied together by a thread wrapped round the cylinder with weights P attached to each end of the thread (Fig. 141). The cylinder weighs Q kgf. The plane of contact of both halves of the cylinder is vertical. Determine the minimum value of P for which both halves of the cylinder will be in equilibrium on a horizontal plane.

Ans. $P=\frac{2}{3}\frac{Q}{\pi}$ kgf.

158. A system consists of a hemisphere with a cylinder of equal density and radius r mounted on the flat side (Fig. 142). Assuming that the system rests in stable equilibrium with the hemisphere on a smooth horizontal plane, determine the maximum height h of the cylinder.

Hint. The centre of gravity of the system must coincide with the centre of the hemisphere. The distance of the centre of gravity of the homogeneous hemisphere from its base is $\frac{3}{8}r$.

Ans. $h=\frac{r}{\sqrt{2}}$.

Part II
KINEMATICS

III. MOTION OF A PARTICLE

10. Equation of Motion and Path of a Particle

159. Find the equation of the path of a particle, if its motion is given by the following equations:

(1) $x=20t^2+5$, $y=15t^2+3$. *Ans.* The straight line $3x-4y=3$ with the origin at the point $x=5$, $y=3$.

(2) $x=5+3\cos t$, $y=4\sin t$. *Ans.* The ellipse $\frac{(x-5)^2}{9}+\frac{y^2}{16}=1$.

(3) $x=at^2$, $y=bt$. *Ans.* The parabola $ay^2-b^2x=0$.

(4) $x=5\cos t$, $y=3-5\sin t$. *Ans.* The circumference of the circle $x^2+(y-3)^2=25$.

(5) $x=3\cos\left(\frac{\pi}{8}+\pi t\right)$, $y=4\sin\left(\frac{\pi}{4}+\pi t\right)$. *Ans.* The ellipse $\frac{x^2}{9}+\frac{y^2}{16}-\frac{xy}{6}\sin\frac{\pi}{8}=\cos^2\frac{\pi}{8}$.

160. The motion of a particle is defined by the equations below. Find the equation of its path, and express the law of motion of the particle along the path starting from the origin.

(1) $x=3t^2$, $y=4t^2$. *Ans.* The straight line $4x-3y=0$; $s=5t^2$.

(2) $x=3\sin t$, $y=3\cos t$. *Ans.* The circumference of the circle $x^2+y^2=9$; $s=3t$.

(3) $x=a\cos^2 t$, $y=a\sin^2 t$. *Ans.* A section of the straight line $x+y-a=0$, for the respective ranges $0\leqslant x\leqslant a$; $s=a\sqrt{2}\sin^2 t$.

(4) $x=5\cos 5t^2$, $y=5\sin 5t^2$. *Ans.* The circumference of the circle $x^2+y^2=25$; $s=25t^2$.

161. The movement of a bridge crane along a shop is defined by the equation $x=t$. A winch which rolls across the crane satisfies the equation: $y=1.5\,t$ (x and y are measured in metres and t in seconds). The chain shortens with velocity $v=0.5$ m/sec. Determine the path of the centre of gravity of the weight, assuming that at the start the centre was located on the horizontal plane Oxy. The axis Oz is directed vertically upwards.

Ans. The path is a straight line: $y=1.5x$; $z=0.5x$.

162. A particle simultaneously performs two harmonic oscillations of equal frequencies but unequal amplitudes and phases. The oscillations are directed along two mutually perpendicular axes:

$$x=a\sin(kt+\alpha),\quad y=b\sin(kt+\beta).$$

Determine the path of the particle.

Ans. The ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{2xy}{ab}\cos(\alpha-\beta)=\sin^2(\alpha-\beta)$.

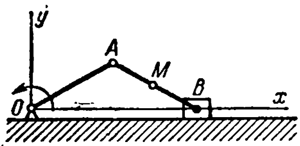


Fig. 143

163. A crank OA rotates with constant angular velocity $\omega=10$ sec^{-1}. $OA=AB=80$ cm. Determine the equation of motion and the path of a particle M at the centre of the connecting rod. Find the equation of motion of the slider B, if at the start the slider was at the extreme right. The axes of coordinates are shown in Fig. 143.

Ans. (1) The path of the particle M is the ellipse: $\frac{x^2}{120^2}+\frac{y^2}{40^2}=1$.

(2) The equation of motion of the slider B is $x=$ $=160\cos 10t$.

164. Determine the equation of motion and the path of a point on a rim of a locomotive wheel of radius $R=1$ m, if the locomotive moves along a straight track with a constant speed of 20 m/sec. It is assumed that the wheel rolls without sliding. Take the origin of the coordinate system to be the initial position of the point on the track and axis Ox along the track.

Ans. The cycloid $x=20t-\sin 20t$; $y=1-\cos 20t$.

165. The motion of a load released by an aircraft is defined by equations: $x=40t$, $y=4.9t^2$, where x, y are measured in metres and t in seconds. Take the origin of the coordinate system to be the point when the load is released. The axis Ox is horizontal and the axis Oy is directed vertically downwards. Determine the equation

of the path of the load. Find also the time of falling and the range of its horizontal flight if the aircraft flies at the altitude $h=3000$ m.

Ans. $y=0.00306\,x^2$; $t=24.74$ sec; $L=989.6$ m.

11. Velocity of a Particle

166. The equation of motion of a frame of a circular eccentric is $x=e\,(1-\cos\omega t)$, where x is measured in centimetres and t in seconds; e is the eccentricity; ω is the angular velocity of the eccentric (e and ω are constants). Determine:

(1) The two next times after $t=0$, when the motion changes its direction.
(2) The first time when the velocity gains its maximum value.
(3) The period of motion.

Ans. (1) $\frac{\pi}{\omega}$ sec; $\frac{2\pi}{\omega}$ sec; (2) $\frac{\pi}{2\omega}$ sec; (3) $T=\frac{2\pi}{\omega}$ sec.

167. A particle performs harmonic oscillations expressed by the law: $x=a\sin kt$. If for $x=x_1$ the velocity is $v=v_1$ while for $x=x_2$ the velocity is $v=v_2$, determine the amplitude a and the circular frequency k of the oscillations.

Ans. $a=\sqrt{\dfrac{v_1^2x_2^2-v_2^2x_1^2}{v_1^2-v_2^2}}$; $k=\sqrt{\dfrac{v_1^2-v_2^2}{x_2^2-x_1^2}}$.

168. The motion of a crank pin is defined by the equations:

$$x=15\sin\frac{\pi}{4}t;\quad y=15\cos\frac{\pi}{4}t$$

(x, y are measured in centimetres and t in seconds). Determine the projections of the crank pin velocity on to the directions x and y, when the end of the pin is on the axes of coordinates. Also find the equation of the hodograph of the velocity.

Ans. When $x=0$, $y=15$ cm $\quad v_x=\frac{15}{4}\pi$ cm/sec, $v_y=0$;

when $x=15$ cm, $y=0$ $\quad v_x=0$, $v_y=-\frac{15}{4}\pi$ cm/sec;

when $x=0$, $y=-15$ cm $\quad v_x=-\frac{15}{4}\pi$ cm/sec, $v_y=0$;

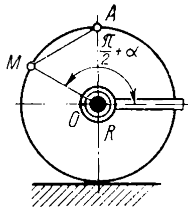
when $x=-15$ cm, $y=0$ $\quad v_x=0$, $v_y=\frac{15}{4}\pi$ cm/sec.

The hodograph is the circumference of the circle: $x_1^2+y_1^2=\frac{225}{16}\pi^2$.

169. A particle describes Lissajous' figure given by the equations: $x=2\cos t$, $y=4\cos 2t$ (x, y are measured in centimetres

and t in seconds). Determine the magnitude and direction of the velocity of the particle when it is on the axis Oy.

Ans. (1) $v=2$ cm/sec; $\cos(v, x)=-1$.
(2) $v=2$ cm/sec; $\cos(v, x)=1$.



170. Find the velocity of the centre M of a connecting rod of the slider crank mechanism and the velocity of the slider for $r=l=a$ and $\varphi=\omega t$, where $\omega=\text{const}$ (Fig. 144).

Ans. (1) $v=\frac{a}{2}\omega\sqrt{8\sin^2\omega t+1}$;
(2) $v=2a\omega\sin\omega t$.

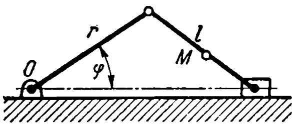


Fig. 144

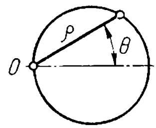


Fig. 145

171. The motion of a load released by an aircraft is given by the equations:

$$x=v_0 t, \quad y=h-\frac{gt^2}{2};$$

axis Ox is horizontal and Oy is directed vertically upwards.

Determine:
(1) The equation of the path. (2) The velocity of the load (magnitude and direction) at the instant when it crosses the axis Ox. (3) The range of flight. (4) The equation of the hodograph of the load velocity and the velocity v_1 of a particle tracing the hodograph.

Ans. (1) $y=h-g\frac{x^2}{2v_0^2}$;

(2) $v=\sqrt{v_0^2+2gh}$; $\cos(v, x)=\frac{v_0}{v}$;

$\cos(v, y)=-\frac{\sqrt{2gh}}{v}$;

(3) $x=v_0\sqrt{\frac{2h}{g}}$

(4) the vertical straight line at a distance v_0 from the origin of the coordinates; $v_{1y}=-g$.

172. A locomotive runs without slipping with a speed $v_0=72$ km/h along a straight track. The radius of its wheels is $R=1$ m.

(1) Determine the magnitude and direction of the velocity of a point M on the rim of the wheel at the instant when the radius of the point M and the direction of the velocity v_0 make the angle $\frac{\pi}{2}+\alpha$.

(2) Plot the hodograph of the velocity of a particle M, and determine the velocity v_1 of the point tracing out this hodograph (Fig. 145).

Ans. (1) The velocity is $v=40\cos\frac{\alpha}{2}$ m/sec and is directed along the straight line MA.

(2) The circumference of the circle $\rho=2v_0\cos\theta$, where $\theta=\frac{\alpha}{2}$ and the radius $r=v_0$ (as shown in Fig. 145); $v_1=\frac{v_0^2}{R}=400$ m/sec².

173. Determine the equation of motion and the path of a particle M on a railway car wheel whose radius $R=0.5$ m. The particle M is 0.6 m from the axis and at start was located on the surface 0.1 m lower than the track. The car rolls along a straight track with velocity $v=10$ m/sec. Find also the instants when the particle M passes its bottom and top positions and also the projections of its velocity on the axes Ox, Oy at these instants. The axis Ox coincides with the direction of the track and the axis Oy passes through the starting bottom position of the particle.

Ans. The oblong cycloid $x=10t-0.6\sin 20t$;

$$y=0.5-0.6\cos 20t;$$

when $t=\frac{\pi k}{10}$ sec, the bottom position of the particle, $v_x=-2$ m/sec, $v_y=0$;

when $t=\frac{\pi}{20}(1+2k)$ sec, the top position of the particle, $v_x=22$ m/sec, $v_y=0$, where $k=0, 1, 2, 3,$

12. Acceleration of a Particle

174. A train runs at a speed of 72 km/h. When the brakes are applied its deceleration is 0.4 m/sec². When and how far from the station should the brakes be applied so that the train stops at the station?

Ans. 50 sec; 500 m.

175. Water drops flow out from a small opening at the bottom of a vertical tube at the uniform rate of one drop per 0.1 sec and acceleration of 981 cm/sec^2. Determine the distance between two adjacent drops exactly 1 second after the latter has left the tube.

Ans. 93.2 cm.

176. A tram runs along a straight track. After starting the distance travelled by the tram is proportional to the cube of time. During the first minute the tram travelled 90 metres. Find the velocity and acceleration for $t=0$ and $t=5$ sec. Sketch the distance, velocity and acceleration curves.

Ans. $v_0=0$; $w_0=0$; $v_5=\frac{15}{8}$ m/min; $w_5=45$ m/min^2.

177. The landing speed of an aircraft is 100 km/h. Determine its deceleration at landing on the track of $l=100$ m length if the deceleration is constant.

Ans. $w=3.86$ m/sec^2.

178. A train moving with uniformly variable motion at an initial velocity of 54 km/h covered a distance of 600 m in the first 30 seconds. If the train travels along a curved track whose radius is $R=1$ km, determine its speed and acceleration at the end of the 30th second.

Ans. $v=25$ m/sec; $w=0.708$ m/sec^2.

179. A slider moves along a rectilinear guide with the acceleration of $w_x=-\pi^2 \sin\frac{\pi}{2}t$ m/sec^2. Derive the equation of motion of the slider if its initial velocity is $v_{0x}=2\pi$ m/sec. The initial position of the slider coincides with its middle point, which is taken as the origin of the coordinate system. Plot the distance, velocity and acceleration curves.

Ans. $x=4\sin\frac{\pi}{2}t$ m.

180. The acceleration of a point on the rim of the flywheel of radius 2 m is defined by the equation: $s=0.1t^3$ (t is measured in seconds and s in metres). Determine the normal and tangential accelerations of the point at the instant when its velocity is $v=30$ m/sec.

Ans. $w_n=450$ m/sec^2; $w_t=6$ m/sec^2.

181. The rectilinear motion of a particle is defined by the law; $s=\frac{g}{a^2}(at+e^{-at})$, where a and g are constants. Find the initial

velocity of the particle and its acceleration as a function of velocity.

Ans. $v_0=0$; $w=g-av$.

182. The motion of a particle is defined by the equations: $x=10\cos 2\pi\frac{t}{5}$, $y=10\sin 2\pi\frac{t}{5}$ (x, y are measured in centimetres, t in seconds). Find the path of the particle, and the magnitude and direction of the velocity and acceleration.

Ans. The path is a circumference of radius 10 cm; the velocity equals $v=4\pi$ cm/sec and it is directed along the tangent when the axis Ox turns through an angle of 90° to the axis Oy; the acceleration $w=1.6\pi^2$ cm/sec² and is directed towards the centre.

183. The motion of a particle is defined by the equations:

$$x=20t^2+5; \quad x=15t^2-3$$

(t is measured in seconds and x, y in centimetres). Determine the magnitude and direction of the velocity and acceleration, when $t=2$ sec and $t=3$ sec.

Ans. When $t=2$ sec, $v=100$ cm/sec;
when $t=3$ sec, $v=150$ cm/sec; $w=\text{const}=50$ cm/sec²;
$\cos(v, x)=\cos(w, x)=0.8$; $\cos(v, y)=\cos(w, y)=0.6$.

184. The motion of a particle is defined by the equations:

$$x=a(e^{kt}+e^{-kt}), \quad y=a(e^{kt}-e^{-kt}),$$

where a and k are constants.

Find the equation of the path of the particle, and its velocity and acceleration as a function of radius-vector $r=\sqrt{x^2+y^2}$.

Ans. The hyperbola: $x^2-y^2=4a^2$; $v=kr$; $w=k^2r$

185. Determine the initial radius of curvature of the path of a particle which is defined by equations: $x=2t$, $y=t^2$ (t is measured in seconds and x, y in metres).

Ans. $\rho_0=2$ m.

186. A wheel rolls without slipping along a horizontal axis Ox (Fig. 146). Determine the magnitude and direction of the acceleration and the radius of curvature of the path of a point on the wheel, if this point describes a cycloid in accordance with the equations:

$$x=20t-\sin 20t, \quad y=1-\cos 20t$$

(t is measured in seconds and x, y in metres). Also find the radius of curvature ρ when $t=0$.

Ans. The acceleration $w=400$ m/sec² is directed along MC to the centre C of the rolling wheel; $\rho=2MA$; $\rho_0=0$.

187. Determine the path of a point M on the connecting rod of a slider crank mechanism (Fig. 147), if $r=l=60$ cm, $MB=\frac{1}{3}l$ and $\varphi=4\pi t$ (t is measured in seconds). Also find the velocity, acceleration and radius of curvature of the path at the instant when $\varphi=0$.

Ans. The ellipse $\frac{x^2}{100^2}+\frac{y^2}{20^2}=1$; $v=80\pi$ cm/sec;

$w=1600\pi^2$ cm/sec²; $\rho=4$ cm.

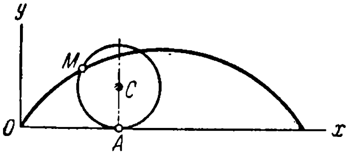


Fig. 146

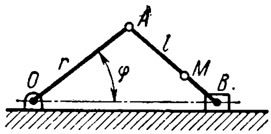


Fig. 147

188. A particle, initially at rest, moves at an angle of $\alpha_0=55°$ to the horizontal with an initial velocity $v_0=1600$ m/sec. The particle is only acted on by gravity, $g=9.81$ m/sec². Determine the maximum height and range of the particle.

Ans. $s=245$ km; $h=87.5$ km.

189. The motion of a particle is defined by the equations: $x=v_0t\cos\alpha_0$; $y=v_0t\sin\alpha_0-\frac{1}{2}gt^2$.

The axis Ox is horizontal and the axis Oy is directed vertically upwards; v_0, g and $\alpha_0<\frac{\pi}{2}$ are constants. Find:

(1) the path of the particle;
(2) the coordinates of the maximum height of the path;
(3) the projections of the velocity on the coordinate axes at the instant when the particle is on the axis Ox.

Ans. (1) The parabola $y=x\tan\alpha_0-\frac{g}{2v_0^2\cos^2\alpha_0}x^2$.

(2) $x=\frac{v_0^2}{2g}\sin 2\alpha_0$; $y=\frac{v_0^2}{2g}\sin^2\alpha_0$.

(3) $v_x=v_0\cos\alpha_0$; $v_y=\pm v_0\sin\alpha_0$.

The upper sign denotes the instant of starting and the bottom one the time $t=\frac{2v_0\sin\alpha_0}{g}$.

190. The motion of a particle is defined by the equations: $x=v_0 t\cos\alpha_0$, $y=v_0 t\sin\alpha_0-\frac{1}{2}gt^2$, where v_0 and α_0 are constants. Find a radius of curvature of the path, when $t=0$, and the first time when $t>0$ and $x=y=0$.

Ans. $\rho=\dfrac{v_0^2}{g\cos\alpha_0}$

191. Determine the motion of a particle (its equation of motion and path) acted on only by gravity, $w=g=9.81$ m/sec², if the initial velocity of the particle is $v_0=1000$ m/sec and is inclined at the angle $\alpha_0=60°$ to the horizontal.

Ans. $x=500t$, $y=866t-4.905t^2$; $y=1.732x-10^{-8}\cdot 1{,}962x^2$;

192. A coastal gun is placed at a height of $h=30$ m above sea level (Fig. 148). A projectile is fired from the gun at the angle of elevation $\alpha_0=45°$, and initial velocity is $v_0=$ $=1000$ m/sec. Neglecting air friction, find how far from the gun the projectile will hit a target located at sea level.

Ans. 102 km.

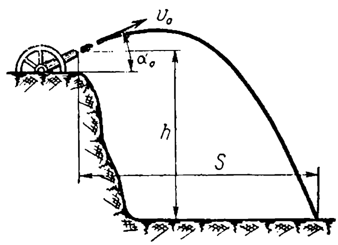


Fig. 148

193. Find the tangential and normal accelerations of a particle whose motion is defined by the equations: $x=\alpha t$; $y=\beta t-\frac{gt^2}{2}$

Ans. $w_t=-\dfrac{g(\beta-gt)}{v}$;

$w_n=\dfrac{g\alpha}{v}$, where v is the velocity of the particle.

194. A train starting from rest at a station accelerates uniformly until a velocity of 72 km/h is attained at the end of the 3rd minute. The track is curved and has a radius of 800 m. Determine the tangential, normal and total accelerations of the train by the end of the 2nd minute.

Ans. $w_t=\frac{1}{9}$ m/sec²; $w_n=\frac{2}{9}$ m/sec²; $w=0.25$ m/sec².

195. The motion of a particle is given in polar coordinates and by the equations: $r=ae^{kt}$ and $\varphi=kt$, where a and k are constants. Determine the equation of the path, the velocity, the acceleration, and the radius of the curvature of the path as functions of its radius-vector r

Ans. $r=ae^{\varphi}$, a logarithmic spiral; $v=kr\sqrt{2}$; $w=2k^2r$; $\rho=r\sqrt{2}$.

IV. SIMPLEST MOTIONS OF A RIGID BODY

13. Rotation of a Rigid Body about a Fixed Axis

196. If the angle of rotation of a steam turbine disk is proportional to the cube of the time and when $t=3$ sec the angular velocity of the disk corresponds to $n=810$ rpm, find the equation of motion.

Ans. $\varphi=\pi t^3$ radians.

197. Find the constant angular acceleration of a body if, starting from rest, it makes 3600 revolutions during the first 2 minutes.

Ans. $\varepsilon=\pi$ sec^{-2}.

198. A shaft rotates with constant acceleration starting from rest. During the first 5 seconds it makes 12.5 revolutions. What is its angular velocity at that instant?

Ans. $\omega=5$ rps $=10\pi$ sec^{-1}

199. The initial angular velocity of a wheel with a fixed axle is 2π sec^{-1}. After 10 revolutions the wheel is brought to rest due to friction in the bearings. Determine the angular acceleration ε of the wheel, assuming it to be constant.

Ans. $\varepsilon=0.1\pi$ sec^{-2}, the rotation is retarded.

200. A watch balance-wheel performs torsional harmonic oscillations (Fig. 149) of period $T=\frac{1}{2}$ sec. The maximum angular amplitude of a point on the rim of the balance-wheel with respect to the position of equilibrium is $\alpha=\frac{\pi}{2}$ radians. Find the angular velocity and angular acceleration of the balance wheel 2 seconds after the instant it passes the position of equilibrium.

Ans. $\omega=2\pi^2$ sec^{-1}; $\varepsilon=0$.

201. A pendulum oscillates in a vertical plane about a fixed horizontal axis O. Starting from the equilibrium position it gains a maximum deflection of $\alpha=\frac{\pi}{16}$ radians in $\frac{2}{3}$ seconds.

(1) Write down the equation of motion of the pendulum, assuming that it performs harmonic oscillations.

(2) Where will the pendulum have its maximum angular velocity and what is its magnitude in this position?

Ans. (1) $\varphi=\frac{\pi}{16}\sin\frac{3}{4}\pi t$ radians;

(2) in plumb position; $\omega_{\max}=\frac{3}{64}\pi^2\ \text{sec}^{-1}$.

202. Determine the velocity v and the acceleration w of the particle on the surface of the earth in the city of Leningrad taking into consideration only the rotation of the earth about its axis, and assuming that the latitude of the city of Leningrad is 60° and the radius of the earth is 6370 km.

Ans. $v=0.232$ km/sec;
$w=0.0169$ m/sec^2.

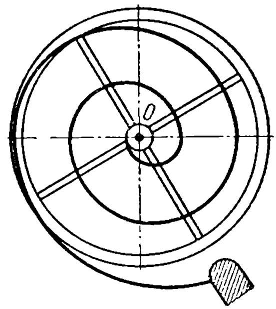


Fig. 149

203. A flywheel of radius $R=2$ m starts from rest and rotates with a uniform acceleration. In ten minutes all the points on the rim of the wheel gain a linear velocity $v=100$ m/sec. Find the velocity, and the normal and tangential accelerations of the points on the rim, when $t=15$ sec.

Ans. $v=150$ m/sec; $w_n=11{,}250$ m/sec^2; $w_t=10$ m/sec^2.

204. The angle of inclination between the total acceleration of a point on the rim of a flywheel and the radius of the latter is 60° At a given instant the tangential acceleration of the point is $w_t=10\sqrt{3}$ m/sec^2. If the radius R of the flywheel is 1 m, find the normal acceleration of the point which is at a distance $r=0.5$ m from the axis of rotation.

Ans. $w_n=5$ m/sec^2.

205. A thread with a load P at one end runs over a wheel mounted on a horizontal axle. At a particular instant the load is allowed to fall from rest with a constant acceleration w_0 thus setting the wheel in motion. Find the total acceleration of the points on the rim of the wheel as a function of the height h through which the load has fallen. Take R as the radius of the wheel.

Ans. $w=\frac{w_0}{R}\sqrt{R^2+4h^2}$.

206. A small ball A suspended from a thread $l=398$ cm long oscillates in the vertical plane about a fixed horizontal axis O in accordance with the equation

$$\varphi=\frac{\pi}{8}\sin\frac{\pi}{2}t$$

(φ is measured in radians and t in seconds). Determine

(1) the first time after starting when the normal acceleration of the ball is zero;

(2) the first time when the tangential acceleration of the ball is zero;

(3) the total acceleration of the ball when $t=\frac{1}{2}$ sec.

Ans. (1) $t=1$ sec; (2) $t=2$ sec; (3) $w=282.95$ cm/sec^2.

14. Conversion of Simplest Motions of a Rigid Body

207. A gear change consists of four gears with appropriate number of teeth: $z_1=10$, $z_2=60$, $z_3=12$, $z_4=70$ (Fig. 150). The function of the gear change is to slow down the rotation of a shaft I and to transfer torque to a shaft II. Determine the reduction ratio of the mechanism.

Ans. $k=35$.

208. A generator with a pulley A is set in motion from rest by means of an endless belt wrapped around a pulley B of the steam engine (Fig. 151). The radii of pulleys are $r_1=75$ cm, $r_2=30$ cm, respectively. After putting the steam engine into operation its angular acceleration equals 0.4π sec^{-2}. Determine the time required for the generator to attain the velocity of 300 rpm. Neglect any sliding between the belt and the pulley.

Ans. 10 sec.

209. Gears *1*, *2*, *3*, *4* and *5* of a rack-and-gear jack are set in motion by a handle A, as shown in Fig. 152. The gears in turn translate the torque to the rack B

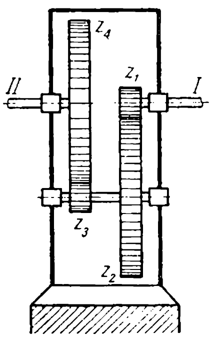


Fig. 150

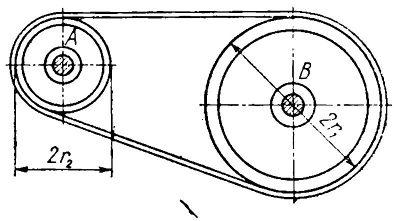


Fig. 151

of the jack. Determine the velocity of the rack if the handle A makes 30 rpm. The number of teeth of the gears are: $z_1=6$, $z_2=24$, $z_3=8$, $z_4=32$. The radius of the fifth gear $r_5=4$ cm.

Ans. $v_B=7.8$ mm/sec.

210. Two identical elliptical gears are used to produce angular velocities which vary periodically (Fig. 153). One gear rotates uniformly about the axle O making 270 rpm, and the other one is

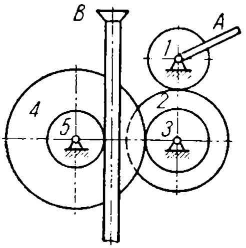


Fig. 152

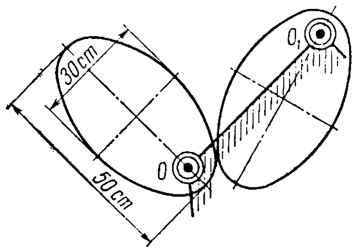


Fig. 153

driven about the axle O_1 by the first gear. The axles O and O_1 are parallel and pass through the foci of the ellipses. The distance OO_1 is 50 cm. The semi-axes of the ellipses are 25 cm and 15 cm, respectively. Determine the minimum and maximum angular velocities of the gear O_1.

Ans. $\omega_{min}=\pi$ sec^{-1}; $\omega_{max}=81\pi$ sec^{-1}.

211. Deduce the law for the driving torque of a pair of elliptical gears with semi-axles a and b (Fig. 154). The angular velocity of the gear I is $\omega_1=$const. The distance between the axles O_1O_2 is $2a$; φ is the angle between the straight line connecting the axis of rotation and the axis N_1M_1 of the elliptic gear I. Both axes pass through the foci of the ellipses.

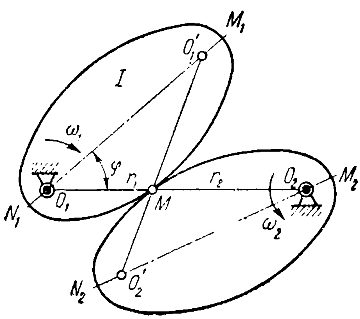


Fig. 154

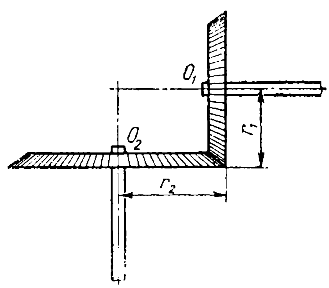


Fig. 155

Ans. $\omega_2 = \frac{a^2-c^2}{a^2-2ac\cos\varphi+c^2}\omega_1$, where c is the eccentricity of the ellipses: $c=\sqrt{a^2-b^2}$.

212. A bevel gear O_1 of radius $r_1=10$ cm is set in rotation from rest by a similar gear O_2 of radius $r_2=15$ cm (Fig. 155). The latter rotates with uniform angular acceleration of 2 rps². Calculate the time required for the bevel gear O_1 to attain an angular velocity corresponding to $n_1=4320$ rpm.

Ans. $t=24$ sec.

213. A driving shaft I of a friction transmission makes 600 rpm and, while in operation, it moves so that the distance varies with time as $d=(10-0.5t)$ cm (t is calculated in seconds). The direction of motion is shown by the arrow (Fig. 156).

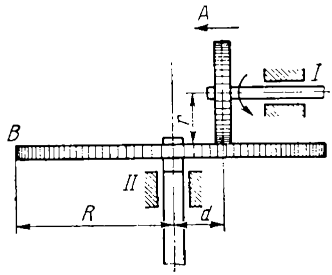


Fig. 156

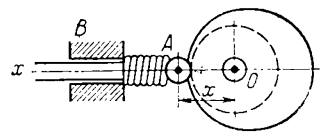


Fig. 157

Determine: (1) the angular acceleration of the shaft II as a function of distance d; (2) the total acceleration of a point on the rim of the wheel B at the instant when $d=r$ The radii of the friction wheels are: $r=5$ cm, $R=15$ cm.

Ans. (1) $\varepsilon=\frac{50\pi}{d^2}$ sec^{-2};

(2) $w=30\pi\sqrt{40{,}000\pi^2+1}$ cm/sec².

214. Find the equation of the contour of a cam which communicates a reciprocal motion to a bar AB (Fig. 157) The motion of the bar is given by the equation: $x=5t+30$ (x is measured in centimetres, and t in seconds). The cam makes 7.5 rpm.

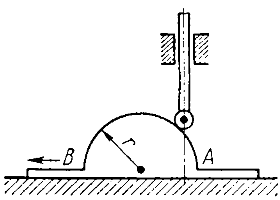


Fig. 158

Ans. $r=\frac{20}{\pi}\varphi+30$, the spiral of Archimedes.

215. A rod rests with one end on the circular contour of a cam of radius $r=30$ cm (Fig. 158). The cam performs reciprocal motion with the velocity of $v=5$ cm/sec. The sinking time of the rod is

$t=3$ sec. If at the initial moment the rod occupies its highest position, find the distance travelled by the rod.

Ans. $h=4.020$ cm.

216. Find the acceleration of a circular cam if, when it is uniformly accelerated from rest, the rod takes 4 seconds to sink from its highest point $h=4$ cm. The radius r of the circular contour of the cam is 10 cm. (See Fig. 158.)

Ans. $w=1$ cm/sec^2.

V. COMPOSITION AND RESOLUTION OF MOTIONS OF A PARTICLE

15. Equations of Motion and Path of the Resultant Motion of a Particle

217. A chart of a vibration recorder moves in the direction Ox with velocity 2 m/sec. The pen which oscillates along the axis Oy traces a sinusoidal line with the length $O_1C=8$ cm and the maximum ordinate $AB=2.5$ cm (Fig. 159). Assuming that the point O_1 of the sinusoidal line corresponds to the position of the pen when $t=O$, find the equation of this recording motion.

Ans. $y=2.5 \sin (50\pi t)$ cm.

218. A tram runs uniformly along a straight horizontal track with a speed $v=18$ km/h. The body of a tram, which is mounted on springs, performs harmonic vibrations of amplitude $a=0.8$ cm and period $T=0.5$ sec. Find the equation of the path of the centre of gravity of the tram body if the mean distance between the centre of gravity and the track equals $h=1.5$ m. When $t=0$, the centre of gravity is at the middle-point and the velocity of vibration is directed upwards. The axis Ox is directed horizontally along the track in the direction of motion and the axis Oy vertically upwards through the centre of gravity at $t=0$.

Ans. $y=1.5+0.008 \sin 0.8\pi x$.

219. Determine the equation of the path of a double pendulum bob which simultaneously makes two interperpen-

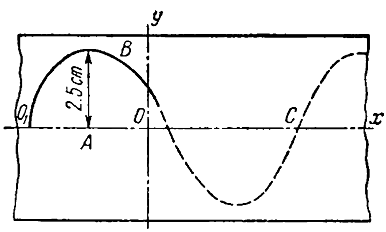


Fig. 159

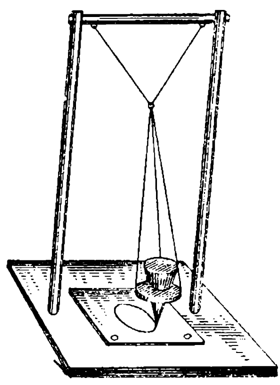


Fig. 160

dicular harmonic vibrations of equal frequency but different amplitudes and phases (Fig. 160). These vibrations are defined by the equations: $x=a\sin(\omega t+\alpha)$, $y=b\sin(\omega t+\beta)$.

Ans. The ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{2xy}{ab}\cos(\alpha-\beta)=\sin^2(\alpha-\beta)$.

220. In some measuring and graduating devices a differential screw, represented in Fig. 161, is used for shifting a pointer. This screw system consists of an axle AB. The portion at A has a screw thread with the pitch h_1 mm and the portion at B has a thread with the pitch $h_2<h_1$. The portion A rotates in a fixed nut C. A part D which envelops the section B does not rotate and is joined to a pointer which slides along the fixed scale.

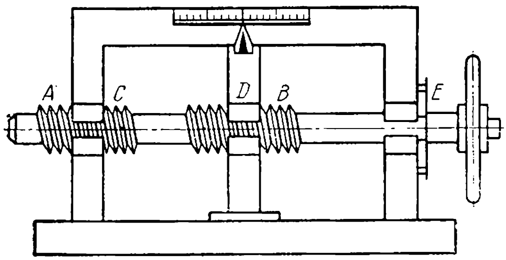


Fig. 161

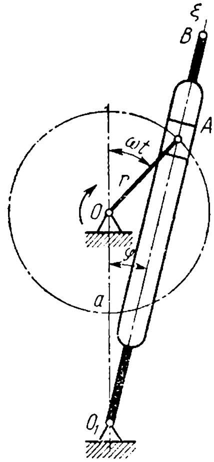


Fig. 162

(1) Determine the displacement of the pointer when the flywheel of the axle makes $1/n$ revolution (the corresponding scale is marked on a disk E); $n=200$, $h_1=0.5$ mm, and $h_2=0.4$ mm. Both threads are either right-handed or left-handed.

(2) What will be the reading on the scale if a left-hand thread is made on the portion A and a right-hand thread on the portion B?

Ans. (1) $s=\frac{1}{n}(h_1-h_2)=0.0005$ mm.

(2) $s=\frac{1}{n}(h_1+h_2)=0.0045$ mm.

221. A mechanism for accelerating a shaper consists of two parallel shafts O and O_1, a crank OA and a rocker O_1B (Fig. 162). The end of the crank OA of length r is hinged to a slider which moves in a slot on the rocker O_1B. If the crank rotates with constant angular velocity ω, derive the equation of the relative motion of

the slider in the slot and the equation of rotation of the rocker itself. The distance between the axes of the shafts $OO_1=a$.

Ans. $\xi=\sqrt{a^2+r^2+2ar\cos\omega t}$; $\tan\varphi=\dfrac{r\sin\omega t}{a+r\cos\omega t}$.

16. Composition of Velocities of a Particle

222. In order to compute the absolute speed of an aircraft in windy weather a straight line of length l is marked on the ground. The ends of the line are clearly seen from above and its direction coincides with that of the wind. At first the aircraft flies along this line down the wind for t_1 sec and then it flies against the wind for t_2 sec. Determine the absolute speed v of the aircraft and the velocity V of the wind.

Ans. $v=\dfrac{1}{2}\left(\dfrac{1}{t_1}+\dfrac{1}{t_2}\right)$ m/sec $=1.8\,l\left(\dfrac{1}{t_1}+\dfrac{1}{t_2}\right)$ km/h;

$V=\dfrac{l}{2}\left(\dfrac{1}{t_1}-\dfrac{1}{t_2}\right)$ m/sec.

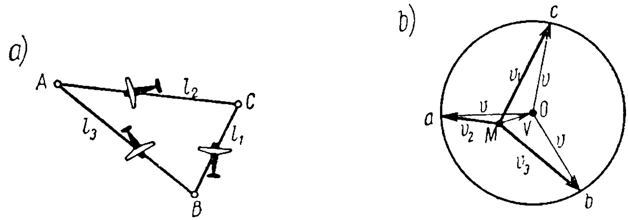


Fig. 163

223. In order to determine the absolute speed v of an aircraft in windy weather a triangular polygon ABC with sides $BC=l_1$, $CA=l_2$, $AB=l_3$ metres is marked on the ground. Each side of the polygon has a determined time of flight t_1, t_2, t_3 sec. Determine the absolute speed v of the aircraft, assuming that its magnitude is constant and the velocity of the wind is V Solve this problem using a graphical method of solution (Fig. 163).

Hint. The speed relative to the wind is called the absolute speed of the aircraft.

Ans. From an arbitrary point M draw three equal vectors $\dfrac{l_1}{t_1}$, $\dfrac{l_2}{t_2}$, $\dfrac{l_3}{t_3}$ parallel to the polygon sides BC, CA, and AB, respectively. The magnitude of velocity v is defined by the radius of a circle drawn through the end points of these vectors. The velocity of wind is defined by the vector $\overline{MO}$.

224. A car runs at 72-km/h speed along the horizontal highway. Falling raindrops trace strips on the side windows of the car at an angle of 40° to the vertical. Determine the absolute velocity v of the falling raindrops.

Ans. $v=23.8$ m/sec.

225. A ship heads due south with the speed of $30\sqrt{2}$ km/h. A second ship sets course in a south-easterly direction with the speed of 30 km/h. Find the magnitude and direction of the velocity of the second ship as seen from an observation post which is located on the deck of the first ship.

Ans. $v_r=30$ km/h and it is headed due north-east.

226. A cam of semi-disk shape with radius r performs a translatory motion. It moves with constant velocity v_0 in the direction of the diameter AB. Determine the velocity of the rod which rests on the cam and slides freely along the groove of the holder. The axis of the latter is perpendicular to the diameter of the cam. A roller of radius ρ is attached to the end point of the rod. Initially the rod is at its highest point. (See Fig. 158.)

Ans. $$v=\frac{v_0^2 t}{\sqrt{(r+\rho)^2-v_0^2t^2}}.$$

227. In a hydraulic turbine the water passes from the guide apparatus to the turbine wheel. To make the flow steady the turbine wheel blades are set in such a way that relative velocity v_r of the water is directed tangentially to the blades (Fig. 164). If the absolute velocity of a drop of water at the inlet is $v=15$ m/sec, find its velocity relative to the outside rim of the wheel. The angle between the absolute velocity and the radius is $\alpha=60°$ $R=2$ m is the radius of the wheel and its angular velocity corresponds to $n=30$ rpm.

Ans. $v_r=10.06$ m/sec, $(\widehat{v_r, R})=41°50'$

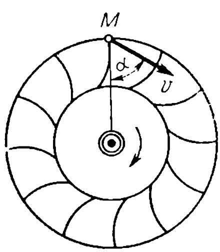


Fig. 164

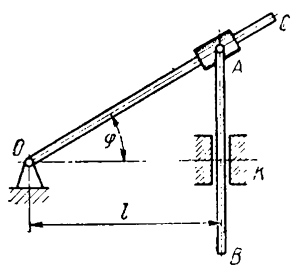


Fig. 165

228. A cylinder of diameter $d=80$ mm is being turned on a lathe. The spindle makes $n=30$ rpm. The velocity of a longitudinal feed is $v=0.2$ mm/sec. Determine the relative velocity of the turning tool v_r relative to the machined part.

Ans. $v_r=125.7$ mm/sec; $\tan\alpha=628$, where α is the angle between v_r and the axis of the turning tool.

229. A crank OC of a rocker mechanism rotates about the axis O which is perpendicular to the plane of the sketch (Fig. 165). A slider A moving along the crank OC sets a rod AB in motion and the latter is constrained to move along the vertical guide K. $OK=l$. Determine the velocity of the slider A relative to the crank OC as a function of the angular velocity ω. Find also the angle of rotation of the crank φ.

Ans. $v_r=\dfrac{l\omega\tan\varphi}{\cos\varphi}$.

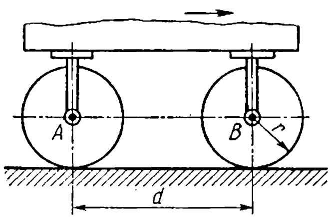


Fig. 166

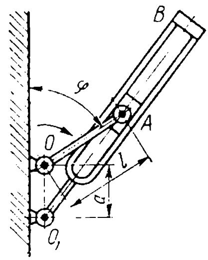


Fig. 167

230. A railway car runs along a straight railway track with a velocity v. Its wheels A and B are rolling without sliding (Fig. 166). The radii of both wheels equal r, and d is the distance between axes. Determine the velocity of the centre of the wheel A with respect to the coordinate system connected to the wheel B.

Ans. The velocity equals $\dfrac{vd}{r}$, is perpendicular to AB and directed downwards.

231. A mechanism consists of two parallel shafts O and O_1, a crank OA and a rocker O_1B (Fig. 167). The end A of the crank OA slides along a groove in the rocker O_1B. The distance between the axes of the shafts OO_1 equals a, the length of the crank OA is l, where $l>a$.

The shaft O rotates with constant angular velocity ω. Find: (1) the angular velocity ω_1 of the shaft O_1 and the relative velocity of the particle A with respect to the rocker O_1B in terms of a variable $O_1A=s$; (2) the maximum and minimum magnitudes of these velocities; (3) the positions of the crank, when $\omega_1=\omega$.

Ans. (1) $\omega_1 = \frac{\omega}{2}\left(1+\frac{l^2-a^2}{s^2}\right)$;

$v_r = \frac{\omega}{2s}\sqrt{(l+s+a)\,(l+s-a)\,(a+l-s)\,(a+s-l)}$;

(2) $\omega_{1\max} = \omega\frac{l}{l-a}$; $\omega_{1\min} = \omega\frac{l}{l+a}$;

$v_{r\max} = a\omega$; $v_{r\min} = 0$;

(3) $\omega_1 = \omega$, when $O_1B \perp O_1O$.

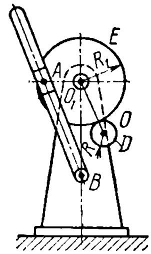


Fig. 168

232. The slide block A of a swinging rocker of a shaper is set in motion by a gear drive (Fig. 168). The latter consists of a gear D and a gear E with a pin serving as the axle of the slide block A. The radii of the gears are: $R=100$ mm, $R_1=350$ mm, $O_1A=300$ mm. The distance O_1B between the axis O_1 of the gear E and the centre of rotation B of the rocker O_1B is 700 mm. If the gears have an angular velocity $\omega=7$ sec^{-1}, determine the angular velocity of the slide block at the instant when O_1A is either vertical (top and bottom positions) or perpendicular to the slide block AB (left and right positions). The points O_1 and B are located on the same vertical line.

Ans. $\omega_I = 0.6$ sec^{-1}; $\omega_{II} = \omega_{IV} = 0$; $\omega_{III} = 1.5$ sec^{-1}.

17. Composition of Accelerations of a Particle Undergoing Translatory Motion of Transport

233. A crank-rocker mechanism of a power hammer consists of a straight rocker which performs a reciprocating motion (Fig. 169). The rocker is set in motion by a slide block A which is connected to the end of a crank $OA=r=40$ cm. The latter rotates uniformly with angular velocity corresponding to $n=120$ rpm. When $t=0$ the rocker is at its lower position. Find the acceleration of the rocker.

Ans. $w = 6320 \cos 4\pi t$ cm/sec^2.

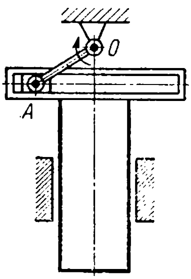


Fig. 169

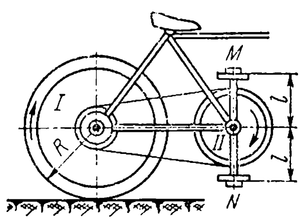


Fig. 170

234. A cyclist rides along a horizontal straight road. The motion of the bicycle (Fig. 170) is defined by the equation: $s=0.1\,t^2$ (s is measured in metres and t in seconds) The following data are given: $R=350$ mm, $l=180$ mm, $z_1=18$ teeth, $z_2=48$ teeth. Determine an absolute acceleration of the pedal axes M and N, assuming that the wheels run without sliding, when $t=10$ sec, if at this instant the position of the crank MN is vertical.

Ans. $w_M=0.860$ m/sec^2; $w_N=0.841$ m/sec^2.

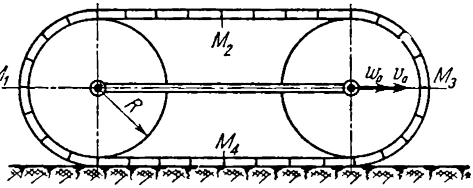


Fig. 171

235. A caterpillar tractor moves without sliding along a straight track with velocity v_0 and acceleration w_0. The radii of its wheels are R (Fig. 171). Neglecting any sliding of the caterpillar over the rims of the wheels, find the velocities and accelerations of points M_1, M_2, M_3 and M_4 on the caterpillar.

Ans. $v_1=v_3=v_0\sqrt{2}$; $v_2=2v_0$; $v_4=0$;

$$w_1=\sqrt{w_0^2+\left(w_0+\frac{v_0^2}{R}\right)^2};\quad w_2=2w_0;$$

$$w_3=\sqrt{w_0^2+\left(w_0-\frac{v_0^2}{R}\right)^2}\quad w_4=0.$$

236. A gear of radius $R=0.5$ m is gripped between two parallel racks sliding in the same direction as the accelerations $w_1=1.5$ m/sec^2 and $w_2=2.5$ m/sec^2 (Fig. 172). Find the linear acceleration w_0 of the centre of the gear O and its angular acceleration. The translatory motion of the system of axes $O\xi\eta$ fixed to the centre of the gear is assumed to be transport.

Ans. $\varepsilon=1$ sec^{-2}; $w_0=2$ m/sec^2.

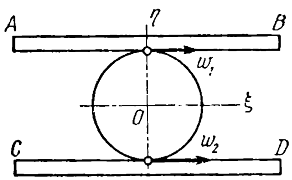


Fig. 172

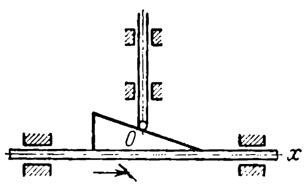


Fig. 173

237. A cam which performs a translatory motion is shaped like a triangle with its driving face inclined at an angle α to the axis Ox (Fig. 173). If the cam moves with a constant acceleration w_0, determine the acceleration of the rod which rests on the cam surface and slides freely in the bearings. The rod is perpendicular to the axis Ox.

Ans. $w = w_0 \tan \alpha$.

238. A cam which performs a translatory motion is in the form of a semi-circle. It slides with constant velocity v_0 in the direction of its diameter AB. Determine the acceleration of the rod which rests on the cam surface perpendicular to its diameter AB and slides freely in the slot of the holder. ρ is a radius of the roller attached at the end of the rod. Initially the rod is at its highest position (see Fig. 158).

Ans. $w = \dfrac{v_0^2 (r+\rho)^2}{\left[(r+\rho)^2 - v_0^2 t^2\right]^{3/2}}$.

239. A trolley runs along a horizontal track with an acceleration $w = 49.2$ cm/sec^2. An electric motor is installed on the trolley. When put into operation its rotor rotates in accordance with the equation $\varphi = t^2$. The angle φ is measured in radians. The radius of the rotor is 20 cm. Determine the absolute acceleration of a point A on the rim of the rotor when $t = 1$ sec, if at this particular instant the point A occupies the position, shown in Fig. 174.

Ans. $w_A = 74.6$ cm/sec^2, it is directed vertically upwards.

240. The data are the same as in Problem 239. Determine the angular velocity of the rotor when the point A, being in the position B, has absolute acceleration equal to zero.

Ans. $\omega = 1.57$ sec^{-1}.

241. The rotation of the shaft of an electric motor is given by the equation $\varphi = \omega t$ (ω = const). A rod OA of length l is attached to the shaft at an angle of 90° (Fig. 175). As the electric motor is not fastened down, it makes horizontal harmonic oscillations on

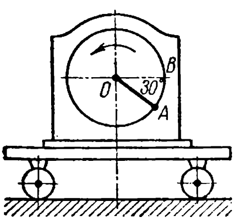


Fig. 174

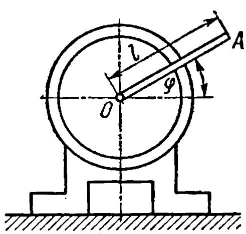


Fig. 175

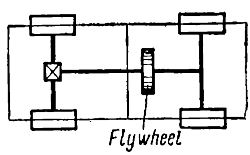


Fig. 176

its base defined by the equation $x=a\sin\omega t$. Determine the absolute acceleration of the point A when $t=\frac{\pi}{2\omega}$ sec.

Ans. $w_A=\omega^2\sqrt{a^2+l^2}$.

242. A car runs along a straight road with the acceleration $w_0=2$ m/sec^2. A rotating flywheel of radius $R=0.25$ m is mounted on the longitudinal shaft (Fig. 176). The instantaneous angular velocity of the flywheel is $\omega=4$ sec^{-1} and its angular acceleration is $\varepsilon=4$ sec^{-2}. Determine the absolute acceleration of the points on the rim of the flywheel at this instant.

Ans. $w=4.58$ m/sec^2.

18. Composition of Accelerations of a Particle Performing Rotational Motion of Transport about a Fixed Axis

243. Two heavy weights A are attached to the ends of a spring of a regulator which rotates about its vertical axis with constant angular velocity according to $n=180$ rpm (Fig. 177). Both weights make harmonic oscillations along the slot MN in such a way that the distance between their centres of gravity and the axis of rotation varies with the law $x=(10+5\sin 8\pi t)$ cm. Determine the acceleration of the centre of gravity of the weight at the instant when the Coriolis acceleration is a maximum. Determine the values of the Coriolis acceleration when the weights are at their extreme positions.

Ans. $w_a=600\pi^2$ cm/sec^2; $w_c=0$.

244. A circular tube of radius $R=1$ m rotates in the clockwise direction about a horizontal axis O with a constant angular velocity $\omega=1$ sec^{-1} In the tube close to a point A a small ball M performs oscillations in such a way that the angle $\varphi=\sin\pi t$ (Fig. 178). Determine the tangential w_t and normal w_n accelerations of the ball, when $t=2\frac{1}{6}$ sec.

Ans. $w_t=-4.93$ m/sec^2; $w_n=13.84$ m/sec^2.

245. An inverted elliptical divider with a fixed crank OO_1 is used as a coupling to transmit torque from a shaft to a parallel one (Fig. 179) The crank AB rotates with angular velocity ω_1 about the axis O_1 and sets a cross-piece and the second shaft in rotation about the axle O. Determine the angular velocity of the cross-piece and the transport and relative (with respect to the cross-piece) velocities and the accelerations (transport, relative and Coriolis) of a point A on the slider when $\omega_1=$const and if $OO_1=AO_1=O_1B=a$.

Ans. $\omega=\frac{\omega_1}{2}$; $\quad v_e=a\omega_1 \sin\frac{\omega_1}{2}t$; $\quad v_r=a\omega_1\cos\frac{\omega_1}{2}t$;

$$w_e=w_r=\frac{a\omega_1^2}{2}\sin\frac{\omega_1}{2}t;\quad w_c=a\omega_1^2\cos\frac{\omega_1}{2}t.$$

246. A compressor with straight air ducts rotates uniformly with angular velocity ω about its axis O which is perpendicular to the plane of the sketch (Fig. 180). The air passes through ducts

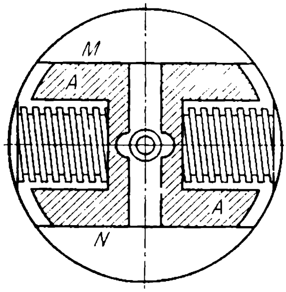


Fig. 177

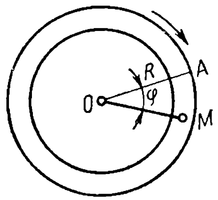


Fig. 178

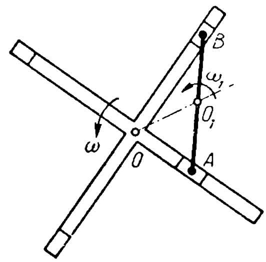


Fig. 179

with constant relative velocity v_r. Find the projections on the coordinate axes of the absolute velocity and acceleration of an air particle which is located at C in the duct AB. The following numerical data are given: the duct AB is inclined at 45° to the radius OC; $OC=0.5$ m; $\omega=4\pi$ sec^{-1}; $v_r=2$ m/sec.

Ans. $v_\xi=7.7$ m/sec; $v_\eta=1.414$ m/sec;
$w_\xi=35.54$ m/sec^2; $w_\eta=-114.5$ m/sec^2.

247. Solve the preceding problem assuming that the air ducts are curvilinear and the radius of curvature of the duct at C equals ρ (Fig. 181). The angle between the normal to the curve AB at C and the radius CO is φ. Radius $CO=r$

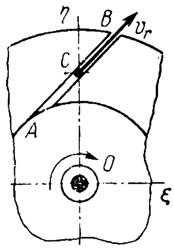


Fig. 180

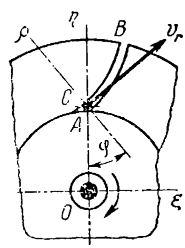


Fig. 181

Ans. $v_\xi = v_r \cos\varphi + r\omega$; $\quad v_\eta = v_r \sin\varphi$; $\quad w_\xi = \left(2v_r\omega - \frac{v_r^2}{\rho}\right)\sin\varphi$;

$$w_\eta = -\left[r\omega^2 + \left(2v_r\omega - \frac{v_r^2}{\rho}\right)\cos\varphi\right].$$

248. Express as a function of time the angular acceleration ε of a swinging rocker in a shaper if a crank of length r performs uniform rotations of angular velocity ω. The distance between the axes of rotation of the crank and the rocker is $a>r$ (See Fig. 162.)

Ans. $\varepsilon = \dfrac{(r^2-a^2)ar\omega^2 \sin\omega t}{(a^2+r^2+2ar\cos\omega t)^2}$.

249. A slide block A performs transport motions together with a rocker which swings with angular velocity ω and angular acceleration ε about the axis O_1 perpendicular to the plane of the rocker. The slide block also performs relative rectilinear motions along the slot on the rocker with the velocity v_r and acceleration w_r. Determine the projections of the absolute acceleration of the slide block on the mobile axes of coordinates connected with the rocker. Denote the projection as a function of the variable distance $O_1A=s$. (See Fig. 167.)

Ans. $w_\xi = w_r - s\omega^2$; $w_\eta = s\varepsilon + 2v_r\omega$, when axes ξ and η are directed along the slot and perpendicular to it respectively.

250. Determine the angular acceleration of a swinging rocker in a crank-rocker mechanism of a shaper at two vertical and two horizontal positions of the crank. The length of the crank is $l=40$ cm, the distance between the axes of the crank and the rocker is $a=30$ cm, the angular velocity of the uniformly swinging crank is $\omega=3$ sec^{-1}. (See Fig. 167.)

Ans. $\varphi=0$ and $\varphi=180°$, $\varepsilon=0$; $\quad \varphi=90°$, $\varepsilon=1.21$ sec^{-2};

$\varphi=270°$, $\varepsilon=1.21$ sec^{-2} (rotation retarded).

251. Using the data of the previous problem, find the acceleration of the relative motion of the slide block of the rocker along its slot. The four positions of the crank remain the same as before.

Ans. $\varphi=0$, $w_r=154.3$ cm/sec^2;

$\varphi=90°$ and $\varphi=270°$, $w_r=103.7$ cm/sec^2;

$\varphi=180°$, $w_r=1080$ cm/sec^2.

252. A cylindrical piece with diameter 80 mm is being turned on a lathe. The spindle makes 30 rpm. The longitudinal feed is 0.2 mm/sec. Determine the relative velocity and relative acceleration of the turning tool with respect to the machined piece, the

transport and Coriolis accelerations of the tool. Referring to the theorem on the composition of accelerations, prove that absolute acceleration of the tool is 0.

Ans. $v_r = 125.7$ mm/sec; $w = 789.5$ mm/sec^2;
$w_r = w_c = 394.8$ mm/sec^2.

253. A disk rotates about the axis O_1O_2 with angular velocity $\omega = 2t$ sec^{-1}. A point M moves towards the rim of the disk according to the law $OM = 4t^2$ cm (Fig. 182). The radius OM

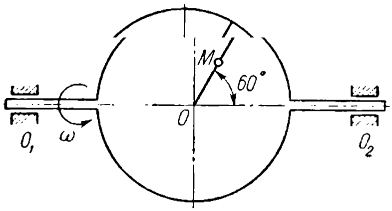


Fig. 182

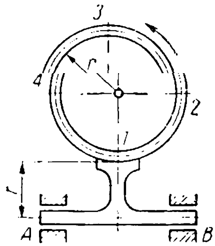


Fig. 183

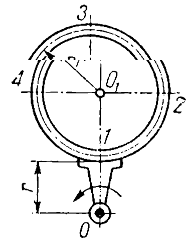


Fig. 184

makes an angle of 60° with the axis O_1O_2. Determine the magnitude of the absolute acceleration of the point M at the instant when $t = 1$ sec.

Ans. $w_M = 35.56$ cm/sec^2.

254. A hollow ring of radius r is rigidly fixed to a shaft AB in such a way that the shaft axis is located on the plane of the axis of the ring, as shown in Fig. 183. The ring contains fluid moving in the clockwise direction with constant relative velocity u. The shaft AB rotates in a clockwise direction, if we consider the axis of rotation from A to B. ω is the constant angular velocity of the shaft. Determine the magnitudes of the absolute accelerations of particles of the fluid at points *1*, *2*, *3* and *4*.

Ans. $w_1 = r\omega^2 - \frac{u^2}{r}$; $w_3 = 3r\omega^2 + \frac{u^2}{r}$; $w_2 = w_4 = 2r\omega^2 + \frac{u^2}{r}$.

255. The problem is the same as the previous one with the exception that the plane of the ring is perpendicular to the axis of the shaft AB (Fig. 184). Determine the magnitudes of the absolute accelerations when: (1) the transport and relative motions are in the same direction; (2) the components of motion are in opposite directions.

Ans. (1) $w_1 = r\omega^2 - \frac{u^2}{r} - 2u\omega$; $w_3 = 3r\omega^2 + \frac{u^2}{r} + 2\omega u$;

$$w_2 = w_4 = \sqrt{\left(\frac{u^2}{r} + 2\omega u + \omega^2 r\right)^2 + 4\omega^4 r^2};$$

(2) $w_1 = r\omega^2 - \frac{u^2}{r} + 2u\omega;\quad w_3 = 3r\omega^2 + \frac{u^2}{r} - 2\omega u;$

$$w_2 = w_4 = \sqrt{\left(\omega^2 r + \frac{u^2}{r} - 2\omega u\right)^2 + 4\omega^4 r^2}.$$

256. A particle M moves uniformly along the generatrix of a circular cone with the axis OA from its vertex to the base with a relative velocity v_r (Fig. 185). Angle $MOA = \alpha$. At the instant $t=0$ the distance $OM_0 = a$. The cone performs uniform rotations about its axis OA with angular velocity ω. Find the absolute acceleration of the particle M.

Ans. The vector of acceleration is located on the plane perpendicular to the axis of rotation and forms the hypotenuse of a triangle with sides: $w_{en} = \omega^2(a + v_r t)\sin\alpha$ and $w_c = 2v_r\omega\sin\alpha$.

257. Referring to the previous problem, find the magnitude of the absolute acceleration of the particle M at

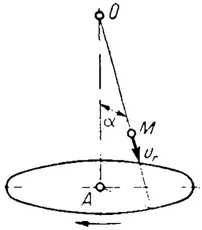


Fig. 185

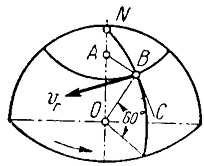


Fig. 186

the time $t = 1$ sec, if it moves along the generatrix of the cone with a constant relative acceleration w_r directed from the vertex of the cone to its base. Assume that the following numerical data are given: $\alpha = 30°$, $a = 15$ cm, $w_r = 10$ cm/sec^2, $\omega = 1$ sec^{-1}. When $t = 0$, the relative velocity of the particle v_r is zero.

Ans. $w = 14.14$ cm/sec^2.

258. A locomotive moves due east along a railway track running in the direction of the parallel of latitude North. The speed of the locomotive is $v_r = 20$ m/sec. Find the Coriolis acceleration w_c of the locomotive.

Ans. $w_c = 0.291$ cm/sec^2.

259. The river Neva flows from east to west along the parallel of latitude 60° North (Fig. 186). It flows with a velocity $v_r = 4$ km/h. Determine the resultant of the projections on to the tangent BC of those component accelerations of the water particles which depend on the speed of flow. Also find the components of the absolute acceleration of the water particles. $R = 64 \times 10^5$ m is the radius of the Earth.

Ans. $w_{BC} = 1396 \times 10^{-5}$ cm/sec^2; $w_e = 1692 \times 10^{-3}$ cm/sec^2;
$w_r = 386 \times 10^{-7}$ cm/sec^2; $w_c = 1616 \times 10^{-5}$ cm/sec^2.

19. Equations of Motion of a Body and Its Particles in a Plane

260. The rule of the ellipsograph is set in motion by a crank OC which rotates with constant angular velocity ω_0 about the axis O (Fig. 187).

Taking the slider B as the pole, find the equations of motion of the rule if $OC=BC=AC=r$

Ans. $x_0=2r\cos\omega_0 t$; $y_0=0$; $\varphi=\omega_0 t$.

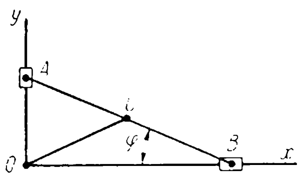


Fig. 187

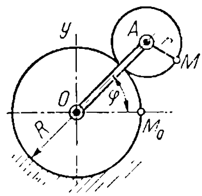


Fig. 188

261. A gear of radius r is in rolling contact with another gear of radius R, as shown in Fig. 188. The first gear is set in motion by the crank OA which rotates with a uniform angular acceleration ε_0 about the axis O of the fixed gear. If for $t=0$ the angular velocity of the crank is $\omega_0=0$ and the initial angle of rotation $\varphi_0=0$ derive the equations of motion of the rolling gear taking its centre A as the pole.

Ans. $x_0=(R+r)\cos\frac{\varepsilon_0 t^2}{2}$; $y_0=(R+r)\sin\frac{\varepsilon_0 t^2}{2}$;

$$\varphi_1=\left(\frac{R}{r}+1\right)\frac{\varepsilon_0 t^2}{2},$$

where φ_1 is the angle of rotation of the rolling gear.

262. A gear II of radius $r_2=12$ cm rolls without sliding inside a fixed gear I of radius $r_1=20$ cm (Fig. 189). Derive the equations of motion of a point M on the gear II with respect to axes O_1x and O_1y passing through the centre of the fixed gear Initially, when $\varphi=0$, the point M was on the axis O_1y being the point of contact of the gears. The rod O_1O_2 which drives the gear II makes $n=270$ rpm.

Ans. $x_M=8\sin 9\pi t-12\sin 6\pi t$; $y_M=8\cos 9\pi t+12\cos 6\pi t$.

263. A gear *II* of radius $r_2=16$ cm rolls without sliding on the outside surface of a fixed gear *I* of radius $r_1=12$ cm (Fig. 190). Obtain the equations of motion of a point M on the gear *II* with respect to axes O_1x and O_1y passing through the centre of the fixed gear. Initially, when $\varphi=0$, the point M was on the axis O_1y being the point of contact of the gears. The crank O_1O_2 makes $n=240$ rpm.

Ans. $x_M=28\sin 8\pi t-16\sin 14\pi t$; $\quad y_M=28\cos 8\pi t-16\cos 14\pi t$.

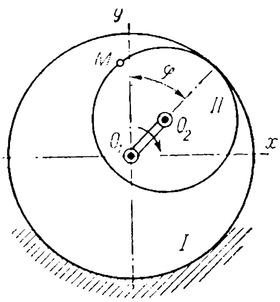


Fig. 189

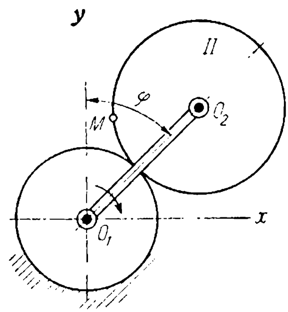


Fig. 190

264. A rod AB, without any initial velocity, falls under the action of gravity and rotates about the centre of gravity C with angular velocity $\omega=\text{const}$. $AC=BC=l$. Find the equation of motion of the point B, if initially the rod AB was in a horizontal position and the point B was on the right-hand side. The acceleration due to gravity is g; the initial position of the point C is assumed to be the origin; the axis Oy is directed vertically upwards and the axis Ox horizontally to the right.

Ans. $x=l\cos\omega t$; $\quad y=\dfrac{gt^2}{2}+l\sin\omega t$.

265. A conchoidograph consists of a rule AB which is hinged at A with a slider moving along the straight slot DE. The rule passes through a pivoted pipe which freely rotates about a fixed axle N. The distance between axes N and Ox equals a. If the distance $AM_1=a$ and $AM_2=\dfrac{a}{2}$, determine the equations of the curves described by points M_1 and M_2 on the rule AB. The coordinate axes are shown in Fig. 191.

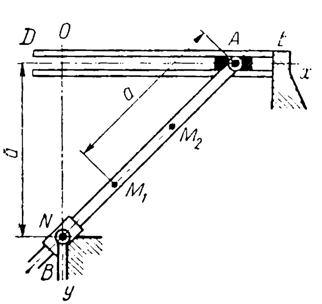


Fig. 191

Ans. (1) $x^2y^2=(a-y)^2(a^2-y^2)$;
(2) $4x^2y^2=(a-y)^2(a^2-4y^2)$.

20. Velocity of a Point of a Body Which Performs Motions in a Plane. Instantaneous Centres of Velocities

266. A rod AB of length 1 m is let go from rest leaning with its ends against two perpendicular straight lines Ox and Oy (Fig. 192). Find the coordinates x and y of the instantaneous centre of velocities at the instant when the angle $OAB = 60°$

Ans. $x = 0.866$ m; $y = 0.5$ m.

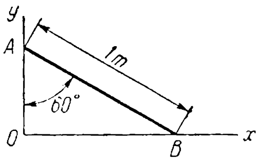


Fig. 192

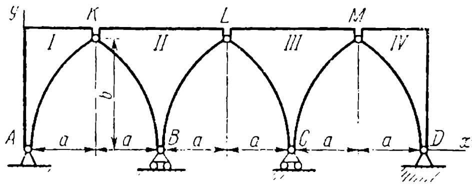


Fig. 193

267. A bridge consists of four parts connected by hinges K, L and M. The supports A and D are held stationary while the supports B and C are fixed on rollers. If at the beginning of deformation the support D is subjected to a horizontal displacement, find the positions of the instantaneous centres of velocities of all parts of the bridge. Dimensions of the bridge and axes are shown in Fig. 193.

Ans. $x_{CI} = y_{CI} = 0$; $x_{CII} = 2a$, $y_{CII} = 2b$; $x_{CIII} = 4a$, $y_{CIII} = 0$; $x_{CIV} = 6a$, $y_{CIV} = 2b$.

268. A straight line AB is free to move in the plane of the sketch (Fig. 194). At a certain time the velocity v_A of the particle A makes an angle of 30° with the straight line AB and its magnitude is 180 cm/sec. At this instant the direction of velocity of the particle B coincides with the straight line AB. Determine the velocity of the particle B.

Ans. $v_B = 156$ cm/sec.

269. A straight rod AB moves in the plane of the sketch (Fig. 195). Its end A moves constantly on the semicircumference CAD while at the same time the rod itself slides on the point C of diameter CD. Determine the veloc-

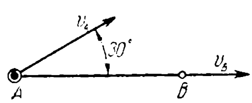


Fig. 194

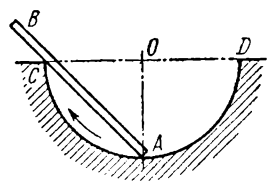


Fig. 195

ity v_c of the point on the rod coinciding with the point C at the time when the radius OA is perpendicular to CD. Assume that at this particular time the velocity of the point A is 4 m/sec.

Ans. $v_C = 2.83$ m/sec.

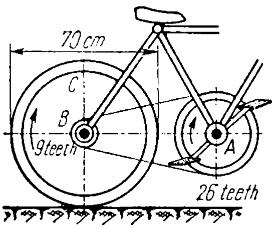


Fig. 196

270. A bicycle transmission consists of a chain passing over a gear A with 26 teeth and a gear B with 9 teeth. The gear B is rigidly connected with the wheel C of diameter 70 cm (Fig. 196) Determine the speed of the bicycle when the gear A makes one revolution per second while the wheel C rolls without sliding along a straight road.

Ans. 22.87 km/h.

271. A wheel of radius $R = 0.5$ m rolls without sliding along a straight road (Fig. 197). The velocity of its centre is constant: $v_0 = 10$ m/sec. Find the velocities of the ends M_1, M_2, M_3 and M_4 of the vertical and horizontal wheel diameters. Determine the angular velocity of the wheel.

Ans. $\omega = 20$ sec^{-1}; $v_1 = 0$; $v_2 = 14.14$ m/sec; $v_3 = 20$ m/sec; $v_4 = 14.14$ m/sec.

272. Two parallel laths move in the same direction with constant velocities $v_1 = 6$ m/sec and $v_2 = 2$ m/sec. A disk with radius $a = 0.5$ is gripped by the laths and rolls between them without sliding (Fig. 198). Find the angular velocity of the disk and the velocity of its centre.

Ans. $\omega = 4$ sec^{-1}; $v_0 = 4$ m/sec.

273. A crank OA rotates with angular velocity $\omega_0 = 2.5$ sec^{-1} about an axle O of a fixed gear of radius $r_2 = 15$ cm. The crank drives a gear A of radius $r_1 = 5$ cm which is mounted on the crank end (Fig. 199). Find the magnitude and direction of velocities of the points A, B, C, D and E on the movable gear, if $CE \perp BD$.

Ans. $v_A = 50$ cm/sec; $v_B = 0$; $v_D = 100$ cm/sec; $v_C = v_E = 70.7$ cm/sec.

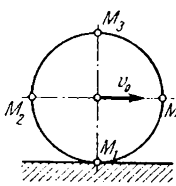


Fig. 197

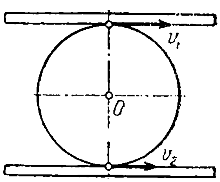


Fig. 198

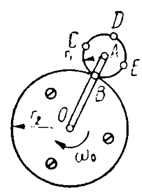


Fig. 199

274. A circle of radius $r_2=9$ cm rolls without sliding on the inner surface of a fixed circle of radius $r_1=18$ cm. A rod AB moving in special guides is pivoted with the rolling circle (Fig. 200). Determine the velocity of the rod AB, when $\varphi=45°$ and the rod O_1O_2 makes $n=180$ rpm.

Ans. $v_B=239.87$ cm/sec.

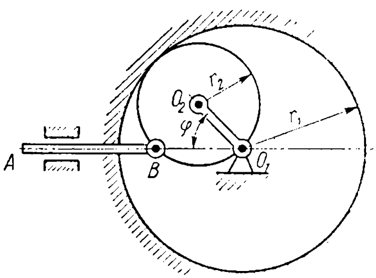


Fig. 200

275. A slider-crank mechanism, shown in Fig. 201, has a crank OA 40 cm long and a connecting rod AB 2 m long. The crank rotates uniformly with an angular velocity corresponding to 180 rpm. Find the angular velocity ω of the connecting rod and the velocity of its centre point M, for four positions of the crank, when the angle AOB equals $0, \frac{\pi}{2}; \pi, \frac{3\pi}{2}$ respectively.

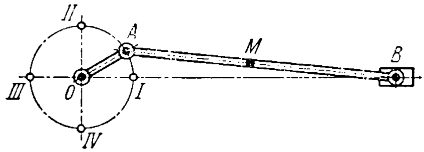


Fig. 201

Ans. *I.* $\omega=-\frac{6}{5}\pi\ \text{sec}^{-1}$; $v=377$ cm/sec;

II. $\omega=0$; $v=754$ cm/sec;

III. $\omega=\frac{6}{5}\pi\ \text{sec}^{-1}$; $v=377$ cm/sec;

IV $\omega=0$; $v=754$ cm/sec.

The minus sign in ω shows that the connecting rod rotates in a direction opposite to that of the crank.

276. Find the velocity of a slider B in an off-centre slider-crank mechanism at two horizontal and two vertical positions of the crank if the crank rotates about a shaft O with angular velocity $\omega=1.5\ \text{sec}^{-1}$ (Fig. 202). Numerical data are as follows: $OA=40$ cm, $AB=200$ cm, $OC=20$ cm.

Ans. $v_1=v_3=6.03$ cm/sec; $v_2=v_4=60$ cm/sec.

277. A rod OB is constrained to rotate about the axle O with a constant angular velocity $\omega=2\ \text{sec}^{-1}$, and so sets the rod AD in motion (Fig. 203). The end A of the rod AD moves along the horizontal axis Ox and the point C along the vertical axis Oy.

Determine the velocity of the point D of the rod when $\varphi=45°$, and find the equation of its path. $AB=OB=BC=CD=12$ cm.

Ans. $v=53.66$ cm/sec; $\left(\frac{x}{12}\right)^2+\left(\frac{y}{36}\right)^2=1.$

278. A linkage shown in Fig. 204 is made up of two rods O_1A and O_2B connected with the rod AB by hinges at A and B. The rods O_1A and O_2B can rotate about fixed points O_1 and O_2. The rods

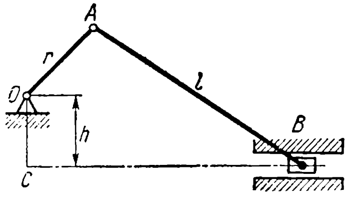


Fig. 202

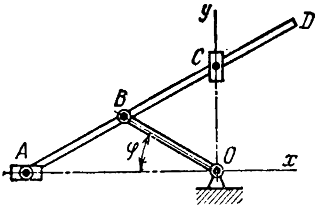


Fig. 203

are in the same plane. The following data are given: the length of the link $O_1A=a$ and its angular velocity is ω. Find, by construction, a point M on AB whose velocity is directed along AB and also determine the magnitude of the velocity v of the point M at the moment when the angle O_1AB equals α.

Ans. $v=a\omega\sin\alpha$.

279. A crossed coupling, shown in Fig. 205, is used to transfer torque from one shaft to a parallel one when a negligible non-coincidence of axes takes place. Two disks A and B are fixed at the end of shafts I and II respectively. Each disk has a diametrical groove a-a and b-b. The disk C with one diametric lug a' and b' at each side is placed between A and B. Each lug fits into a groove of a corresponding disk in such a way that the lug can slide in it freely. The lugs of the disk C are mutually perpendicular.

Prove that the angular velocities of both shafts are equlal and find the instantaneous centre of velocities of the disk C.

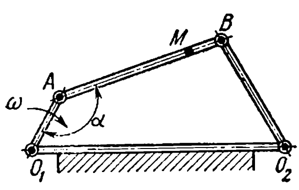


Fig. 204

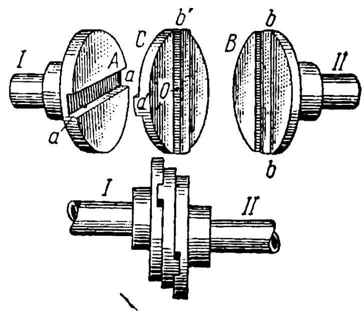


Fig. 205

Ans. The instantaneous centre of velocities of the disk C coincides with the point of intersection of the straight lines which pass through the centres O_1 and O_2 on the axes of the shafts I and II, and are perpendicular to the lugs a' and b' The motion of the disk C is the same as the motion of a right-angled triangle $a'Ob'$ the sides of which constantly pass through the stationary centres O_1 and O_2 (O is centre of the disk C).

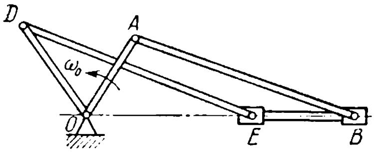


Fig. 206

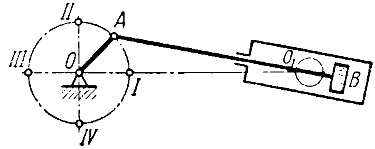


Fig. 207

280. The sliders B and E of a double slider-crank are connected by the rod BE. A driver crank OA and a driven crank OD rocks about the same fixed axis O, perpendicular to the plane of the sketch (Fig. 206). Determine the instantaneous angular velocities of the driven crank OD and the connecting rod DE at the moment when OA is perpendicular to the guide of the sliders. Its instantaneous angular velocity is $\omega_0=12$ sec^{-1}. The numerical data are as follows: $OA=10$ cm; $OD=12$ cm; $AB=26$ cm; $EB=12$ cm; $DE=12\sqrt{3}$ cm.

Ans. $\omega_{OD}=10\sqrt{3}$ sec^{-1}; $\omega_{DE}=\frac{10}{3}\sqrt{3}$ sec^{-1}.

281. In a steam engine with a rocking cylinder the crank OA is 12 cm long (Fig. 207). The distance between the axis of the shaft and the axis of the cylinder journals is $OO_1=60$ cm. A connecting rod AB is 60 cm long. If the angular velocity of the crank is $\omega=5$ sec^{-1}=const, determine the velocity of the piston at four positions of the crank which are shown in Fig. 207

Ans. $v_1=15$ cm/sec, $v_3=10$ cm/sec; $v_2=v_4=58.88$ cm/sec.

282. Determine the velocity of the bar DE of the steam engine at the two vertical and the two horizontal positions of the crank OA (Fig. 208) The angular velocity of the crank is ω=const=20 sec^{-1}. $OA=40$ cm; $AC=20\sqrt{37}$ cm, $CB=20\sqrt{37}$ cm.

Ans. $v_1=400$ cm/sec, when $\alpha=0°$;
$v_2=0$, when $\alpha=90°$;
$v_3=400$ cm/sec, when $\alpha=180°$;
$v_4=0$, when $\alpha=270°$.

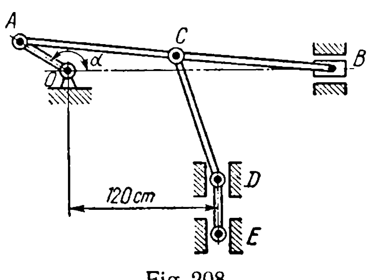


Fig. 208

283. A movable crusher jaw AB of length 60 cm, rotating about centre A, is set in motion by a 10-cm crank OE. The latter makes 100 rpm. The torque force is transmitted to a jaw by means of levers BC, CD, each 40 cm long, and CE. Find the angular velocity of AB when it is in the position, shown in Fig. 209.

Ans. $\omega = 1.852\ \text{sec}^{-1}$.

284. A gear K 20 cm in diameter and a crank OA 20 cm long are mounted on the axle O (Fig. 210). They are not connected together. A gear L 20 cm in diameter is tightly mounted on the

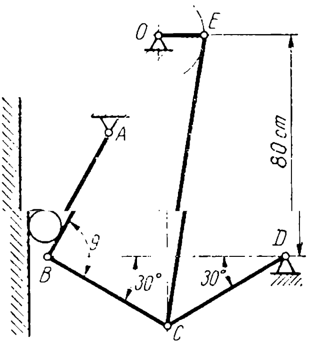


Fig. 209

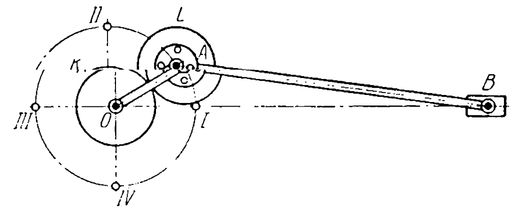


Fig. 210

connecting rod AB which is 1 m long. The gear L is meshed with the gear K which rotates with uniform angular velocity corresponding to $n = 60$ rpm causing the connecting rod AB and the crank OA to rotate. The system is in the vertical plane. Determine the angular velocity ω_1 of the crank OA at the two horizontal and two vertical positions.

Ans. $I.\ \omega_1 = \dfrac{10}{11}\pi\ \text{sec}^{-1}$; $III.\ \omega_1 = \dfrac{10}{9}\pi\ \text{sec}^{-1}$;

$II.\ \omega_1 = \pi\ \text{sec}^{-1}$; $IV\ \ \omega_1 = \pi\ \text{sec}^{-1}$.

285. Fig. 211 represents a Watt's planetary mechanism in which a crank is freely mounted on the axle of the gear of radius $R = 25$ cm. The gear rotates about the same axis with angular velocity $\omega_0 = \text{const} = 10\ \text{sec}^{-1}$. A crank OA is joined to a connecting rod AB 150 cm long which is connected to the gear

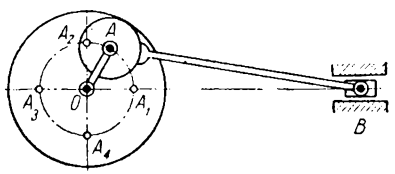


Fig. 211

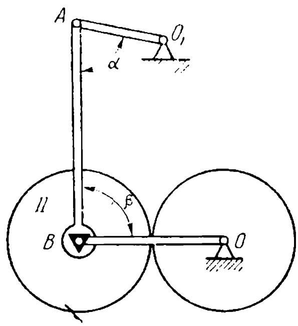


Fig. 212

of radius $r = 10$ cm. Find the angular velocity of the crank at the two vertical and the two horizontal positions.

Ans. $\omega_1 = 17.81$ sec^{-1}; $\omega_2 = \omega_4 = 16.67$ sec^{-1}; $\omega_3 = 15.62$ sec^{-1}.

286. A Watt's straight-line mechanism has a rocker arm O_1A which rotates about the axis O_1 (Fig. 212). By means of a connecting rod AB the rocker arm transmits a torque to a crank OB, freely mounted on the axle O. A gear I is also mounted on the same axle O. A connecting rod AB is rigidly joined to the gear II. Given that $r_1 = r_2 = 30\sqrt{3}$ cm, $O_1A = 75$ cm, $AB = 150$ cm, and the angular velocity of the rocker arm is $\omega_0 = 6$ sec^{-1}, find the angular velocities of the crank OB and the gear I at the instant when $\alpha = 60°$, $\beta = 90°$

Ans. $\omega_{OB} = 3.75$ sec^{-1}; $\omega_I = 6$ sec^{-1}.

287. A 30-cm crank OA rotates about an axle O with angular velocity $\omega_0 = 0.5$ sec^{-1}. A gear of radius $r_2 = 20$ cm rolls without sliding on a fixed gear of radius $r_1 = 10$ cm and

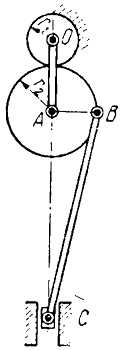


Fig. 213

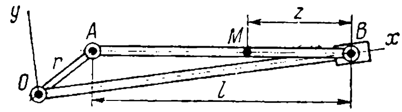


Fig. 214

transmits torque to a connecting rod $BC = 20\sqrt{26}$ cm (Fig. 213). Determine the angular velocity of the connecting rod and velocities of the points B and C at the instant when the radius AB is perpendicular to the crank OA.

Ans. $\omega_{\text{connecting rod}} = 0.15$ sec^{-1};
$v_B = 21.2$ cm/sec; $v_C = 18$ cm/sec.

288. Fig. 214 represents a slider-crank mechanism. Its shaft rotates uniformly with angular velocity ω and the length of the crank r is small compared to the length of a connecting rod l. Find approximate expressions for the projections of the velocity and acceleration of any point M of the crank AB on the coordinate axes. The position of the point M is defined by its distance from the position of a slipper pin. $MB = z$.

H i n t. The root $\sqrt{1 - \left(\frac{r}{l}\sin\varphi\right)^2}$, where $\varphi = \omega t$ denotes the angle AOB, is obtained while solving this problem. Expand this root in a power series and retain only the first two terms.

Ans. $v_x = -\omega\left[r\sin\varphi + \frac{(l-z)\,r^2}{2l^2}\sin 2\varphi\right]$; $\quad v_y = \frac{zr}{l}\,\omega\cos\varphi$;

$$w_x = -\omega^2\left[r\cos\varphi + \frac{(l-z)\,r^2}{l^2}\cos 2\varphi\right] \qquad w_y = -\frac{zr}{l}\,\omega^2\sin\varphi.$$

21. Space and Body Centrodes

289. Determine the space and body centrodes of the pulleys A and B of the polyspast, shown in Fig. 215. The radii of the pulleys A and B are r_A and r_B, respectively. It is assumed that the weight C performs translations in the vertical direction.

Ans. The body centrodes are: the circumference of a circle of radius r_A for the pulley A, and the circumference of a circle of radius $\frac{1}{3}r_B$ for the pulley B. The space centrodes are: the vertical lines tangent to the right-hand sides of the body centrodes.

290. Find by geometry the space and body centrodes of the connecting rod AB whose length is equal to that of the crank AB: $OA = AB = r$ (Fig. 216).

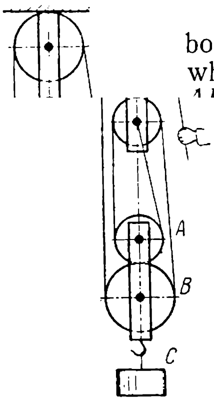


Fig. 215

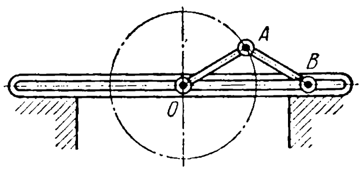


Fig. 216

Ans. The space centrode is the circumference of a circle of radius $2r$ with the centre at O whereas the body centrode is the circumference of a circle of radius r with the centre at A on the crank pin.

291. Find graphically the body and space centrodes of the connecting rod in a slider-crank mechanism, if the length of the connecting rod is twice that of the crank: $\frac{r}{l} = \frac{1}{2}$.

292. A rod AB moves in such a way that the particle A traces out a circle of radius r with centre at O (Fig. 217). The rod constantly passes through the given point N located on the same circle. Find the centrodes of the rod.

Ans. The space centrode is the circle of radius r with centre at O. The body centrode is the circle of radius $2r$ with centre at A.

293. Find the space and body centrodes of side CD of the anti-parallelogram which rests on a bigger side AB if $AB=CD=b, AD==BC=a$ and $a<b$.

Ans. The space centrode is the hyperbola with foci at A and B. The body centrode is the identical hyperbola with foci at C and D. The actual semi-axes of the hyperbolae are equal to $\frac{a}{2}$ (Fig. 218).

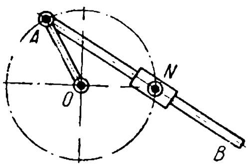


Fig. 217

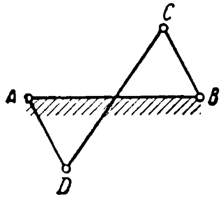


Fig. 218

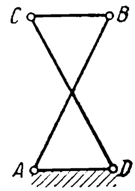


Fig. 219

294. Find the space and body centrodes of side BC of the anti-parallelogram resting on the smaller side AD if $AB=CD=b, AD=CB=a$ and $a<b$ (Fig. 219)

Ans. The space centrode is the ellipse with foci at A and D with semi-axes $\frac{b}{2}$ and $\frac{1}{2}\sqrt{b^2-a^2}$. The body centrode is the identical ellipse with foci at B and C.

295. A circle of radius 10 cm rolls without sliding on the circumference of another circle of radius 20 cm (Fig. 220). Locate the body and space centrodes. Determine the velocities of the vertices A, B and C of a square inscribed in the smaller circle at the moment when the vertex A is in contact with the larger circle.

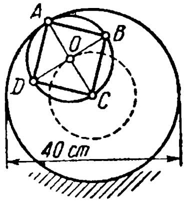


Fig. 220

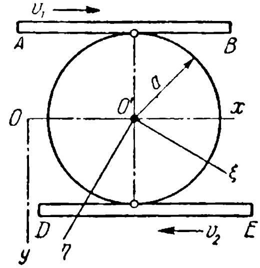


Fig. 221

The motion of the centre of the smaller circle $ABCD$ is uniform and traces out one circumference per second.

Ans. $v_A=0$; $v_B=88.84$ cm/sec; $v_C=125.66$ cm/sec.

296. Two parallel racks AB and DE move in opposite directions with constant velocities v_1 and v_2. A disk of radius a is constrained to move between these racks without sliding. Its motion is caused by the movement and friction of the racks. Determine: (1) the equation of the centrodes of the disk; (2) the velocity v_0 of the disk centre O', and (3) the angular velocity ω of the disk. The coordinate axes are shown in Fig. 221.

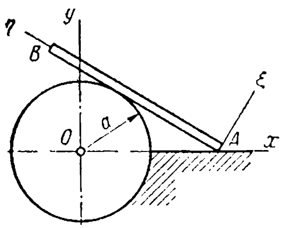


Fig. 222

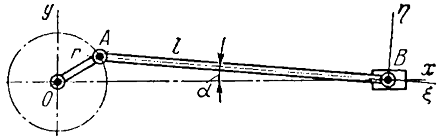


Fig. 223

Ans. (1) $y_C=a\dfrac{v_1-v_2}{v_1+v_2}$; $\xi_C^2+\eta_C^2=a^2\left(\dfrac{v_1-v_2}{v_1+v_2}\right)^2$;

(2) the velocity of the disk centre is in the same direction as the largest velocity

$$v_0=\frac{v_1-v_2}{2};$$

(3) $$\omega=\frac{v_1+v_2}{2a}.$$

297. Set up equations for the space and body centrodes of AB which rests on a circle of radius a, if the end A of the rod along the line Ox passing through the centre of the circle. T ordinate axes are shown in Fig. 222.

Ans. $x_C^2(x_C^2-a^2)-a^2y_C^2=0$; $\eta_C^2=a\xi_C$.

298. Find approximate equations for the space and centrodes of the connecting rod AB of the slider crank mech shown in Fig. 223. The length of the connecting rod $AB=$ large compared with the crank $OA=r$ that for the angle A it is possible to assume $\sin\alpha=\alpha$ and $\cos\alpha=1$.

Ans. $(x_C-l)^2(x_C^2+y_C^2)=r^2x_C^2$; $l^2\xi_C^2(l^2+\eta_C^2)=r^2\eta_C^4$.

22. Accelerations of a Point on a Body Which Performs Motions in a Plane. Instantaneous Centres of Accelerations

299. Show that the projections of the accelerations of two points of a plane body on the straight line connecting these two points are equal at the instant, when $\omega=0$.

300. A wheel rolls without sliding down the inclined plane, as shown in Fig. 224. At a certain moment the velocity of the centre of the wheel is $v_0=1$ m/sec, and its acceleration is $w_0=3$ m/sec^2.

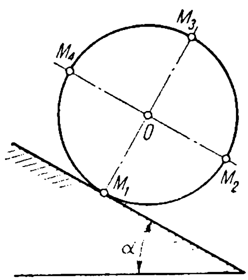


Fig. 224

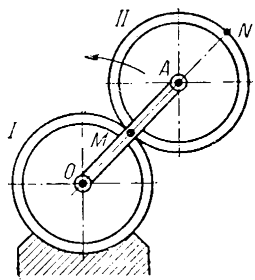


Fig. 225

If the radius of the wheel is $R=0.5$ m, find the accelerations of the ends of two mutually perpendicular diameters of the wheel, when one of the diameters is parallel to the plane.

Ans. $w_1=2$ m/sec^2; $w_2=3.16$ m/sec^2; $w_3=6.32$ m/sec^2; $w_4=5.83$ m/sec^2.

301. A gear of radius $R=12$ cm is driven by a crank OA rotating about an axle O of a fixed gear with the same radius (Fig. 225). The crank rotates with an angular acceleration $\varepsilon_0=8$ sec^{-2} and its instantaneous angular velocity is $\omega_0=2$ sec^{-1}. Determine: (1) the acceleration of a certain point on the rolling gear which, at this moment, coincides with the instantaneous centre of velocities; (2) the acceleration of the point N which is diametrically opposite to the first point, and (3) the position K of the instantaneous centre of accelerations.

Ans. (1) $w_M=96$ cm/sec^2; (2) $w_N=480$ cm/sec^2; (3) $MK=4.24$ cm; $\measuredangle AMK=45°$

302. A gear I of radius r rolls on the inner surface of a gear II of radius $R=2r$ (Fig. 226) The crank OO_1, which sets the gear II in motion, has constant angular velocity ω_0. Find the position of the instantaneous centre of acceleration and the velocity v_K of the point which coincides with the centre at this time. Also find the acceleration w_C of the point which coincides with the instantaneous centre of velocities at this time.

Ans. The instantaneous centre of accelerations coincides with the centre O of the fixed gear.
$v_K = 2r\omega_0$; $w_C = 2r\omega_0^2$

303. Referring to the previous problem, find the radius of a curved path of the point M on the rolling gear when M is at the maximum and minimum distances from the centre O. $O_1M = a$.

Ans. $\rho_1 = \frac{(r-a)^2}{r+a}$; $\rho_2 = \frac{(r+a)^2}{r-a}$.

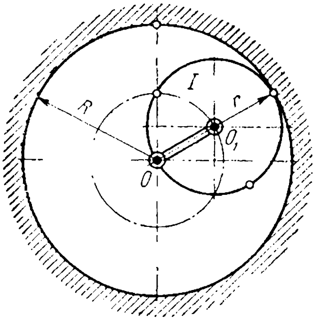


Fig. 226

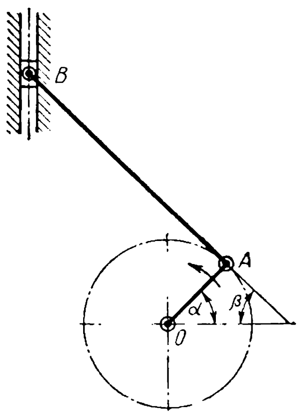


Fig. 227

304. Find the instantaneous centre of accelerations, the angular velocity and the angular acceleration of a plane body if at a certain moment the following data are known: the acceleration of a point A is $w_A = 15$ cm/sec^2 and the acceleration of a point B is $w_B = 10$ cm/sec^2 while the accelerations w_A and w_B are perpendicular to the straight line AB and are both directed to the same side; $AB = 10$ cm.

Ans. The instantaneous centre of accelerations is on the straight line AB at the distance $AK = 30$ cm; $\omega = 0$; $\varepsilon = 0.5$ sec^{-2}.

305. A 20-cm crank OA uniformly rotates with angular velocity $\omega_0 = 10$ sec^{-1} and drives a connecting rod AB of length 100 cm. A slider B moves in the vertical direction (Fig. 227). Find the angular velocity and angular acceleration of the connecting rod as well as the acceleration of the slider B when the crank and connecting rod are mutually perpendicular and make angles $\alpha = 45°$ and $\beta = 45°$ with the horizontal axis.

Ans. $\omega = 2$ sec^{-1}; $\varepsilon = 16$ sec^{-2}; $w_B = 565.6$ cm/sec^2.

306. A crank OA rotates with a constant angular velocity ω_0. It is joined to a connecting rod AB lying in a parallel plane. AB has a gear of radius r tightly mounted on its end A. This gear drives another gear of radius $r_1 = r$. The latter rides loosely on the

shaft O. Find the angular velocity ω and angular acceleration ε of this gear when the crank is in the horizontal and vertical positions. $AB=l$. (See Fig. 210.)

Ans. (1) $\omega=2\left(1+\frac{r}{l}\right)\omega_0$; $\varepsilon=0$;

(2) $\omega=2\omega_0$; $\varepsilon=\frac{2r\omega_0^2}{\sqrt{l^2-4r^2}}$, the rotation is retarded.

307. A crank OA of the eccentric crank mechanism rotates about its end O with a constant angular velocity ω_0. The following data are given: $OA=r$, $AB=l$, the distance between the crank axis O and the line of motion of the slider is $OC=h$. (See Fig. 202.) Determine the angular velocity and angular acceleration of the connecting rod and the velocity and acceleration of the slider B when the crank OA is in two positions: one horizontal to the right and the other vertical upwards.

Ans. (1) $\omega=\frac{r\omega_0}{\sqrt{l^2-h^2}}$; $\varepsilon=\frac{hr^2\omega_0^2}{(l^2-h^2)^{3/2}}$; $v_B=\frac{hr\omega_0}{\sqrt{l^2-h^2}}$;

$w_B=r\omega_0^2\left[1+\frac{rl^2}{(l^2-h^2)^{3/2}}\right]$.

(2) $\omega=0$; $\varepsilon=\frac{r\omega_0^2}{\sqrt{l^2-(r+h)^2}}$, the rotation is retarded;

$w_B=r\omega_0$; $w_B=\frac{r(r+h)\omega_0^2}{\sqrt{l^2-(r+h)^2}}$

308. An antiparallelogram consists of two cranks AB and CD each 40 cm long and a rod BC of length 20 cm pivoted to them. The distance between two fixed axes A and D is 20 cm. The crank AB rotates with a constant angular velocity ω_0. Determine the angular velocity and angular acceleration of the rod BC at the instant when the angle ADC equals 90° (Fig. 228).

Ans. $\omega_{BC}=\frac{8}{3}\omega_0$, the rotation is retarded; $\varepsilon_{BC}=\frac{20}{9}\omega_0^2$.

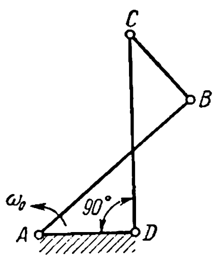


Fig. 228

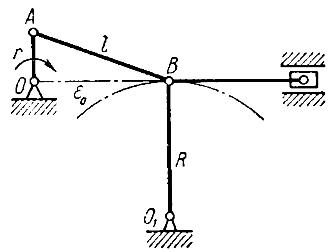


Fig. 229

309. Taking the data of Problem 280, determine the acceleration of a crank pin D of the driven crank OD. The position of the mechanism is as shown in Fig. 206.

Ans. $w_D = 5240$ cm/sec^2.

310. Determine the velocity and acceleration (normal and tangential) of the point B (Fig. 229) at the instant when links OA and O_1B are vertical. The crank rotates with a constant angular acceleration $\varepsilon_0 = 5$ sec^{-2}, and its instantaneous angular velocity is $\omega_0 = 10$ sec^{-1}. $OA = r = 20$ cm, $O_1B = 100$ cm, $AB = l = 120$ cm.

Ans. $v_B = 200$ cm/sec; $w_{Bn} = 400$ cm/sec^2;
$w_{Bt} = 370.45$ cm/sec^2.

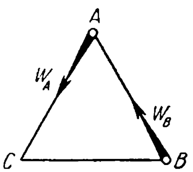


Fig. 230

311. An equilateral triangle ABC moves in the plane of the drawing. The accelerations of the vertices A and B at this particular time are 16 cm/sec^2 and directed along the sides of the triangle (Fig. 230). Determine the acceleration of the third vertex C of the triangle.

Ans. $w_C = 16$ cm/sec^2 and is directed along CB from C to B.

312. Fig. 207 represents a steam engine with a rocking cylinder resting on the journals O_1O_1. The crank OA is 12 cm long and the connecting rod AB is 60 cm long. The distance between the axis of the shaft and the axis of the cylinder journals is $OO_1 = 60$ cm. Determine the acceleration of the piston B and the radius of its path at the following two positions: (1) the crank and connecting rod are mutually perpendicular, and (2) the crank occupies the position III. The angular velocity of the crank is $\omega_0 = \text{const} = 5$ sec^{-1}.

Ans. (1) $w = 6.24$ cm/sec^2; $\rho = 576$ cm;
(2) $w = 258.3$ cm/sec^2; $\rho = 0.39$ cm.

23. Composition of Motions of a Body in a Plane

313. A crank III connects the axles O_1 and O_2 of two gears I and II which are in mesh. The meshing may be external or internal, as shown in Fig. 231. The gear I is fixed and the crank III rotates about the axle O_1 with angular velocity ω_3. Assuming that the radii of the gears are r_1 and r_2, respectively, calculate the absolute angular velocity ω_2 of the gear II and also find its angular velocity ω_{23} with respect to the crank.

Ans. External meshing:

$$\omega_2 = \omega_3 \frac{r_1 + r_2}{r_2}; \quad \omega_{23} = \omega_3 \frac{r_1}{r_2}$$

Internal meshing:

$$\omega_2 = -\omega_3 \frac{r_1 - r_2}{r_2} \qquad \omega_{23} = -\omega_3 \frac{r_1}{r_2}$$

The minus sign denotes that the bodies rotate in opposite directions.

314. A crank OA rotates about the axle O of a fixed gear (with $z_0 = 60$ teeth) with angular velocity corresponding to $n_0 = 30$ rpm. The axle of a double gear (with $z_1 = 40$ and $z_2 = 50$ teeth, respectively) is mounted on the crank OA. Calculate the number of revolutions per minute of the last gear with $z_3 = 25$ teeth (Fig. 232).

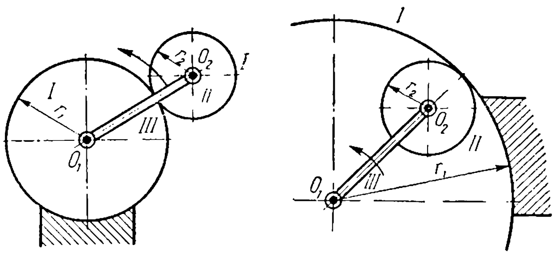


Fig. 231

Ans. $n_3 = n_0 \left(1 - \frac{z_0 z_2}{z_1 z_3}\right) = -60$ rpm.

The minus sign denotes that the bodies rotate in opposite directions.

315. A speed reducer consists of three gears. The first gear (with $z_1 = 20$ teeth) is mounted on a driving shaft I which makes $n_1 = 4500$ rpm. The second gear ($z_2 = 25$) fits loosely on the frame firmly keyed to the driven shaft II (Fig. 233). The third gear with the internal engagement is fixed. Determine the number of revolutions per minute of the driven shaft and the running gear.

Ans. $n_{II} = 1000$ rpm;
$n_2 = -1800$ rpm.

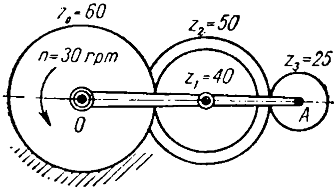


Fig. 232

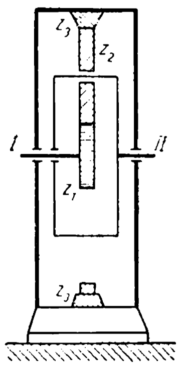


Fig. 233

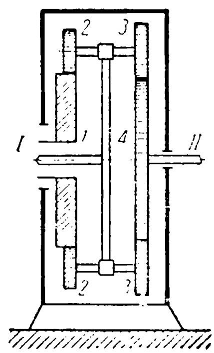


Fig. 234

316. Fig. 234 represents a speed reducer with differential transmission. A driving shaft rotates with angular velocity $\omega_I = 120$ sec^{-1}. It is connected with a crank on which twin transmission gears are mounted. A gear *I* rotates with angular velocity $\omega_1 = 180$ sec^{-1} and has $z_1 = 80$ teeth. Running gears have $z_2 = 20$ and $z_3 = 40$ teeth, respectively. A gear on the driven shaft has $z_4 = 60$ teeth. The gear *I* and the driving shaft rotate in the same direction. Find the angular velocity ω_{II} of the driven shaft.

Ans. $\omega_{II} = 280$ sec^{-1}.

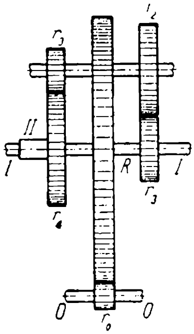


Fig. 235

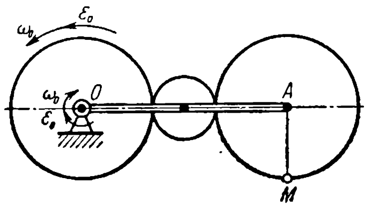


Fig. 236

317. A gear of radius R of a cylindrical differential transmission is fit loosely on a shaft *I-I* (Fig. 235). It has twin gears of radii r_2 and r_3, respectively. The gear R is driven by a gear of radius r_0. The gears of radii r_2 and r_3 are in mesh with the gears of radii r_1 and r_4 which are keyed to the shafts *I-I* and *II*, respectively. The latter is of a sleeve type. Assuming that the angular velocities of rotations of the shafts *I-I* and *O-O* are n_1 and n_0, find the angular velocity of the shaft *II*. Both shafts rotate in the same direction.

Ans. $n_2 = \left(n_1 + n_0 \frac{r_0}{R}\right) \frac{r_1 r_3}{r_2 r_4} - n_0 \frac{r_0}{R}$.

318. An epicyclic gear is shown in Fig. 236. The driving gear of radius R rotates in a counter-clockwise direction with angular velocity ω_0 and angular acceleration ε_0. A crank of length $3R$ rotates about its axis in a clockwise direction with the same angular velocity and the same angular acceleration. Find the velocity and acceleration of the point M on the driven gear of radius R, when it is in the position shown in the figure.

Ans. $v = R\omega_0\sqrt{10}$; $w = R\sqrt{10(\varepsilon_0^2 + \omega_0^4) - 12\omega_0^2\varepsilon_0}$.

24. Rotation of a Rigid Body about a Fixed Point

319. A circular cone with the height $h=4$ cm and the radius of the base $r=3$ cm rolls without sliding on a plane, as shown in Fig. 237 It has a fixed vertex at O. Find the angular velocity of the cone, and the coordinates of a point describing the hodograph of the angular velocity. Also determine the angular acceleration of the cone, if the velocity of the centre of the base is $v_C=48$ cm/sec$=$const.

Ans. $\omega=20\ \text{sec}^{-1}$; $x_1=20\cos 15t$;
$y_1=20\sin 15t$; $z_1=0$; $\varepsilon=300\ \text{sec}^{-2}$.

320. A circular cone A runs 120 times per minute round the circumference of a fixed cone B (Fig. 238). The height of the cone OO_1 is 10 cm. Find the transport angular velocity ω_e of the rotation about the axis z, the relative angular velocity ω_r of the cone about the axis OO_1, the absolute angular velocity ω_a and the absolute angular acceleration ε_a of the cone. Also determine the velocities and accelerations of points C and D of the rolling cone.

Ans. $\omega_e=4\pi\ \text{sec}^{-1}$; $\omega_r=6.92\pi\ \text{sec}^{-1}$;
$\omega_a=8\pi\ \text{sec}^{-1}$, it is directed along OC;
$\varepsilon_a=27.68\pi^2\ \text{sec}^{-2}$, it is directed parallel to the axis x;
$v_C=0$; $v_D=80\pi$ cm/sec, it is directed parallel to the axis x;
$w_C=320\pi^2\ \text{cm/sec}^2$, it is directed perpendicular to OC in the plane Oyz;
the projections of the acceleration of the point D:
$w_{Dy}=-480\pi^2\ \text{cm/sec}^2$;
$w_{Dz}=-160\sqrt{3}\pi^2\ \text{cm/sec}^2$.

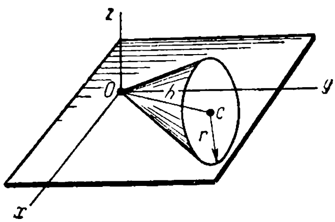


Fig. 237

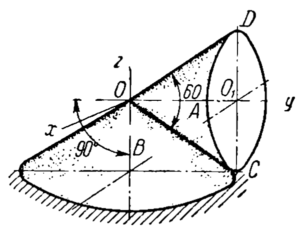


Fig. 238

321. A disk OA of radius $R=4\sqrt{3}$ cm rotating about a fixed point O rolls round a stationary cone of semi-angle 30° at a vertex (Fig. 239) Find the angular velocity of rotation of the disk about its axis of symmetry if the acceleration w_A of a point A on the disk has a constant magnitude and equals 48 cm/sec².

Ans. $\omega=2$ sec^{-1}.

322. A body rotates about a fixed point. At a particular instant its angular velocity is given by a vector whose projections on coordinate axes equal: $\sqrt{3}$, $\sqrt{5}$, $\sqrt{7}$ Find, at this particular moment, the velocity v of a point of the body which is defined by coordinates $\sqrt{12}$, $\sqrt{20}$, $\sqrt{28}$.

Ans. $v=0$.

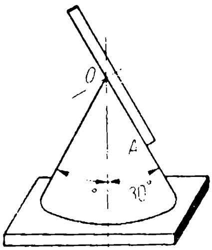


Fig. 239

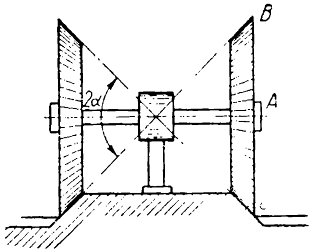


Fig. 240

323. Find the velocities and accelerations of points C and B on the conical roller which spins without sliding along the horizontal conical circular support (Fig. 240). The radius of the roller base is $R=10\sqrt{2}$ cm and the angle at the vertex is $2\alpha=90°$ The velocity of motion of the centre of the roller A along its path is $v_A=20$ cm/sec.

Ans. $v_C=0$; $w_C=40$ cm/sec²;
$v_B=40$ cm/sec; $w_B=40\sqrt{5}$ cm/sec².

324. Find the equations of the instantaneous axis and the magnitude of angular velocity ω of a body assuming that the projections of the velocity of a point M_1 (0, 0, 2) on the coordinate axes connected with the body are:

$$v_{x1}=1 \text{ m/sec};\quad v_{y1}=2 \text{ m/sec};\quad v_{z1}=0;$$

the direction of velocity of a point M_2 (0, 1, 2) is defined by the cosines of the angles formed with the coordinate axes:

$$-\frac{2}{3},\ +\frac{2}{3},\ -\frac{1}{3}.$$

Ans. $x+2y=0$; $3x+z=0$; $\omega=3.2$ sec^{-1}.

325. The rotation of a body about a fixed point is described by Euler's angles and defined by the equations: $\varphi = nt$; $\psi = \frac{\pi}{2} + ant$; $\theta = \frac{\pi}{3}$. Determine the projections of angular velocity and angular acceleration of the body on fixed axes if a and n are constant. Explain also that value of the parameter a when the plane Oxy is a fixed axoid of the body.

Ans. $\omega_x = \frac{n\sqrt{3}}{2}\cos ant$; $\omega_y = \frac{n\sqrt{3}}{2}\sin ant$; $\omega_z = n\left(a + \frac{1}{2}\right)$;

$\varepsilon_x = -\frac{an^2\sqrt{3}}{2}\sin ant$; $\varepsilon_y = \frac{an^2\sqrt{3}}{2}\cos ant$;

$\varepsilon_z = 0$; $a = -\frac{1}{2}$.

25. Composition of Rotations of a Rigid Body about Intersecting Axes

326. Fig. 241 represents two bevel gears whose axles are rigidly fixed with the angles α and β at the vertices. The first gear rotates with the velocity ω_1. When $\alpha = 30°$, $\beta = 60°$, and $\omega_1 = 10$ rpm, determine the velocity ω_2 of the second gear.

$$Ans.\quad \omega_2 = \omega_1 \frac{\sin\frac{\alpha}{2}}{\sin\frac{\beta}{2}} = 5.16 \text{ rpm}.$$

327. A ball crusher has a hollow spherical body *II* (with the balls and crushing material inside), which is mounted on the axle CD (Fig. 242). At the end of the axle a bevel gear E of radius r is wedged. The axle CD rests on bearings in the frame *I* forming a single unit together with the axle AB. The unit is set in motion by a handle G. A gear E is in mesh with a fixed gear F of radius R. If the handle of the ball crusher rotates with angular velocity ω_0 determine the absolute angular velocity of the ball crusher. The

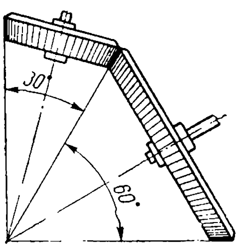


Fig. 241

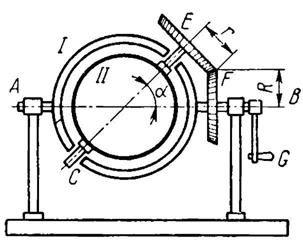


Fig. 242

angle between axes AB and CD is α. Also find the absolute angular acceleration of the ball crusher if the angular velocity of the handle is $\omega_0 = \text{const}$.

Ans. $\omega_A = \frac{\omega_0}{r}\sqrt{r^2+R^2+2rR\cos\alpha}$; $\quad \varepsilon = \omega_0^2 \frac{R}{r}\sin\alpha$.

328. A differential friction gear consists of two disks AB and DE with centres on the same axis of rotation (Fig. 243) A wheel MN of radius $r=5$ cm is squeezed between the disks. The axle HI of the wheel MN is perpendicular to the axle of the disks. Deter-

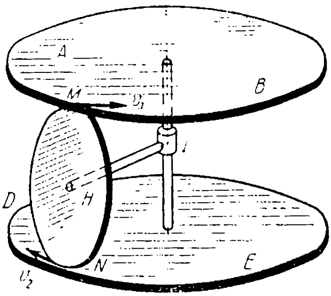


Fig. 243

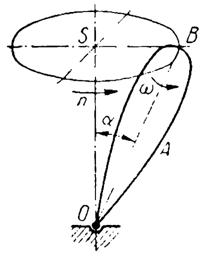


Fig. 244

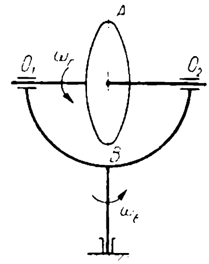


Fig. 245

mine for the wheel MN: the velocity v of the centre H and the angular velocity ω_r of rotation about the axle HI. The velocities of the points of contact between the wheel and the disks are: $v_1 = 3$ m/sec, $v_2 = 4$ m/sec; $r = 5$ cm.

Ans. $v = 0.5$ m/sec; $\quad \omega_r = 70\ \text{sec}^{-1}$.

329. Taking the data of the previous problem and assuming that the length HI is $\frac{1}{14}$ m, determine the absolute angular velocity and absolute angular acceleration of the wheel MN.

Ans. $\omega = \sqrt{4949}\ \text{sec}^{-1}$; $\quad \varepsilon = 490\ \text{sec}^{-2}$.

330. A merry-go-round A rotates about the axis OB with constant angular velocity $\omega_1\ \text{sec}^{-1}$. The axis OB describes a cone (Fig. 244). A vertex B makes n rotations per minute. The angle $BOS = \alpha$. Find the angular velocity ω and the angular acceleration ε of the rotating body.

Ans. $\omega = \sqrt{\omega_1^2 + \left(\frac{\pi n}{30}\right)^2 + 2\omega_1 \frac{\pi n}{30}\cos\alpha}$; $\quad \varepsilon = \omega_1 \frac{\pi n}{30}\sin\alpha$.

331. A disk of radius R rotates with constant angular velocity ω_r about the horizontal axle O_1O_2 (Fig. 245). The latter also rotates with constant angular velocity ω_e about the vertical axis.

Find the velocities and accelerations of points A and B on the extremities of the vertical diameter of the disk.

Ans. $v_A = v_B = R\omega_r$; $w_A = w_B = R\omega_r\sqrt{4\omega_e^2 + \omega_r^2}$.

332. A square frame rotates about the axle AB (Fig. 246) making 2 rpm. A disk rotates with angular velocity corresponding to 2 rpm about the axle BC, which coincides with the diagonal of the frame. Determine the absolute angular velocity and angular acceleration of the disk.

Ans. $\omega = 0.39$ sec^{-1}; $\varepsilon = 0.031$ sec^{-2}.

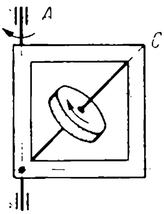


Fig. 246

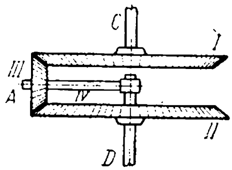


Fig. 247

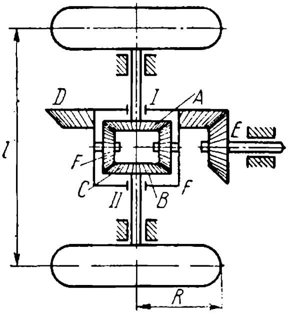


Fig. 248

333. A differential friction gear consists of a bevel gear *III* (satellite) fit loosely on the crank *IV* The latter rotates about a fixed axis CD (Fig. 247). The satellite is in mesh with bevel gears *I* and *II* rotating about the same axis CD with angular velocities $\omega_1 = 5$ sec^{-1} and $\omega_2 = 3$ sec^{-1}. They rotate in the same direction. The radius of the satellite is $r = 2$ cm and the radii of both gears *I* and *II* are $R = 7$ cm. Determine the angular velocity ω_4 of the crank *IV*, the angular velocity ω_{34} of the satellite with respect to the crank. Find the velocity of the point A.

Ans. $v_A = 28$ cm/sec, $\omega_4 = 4$ sec^{-1}, $\omega_{34} = 3.5$ sec^{-1}.

334. The problem is the same as the previous one, except that the bevel gears *I* and *II* rotate in opposite directions with angular velocities: $\omega_1 = 7$ sec^{-1}, $\omega_2 = 3$ sec^{-1}. Determine v_A, ω_4 and ω_{34}. $R = 5$ cm, $r = 2.5$ cm.

Ans. $v_A = 10$ cm/sec; $\omega_4 = 2$ sec^{-1}; $\omega_{34} = 10$ sec^{-1}.

335. An automobile runs along a curved road. Its external wheels rotate quicker than the internal ones as they cover longer distance in comparison with the internal wheels. To avoid breaking the rear driving axle, a differential transmissions gear of the following arrangement is used (Fig. 248). The rear axle with two wheels on it is made of two separate parts *I* and *II*. At the end of each semi-axle two gears A and B are tightly fit. A gear

box C with a tightly fixed bevel gear D rotates on bearings about the shaft of these axles. The gear box is driven by the main (longitudinal) shaft which is set in motion by the motor through the gear E.

The two bevel gears F (satellites) transfer the torque of the gear box C to the gears A and B. The satellites rotate freely about the axles, which are installed in the gear box perpendicular to the rear axle I-II.

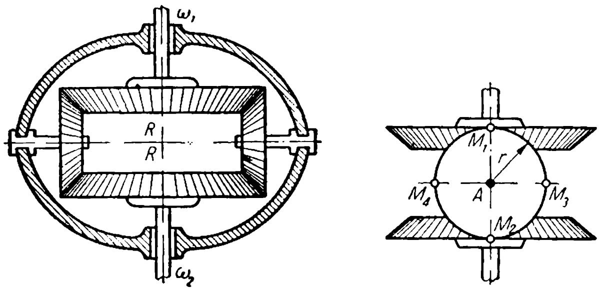


Fig. 249

Find the angular velocities of the rear wheels as a function of the angular velocity of the gear box C. Determine the angular velocity ω_r of the satellites with respect to the gear box if the automobile runs with a speed $v=36$ km/h along the curved road of radius $\rho=5$ m. The radius of the wheels of the rear axle is $R=0.5$ m, the distance between them is $l=2$ m. The radii of the gears A and B are twice that of the satellites: $R_0=2r$

Ans. $\omega_1=24\ \text{sec}^{-1}$; $\omega_2=16\ \text{sec}^{-1}$; $\omega_r=8\ \text{sec}^{-1}$.

336. Fig. 249 represents a differential transmission which connects the semi-axles of the automobile. It consists of two bevel gears of equal radii $R=6$ cm, which are mounted on semi-axles. When the automobile runs on curves the bevel gears rotate in the same direction with constant angular velocities $\omega_1=6\ \text{sec}^{-1}$ and $\omega_2=4\ \text{sec}^{-1}$, respectively. A rolling satellite of radius $r=3$ cm is squeezed between the two bevel gears. The axle on which the satellites are loosely fit is fixed in the housing and can rotate with it about the rear axle. Find the acceleration of four points M_1, M_2, M_3 and M_4 on the satellite with respect to the body of the automobile. The points are located on the ends of two diameters in the way shown in Fig. 249.

Ans. $w_1=210.4\ \text{cm/sec}^2$; $w_2=90.8\ \text{cm/sec}^2$;
$w_3=w_4=173.4\ \text{cm/sec}^2$.

337. Fig. 250 represents a differential of a gear-cutting machine. An accelerating gear 4 with the gear I firmly fixed to it is loosely fit on a driving shaft a. The head carrying the axle CC of

satellites *2-2* is fastened to the end of the driving shaft *a*. Determine the angular velocity of a driven shaft *b* with a gear *3* rigidly wedged to it, if: (1) the angular velocity of the driving shaft is ω_a and the angular velocity of the accelerating gear is $\omega_4=0$; (2) the angular velocity of the driving shaft is ω_a, the accelerating gear and the driving shaft rotate in the same direction with angular velocity ω_4; (3) the accelerating gear and the driving shaft rotate

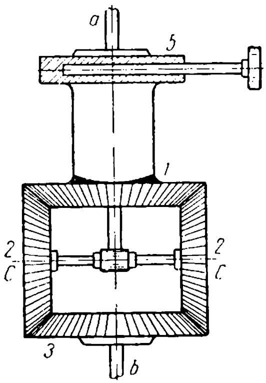


Fig. 250

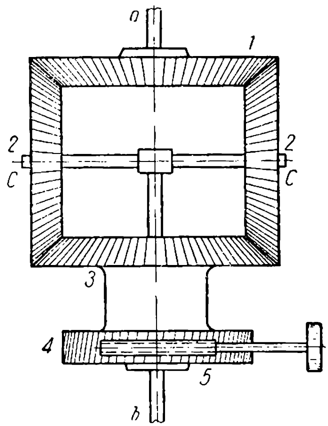


Fig. 251

in the same direction with equal angular velocities $\omega_4=\omega_a$; (4) the accelerating gear and the driving shaft rotate in the same direction while $\omega_4=2\omega_a$; (5) the angular velocity of the driving shaft is ω_a and the accelerating gear rotates in the opposite direction with angular velocity ω_4.

Ans. (1) $\omega_b=2\omega_a$; (2) $\omega_b=2\omega_a-\omega_4$; (3) $\omega_b=\omega_a$; (4) $\omega_b=0$; (5) $\omega_b=2\omega_a+\omega_4$.

338. The problem is the same as the previous one, except that the velocity of a driving shaft is $\omega_a=60$ rpm. Determine the required velocity of the accelerating gear to keep the driven shaft stationary

Ans. $\omega_4=120$ rpm.

339. A machine-tool differential has a bevel gear *1* which is keyed to a driving shaft *a* (Fig. 251) A head, carrying the axle *CC* of satellites *2-2* is wedged at the end of the driving shaft *b*. On the same shaft a bevel gear *3* is loosely fit constituting one whole unit with a worm gear *4*. Determine the velocity ratio when the worm *5* is held stationary together with the gears *4* and *3*, assuming that all the bevel gears are of the same radius.

Ans. $\dfrac{\omega_b}{\omega_a}=0.5$.

340. Fig. 252 shows a ball crusher with a hollow sphere of $d=10$ cm which is mounted on the axle AB. A gear with $z_4=$ $=28$ teeth is wedged to the same axle AB which is fixed in bearings a and b in the rotating frame I. The latter together with the axle CD constitutes one whole unit, which is set in motion by a handle III. A crusher is constrained to rotate about the axle AB by gears with $z_1=80$, $z_2=43$, $z_3=28$ teeth respectively. The first of the three gears is held stationary. Determine the absolute angular velocity, and angular acceleration of the crusher and the velocities and accelerations of the points E and F on the axle CD at a given instant, assuming that the handle is rotated with constant angular velocity $\omega=4.3$ sec^{-1}.

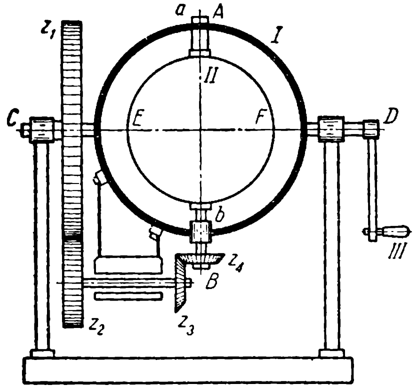


Fig. 252

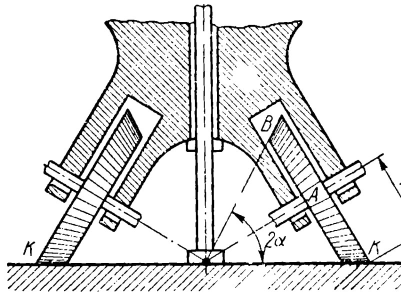


Fig. 253

Ans. $\omega_a=9.08$ sec^{-1}; $\varepsilon=34.4$ sec^{-2};
$v_E=v_F=40$ cm/sec; $w_E=w_F=468$ cm/sec^2.

341. A turning part of a bridge is mounted on bevel gears K serving as rollers (Fig. 253). The axles of the fixed bevel gears are inclined in a circular frame L in such a way that lines continued along their direction intersect at the geometrical centre of a flat bearing gear, on which the bevel gears K roll. If the radius of the roller base is $r=25$ cm, the angle at the vertex is 2α, where $\cos\alpha=84/85$, find the angular velocity and angular accelerations of the roller as well as the velocities and accelerations of points A, B and C (A is the centre of a gear BAC). The angular velocity of the circular frame about a vertical axle is $\omega_0=\text{const}=0.1$ sec^{-1}.

Ans. $\omega=0.646$ sec^{-1}, $\varepsilon=0.0646$ sec^{-2};
$v_A=15.92$ cm/sec, $v_B=31.84$ cm/sec, $v_C=0$;
$w_A=1.595$ cm/sec^2, $w_B=11.06$ cm/sec^2;
$w_C=10.54$ cm/sec^2.

PART III
DYNAMICS

VIII. DYNAMICS OF A PARTICLE

26. Determination of Force Acting During Motion

342. A bucket which weighs 280 kgf descends into a mine with uniform acceleration. During the first 10 sec it drops 35 m. Find the tension in the cable holding the bucket.

Ans. 260 kgf.

343. A body which weighs $P=$ 3 kgf rests on a table. A string is tied with one end to the body and with its free end is held by the hand. Determine the acceleration which is required for breaking the string while lifting the body vertically upwards assuming that the latter breaks when $T=4.2$ kgf.

Ans. $w=3.92$ m/sec^2.

344. Fig. 254 shows the velocity graph of the upward motion of a lift weighing 480 kgf. Find the tensions T_1, T_2, T_3 in the cable holding the lift during the three periods of time: (1) from $t=0$ to $t=2$ sec; (2) from $t=2$ sec to $t=8$ sec; (3) from $t=8$ sec to $t=10$ sec.

Ans. $T_1=602.4$ kgf; $T_2=480$ kgf; $T_3=357.6$ kgf.

345. At track curvatures the outer rail is higher than the inner one. This is done so for the purpose to direct the pressure of a running train perpendicular to the track. Determine the magnitude h of the elevation of the outer rail over the inner one in a curve of radius 400 m, when the speed of the train is 10 m/sec and the rail gauge is 1.6 m.

Ans. $h=4.1$ cm.

346. A weight $M=1$ kgf is suspended by a thread 30 cm from a fixed point O. It represents a conical pendulum as it traces a circular path on a horizontal plane. The thread forms an angle of 60° with the vertical. Determine the velocity of the weight and the tension T in the thread (Fig. 255).

Ans. $v=210$ cm/sec; $T=2$ kgf.

347. An automobile weighing $Q=1000$ kgf runs over an arched

bridge with the speed $v=10$ m/sec. The radius of the curvature of the bridge at its mid-point is $\rho=50$ m. Determine the pressure which the automobile exerts on the bridge at the instant when it passes over the centre of the bridge.

Ans. 796 kgf.

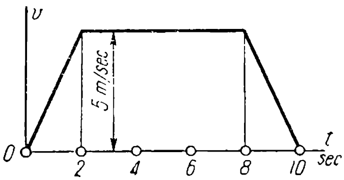


Fig. 254

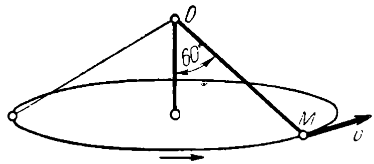


Fig. 255

348. A body is weighed on a spring balance in an ascending lift. The registered weight is 5 kgf and the tension in the spring is 5.1 kgf (a reading on the balance). Determine the acceleration of the lift.

Ans. 0.196 m/sec².

349. A tram body, together with the load, weighs $Q_1=10{,}000$ kgf and the total weight of the bogie and wheels is $Q_2=1000$ kgf Find the maximum and the minimum values of force acting on the rails of the horizontal straight track, assuming that the tram body oscillates harmonically in a vertical direction on its springs, and its motion is defined by the law: $x=2\sin 10t$ cm.

Ans. $N_1=13{,}040$ kgf; $N_2=8960$ kgf.

350. A piston of a steam engine is oscillating horizontally in accordance with the law: $x=r\left(\cos\omega t+\frac{r}{4l}\cos 2\omega t\right)$ cm, where r is a length of the crank, l is a length of the connecting rod and ω is a constant angular velocity of the shaft. Find the maximum value of the force acting on the piston if the weight of the latter is Q.

Ans. $P=\frac{Q}{g}r\omega^2\left(1+\frac{r}{l}\right)$.

351. The motion of a 2-gf particle is defined by the equations: $x=3\cos 2\pi t$ cm; $y=4\sin\pi t$ cm, where t denotes the time in seconds. Determine the projections of the force acting on the particle as a function of its coordinates.

Ans. $X=-0.08x$ gf; $Y=-0.02y$ gf.

352. A ball of mass 1 g falls under the action of gravity encountering an air resistance so that its equation of motion is $x=490t-245(1-e^{-2t})$, where x is measured in centimetres and t

in seconds. The axis Ox is directed vertically downwards. Assuming that $g=980$ cm/sec^2, determine in dynes the force R of the air resistance encountered by the ball as a function of its velocity v.

Ans. $R=2v$.

353. A table of a planer weighs $Q_1=700$ kgf, a machined piece weighs $Q_2=300$ kgf, the velocity of motion of the table is $v=0.5$ m/sec and the racing time is $t=0.5$ sec. Determine the force required for racing (assuming that the motion is uniformly accelerated) and for continuing the uniform motion of the table if the coefficient of friction for racing is $f_1=0.14$ and for uniform motion is $f_2=0.07$.

Ans. $P_1=242$ kgf; $P_2=70$ kgf.

354. $v=1$ m/sec is the velocity with which a crab and a weight $Q=10{,}000$ kgf travel along the horizontal truss of a bridge crane. The distance between the centre of gravity of the weight and the point of suspension is $l=5$ m. When the crab is brought suddenly to rest, the weight tends to move by reason of its inertia and then it starts swinging about the point of suspension. Determine the maximum tension in the cable.

Ans. $S=10{,}200$ kgf.

355. A train weighs 200,000 kgf without the locomotive. Running with a uniform acceleration along a horizontal track it attains a speed of 54 km/h in 60 sec from the start. Determine the tension in the coupling between the train and the locomotive on the run, if the frictional resistance equals 0.005 of the weight of the train.

Ans. 6100 kgf.

356. An aircraft weighing 2000 kgf flies horizontally with an acceleration of 5 m/sec^2 and an instantaneous speed of 200 m/sec. The air drag at this speed is proportional to the square of the speed; when the speed of 1 m/sec is attained the air drag equals 0.05 kgf. Assuming that the force of resistance is directed opposite to the velocity, determine the tractive force of the propeller, if the angle between the flight direction and the tractive force is 10°

Ans. $F=3080$ kgf.

357. A weight $P=5$ kgf is suspended from a spring and performs harmonic oscillations. Determine the magnitude of the force c which must be applied to the spring to produce an extention by 1 cm, if the weight P makes 6 full oscillations per 2.1 sec. Neglect any resistance.

Ans. $c=1.65$ kgf/cm.

358. An aircraft dives verticaly and attains a speed of 1000 km/h. Reaching this speed the pilot pulls the aircraft out tracing an arc of radius $R=600$ m in vertical plane. The weight of the pilot is 80 kgf. He is subjected to pressure from the seat during this flight. Find the maximum pressure exerted on the pilot.

Ans. 1130 kgf.

359. A locomotive weighing $Q=180{,}000$ kgf passes over a bridge with the speed $v=72$ km/h. At the instant when the locomotive is on the middle part of the bridge, its bending deflection is $h=0.1$ m. Determine the magnitude of the additional pressure on the bridge at any given time, considering that the bridge is a massless single-span beam of uniform cross section of length $L=100$ m, whose ends are hinged. Neglect the dimensions of the locomotive.

Ans. $\dfrac{12\,Qhv^2}{gL^2}=880$ kgf.

360. A governor, shown in Fig. 256, has two 30-kgf weights A which can slide along the horizontal line MN. The weights are attached to the ends of two springs and the latter are fixed at the points M and N The centres of gravity of both weights coincide with the ends of the springs, which are 5-cm distance from the axis O when the springs are not strained. The axis O is perpendicular to the plane of the sketch. A force of 20 kgf changes the length of the spring by 1 cm. Determine the distances between the centres of gravity of the weights and the axis O when the governor rotating uniformly makes 120 rpm.

Ans. 6.58 cm.

361. A particle of mass m moves along an ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. The acceleration of the particle is parallel to the axis y. When $t=0$, the coordinates of the

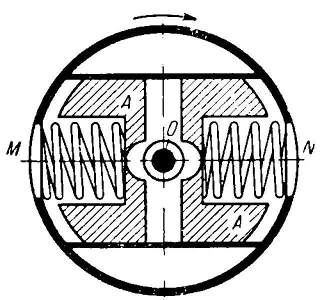


Fig. 256

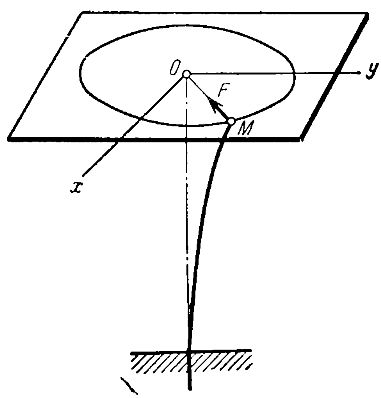


Fig. 257

particle are $x=0$, $y=b$ and the initial velocity is v_0. Determine the magnitude of the force acting on the moving particle at each point of its path.

Ans. $F_y=-m\dfrac{v_0^2b^4}{a^2y^3}$.

362. A ball of mass m is attached to the end of a vertical flexible rod whose lower end is squeezed into a fixed hole. When small deviations of the rod from its vertical equilibrium positions take place (Fig. 257), it may be assumed that the ball centre moves in a horizontal plane Oxy, passing through the upper equilibrium position of the ball centre. Determine the law of alteration of the force acted on the ball by the flexible rod, if the ball moves in accordance with the equations:

$$x=a\cos kt;\quad y=b\sin kt;$$

where a, b, and k are constants.

Ans. $F=mk^2r$, where $r=\sqrt{x^2+y^2}$.

27. Differential Equations of Motion

(a) Rectilinear Motion

363. A heavy body slides down along the smooth surface inclined at 30° to the horizontal. Determine the time taken by the body to pass a distance of 9.6 m if its initial velocity is 2 m/sec.

Ans. 1.61 sec.

364. A body of weight P is given a push and moves along a rough horizontal plane. It travels the distance $s=24.5$ m during 5 sec and then comes to rest. Determine the coefficient of friction f.

Ans. $f=0.2$.

365. Assuming that the resistance of compression is constant, find the recoil length of the barrel of a field gun, if the initial velocity of recoil is 10 m/sec and its length is 1 m.

Ans. 0.2 sec.

366. A ball of weight $P=10$ kgf and radius $r=8$ cm is dropped from a height and falls to the ground. The air resistance is $R=k\sigma v^2$, where v is the velocity of falling and σ is the area of projection of the falling body on a surface perpendicular to the direction of motion; k is a numerical coefficient which depends on the shape of the body, and its value for the ball is 0.024 kgf·sec²/m⁴. Determine the maximum velocity of the falling ball.

Ans. $v=144$ m/sec.

367. Two geometrically identical homogeneous balls are manufactured of different materials. Their specific weights are γ_1 and γ_2, respectively. Both balls fall down to the ground. Assuming that the air resistance is proportional to the square of the velocity, determine the ratio of the maximum velocities of the balls.

Ans. $\dfrac{v_{1\,max}}{v_{2\,max}}=\sqrt{\dfrac{\gamma_1}{\gamma_2}}$.

368. A heavy particle M moves up a rough surface of inclination $\alpha=30°$ to the horizontal. Initially the velocity of the particle is $v_0=15$ m/sec. The coefficient of friction is $f=0.1$. Determine the distance travelled by the particle before it comes to rest and the time taken.

Ans. $s=\dfrac{v_0^2}{2g(f\cos\alpha+\sin\alpha)}=19.55$ m;

$T=\dfrac{v_0}{g(f\cos\alpha+\sin\alpha)}=2.61$ sec.

369. A skier moves down an inclined slope of angle 45° without using his stick. The coefficient of friction between the skis and snow is $f=0.1$. The air resistance to the skier's motion is $R=\alpha v^2$, where $\alpha=$const and v is the skier's speed. When the skier attains the speed of 1 m/sec the air resistance is 0.0635 kgf. If the skier weighs 90 kgf, determine the maximum speed that he can attain. Calculate the maximum speed the skier can attain if by using the best grease for his skis the coefficient of friction is reduced to 0.05.

Ans. $v_{1\,max}=108$ km/h; $v_{2\,max}=111$ km/h.

370. A ship, travelling at 1 m/sec, overcomes the water resistance which is proportional to the square of the speed $\alpha=120$ kgf. The thrust of the propellers is directed along the line of motion and changes in accordance with the law: $T=T_0\left(1-\dfrac{v}{v_s}\right)$ kgf, where $T_0=120{,}000$ kgf is the thrust at the instant when the ship is at rest; $v_s=$const$=33$ m/sec. What is the maximum speed that can be attained by the ship?

Ans. $v_{max}=20$ m/sec$=72$ km/h.

371. An aircraft flies horizontally. The air resistance is proportional to the square of the speed. At a speed of 1 m/sec the air resistance equals 0.05 kgf. The tractive force is constant and equals 3080 kgf and it makes an angle of 10° with the direction of the movement of the aircraft. Determine the maximum speed of the aircraft.

Ans. $v_{max}=246$ m/sec.

372. A railway car rolls with constant speed down a straight track inclined at an angle of $\alpha=10°$ to the horizontal. Assuming that the frictional resistance is proportional to the normal pressure, determine the acceleration of the car and its speed 20 sec from the start. It is assumed that the car started to roll from rest down the track inclined at the angle $\beta=15°$ Calculate also the distance travelled by the car during this period.

Ans. $w=\dfrac{\sin(\beta-\alpha)}{\cos\alpha}g=0.87\ \text{m/sec}^2;$

$$v=\frac{\sin(\beta-\alpha)}{\cos\alpha}gt=17.4\ \text{m/sec};$$

$$s=\frac{g\sin(\beta-\alpha)}{\cos\alpha}\frac{t^2}{2}=174\ \text{m}.$$

373. A plane starts diving without any initial vertical speed. The air resistance is proportional to the square of the speed. Find the dependence between the vertical speed at a given instant, the path travelled, and the maximum speed of diving.

Ans. $v=v_{\max}\sqrt{1-e^{-2gs/v^2_{\max}}}$.

374. A body of weight p is projected vertically upwards with a velocity v_0. The air resistance is defined by the formula k^2pv^2, where v is the velocity of the body Determine the height H travelled by the body and the time T elapsed.

Ans. $H=\dfrac{\ln\left(v_0^2k^2+1\right)}{2gk^2};\quad T=\dfrac{\arctan kv_0}{kg}$

375. A 2-kgf body is projected vertically upwards with the velocity of 20 m/sec. It is acted on by air resistance which, measured in kilograms, equals $0.04v$ at the velocity v m/sec; $g=9.8\ \text{m/sec}^2$ How long does it take the body to reach its highest point?

Ans. 1.7 sec.

376. A mine train, of weight Q measured in kgf, and velocity v, measured in m/sec, runs in a mine. The resistance to the motion of the train at low speeds is defined by the empirical formula:

$$R=(2.5+0.05v)Q\ \text{kgf}..$$

Determine the time elapsed and the distance covered when the train attains the velocity $v=12$ km/h, travelling along a horizontal track The weight of the locomotive and the train equals $Q=40{,}000$ kgf and the tractive force of the electric locomotive is $F=200$ kgf. Find also the magnitude of the tractive force N when the train maintains uniform motion.

Ans. $t=141$ sec; $s=245$ m; $N=106.6$ kgf.

377. What should be the tractive force T=const of the propeller to increase the speed of an aircraft from v_0 m/sec to v_1 m/sec after flying s metres horizontally? The tractive force of the propeller is directed along the line of motion of the aircraft. The frontal resistance is directed opposite to the velocity, is proportional to the square of the speed, and equals α kgf when the speed equals 1 m/sec. The aircraft weighs P kgf.

Ans. $T=\alpha\dfrac{\left(v_0^2-v_1^2e^{\frac{2\alpha gs}{P}}\right)}{1-e^{\frac{2\alpha gs}{P}}}$ kgf.

378. A ship of 10,000,000 kgf travels at a speed of 16 m/sec. A water resistance is proportional to the square of the speed of the ship and equals 30,000 kgf at the speed of 1 m/sec. What distance will the ship travel before attaining the speed of 4 m/sec? What time is required to travel this distance?

Ans. $s=47.1$ m; $T=6.38$ sec.

379. A body falls from a height without any initial velocity. The air resistance is $R=k^2pv^2$, where v is the velocity of the body, and p is its weight. Determine the velocity of the body attained after time t. Also find the limiting velocity.

Ans. $v=\dfrac{1}{k}\dfrac{e^{kgt}-e^{-kgt}}{e^{kgt}+e^{-kgt}}$; $\quad v_\infty=\dfrac{1}{k}$.

380. A small heavy ball moves along an imaginary straight tunnel dug through the centre of the earth. The magnitude of the gravitational force inside the earth is proportional to the distance between the particle and the centre of the earth and is directed towards the centre. The ball is dropped from rest into the tunnel at the surface of the earth. Describe the motion of the ball. Find its velocity when passing through the centre of the earth and the time required to reach the centre. The radius of the earth is $R=637\times10^6$ cm and the acceleration due to gravity at the earth's surface is $g=980$ cm/sec^2.

Ans. The distance between the ball and the centre of the earth varies as:

$$x=R\cos\sqrt{\frac{g}{R}}\,t;\quad v=7.9\text{ km/sec};\quad T=21.1\text{ min}.$$

381. A body is dropped from rest at the height h to the ground. It is assumed that gravity is proportional to the inversed square of the distance between the body and the centre of the earth. Determine the time T, taken to reach the surface of the earth, and the velocity v attained by the body during this time. The radius of

the earth is R and the acceleration due to gravity at the earth's surface is g. Neglect the air resistance.

Ans. $v=\sqrt{\dfrac{2gRh}{R+h}}$; $T=\dfrac{1}{R}\sqrt{\dfrac{R+h}{2g}}\left(\sqrt{Rh}+\dfrac{R+h}{2}\arccos\dfrac{R-h}{R+h}\right)$.

382. A railway car weighing $Q=9216$ kgf is constrained to move along a horizontal track under the action of a wind blowing in the direction of the track. The frictional resistance to the car motion is 1/200 of its weight. The pressure of the wind is $P=kSu^2$ kgf, where S is the area, 6 m², of the rear wall of the car, and u is the velocity of the wind relative to the car, $k=0.12$. The absolute velocity of the wind is $w=12$ m/sec. The initial velocity of the car is 0. Determine: (1) the maximum velocity v_{max} of the car; (2) the time T taken to reach this velocity; (3) the distance x_1 travelled by the car before reaching a velocity of 3 m/sec.

Ans. (1) $v_{max}=4$ m/sec; (2) $T=\infty$; (3) $x_1=187$ m.

(b) Curvilinear Motion

383. A 18-kgf body moves in the air and traces out the paths, represented in Fig. 258. Its initial velocity is $v_0=700$ m/sec. There are two cases: (1) when the direction of the dropping of the body makes an angle of 45° with the horizontal, and (2) when the angle equals 75° For both cases determine the increase in the altitude reached (in kilometres), and increase in the range of flight, if there is no air resistance.

Ans. The increase in altitude equals (1) 7.5 km; (2) 12 km; the increase in the range of flight equals (1) 36.5 km; (2) 16.7 km.

384. A plane A flies at an altitude of 4000 m with the horizontal speed of 500 km/h. At what distance x, measured on the horizontal line from a given point B (see Fig. 259), should a weight be dropped from the plane to reach this

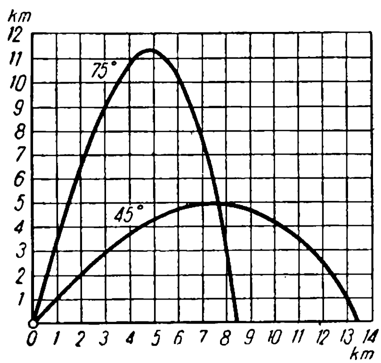


Fig. 258

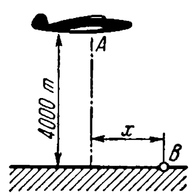


Fig. 259

point B? The weight is dropped without any initial velocity. The air resistance may be neglected.

Ans. $x=3960$ m.

385. A body is projected at an angle α to the horizontal. Its horizontal range is l_α Find the horizontal range when the angle of projection equals $\frac{\alpha}{2}$ Neglect the effect of the air resistance.

Ans. $l_{\frac{\alpha}{2}}=\frac{l_\alpha}{2\cos\alpha}$.

386. Find the range of flight of a body if the radius of curvature of its path at its highest point is $\rho=16$ km and $\alpha=30°$ is the angle of projection. Neglect air resistance.

Ans. $x_{max}=2\rho\tan\alpha=18{,}480$ m.

387. Several particles are simultaneously thrown in vertical plane from one point. Having the same initial velocity v_0, all particles are projected at different angles to the horizontal. Find geometrical positions of all particles at some time t.

Ans. A circle of radius v_0t with the centre, located on the vertical through the point of projection, is $\frac{1}{2}gt^2$ lower than the position of this point.

388. A particle of weight P is projected at angle α to the horizontal with the initial velocity v_0. It moves under the action of gravity and air resistance R. Determine the maximum height h attained by the body above the initial position, assuming that the resistance is proportional to the velocity $R=kpv$. Derive the equation of the path of the particle. Also determine the distance s along the horizontal where the particle attains its highest position.

Ans. $h=\frac{v_0\sin\alpha}{gk}-\frac{1}{gk^2}\ln(1+kv_0\sin\alpha)$;

$$x=\frac{v_0\cos\alpha}{kg}(1-e^{-kgt});$$

$$y=\frac{1}{kg}\left(v_0\sin\alpha+\frac{1}{k}\right)(1-e^{-kgt})-\frac{t}{k};$$

$$s=\frac{v_0^2\sin 2\alpha}{2g(kv_0\sin\alpha+1)}.$$

389. A vertical pipe is placed in the centre of a round basin and its top is tightly sealed. On the pipe at a height of one metre some small holes are made which eject the water at different angles φ to the horizontal $\left(\varphi<\frac{\pi}{2}\right)$ the initial velocity of the jet

is $v_0=\sqrt{\frac{4g}{3\cos\varphi}}$ m/sec, where g is the acceleration due to gravity. Determine the smallest radius R of the basin for which all the water falls into it, independent of the height of its wall.

Ans. $R=2.83$ m.

390. A heavy particle of mass m grams is attracted to a fixed centre O by a force directly proportional to the distance. The motion is performed in vacuum. The attractive force per unit distance is k^2m dynes. Find the path of the particle at the moment, when $t=0$: $x=a$; $\dot{x}=0$; $y=0$; $\dot{y}=0$, and the axis Oy is directed vertically downwards.

Ans. A harmonic oscillation: $x=a\cos kt$; $y=\frac{g}{k^2}(1-\cos kt)$

along the intercept of the straight line: $y=\frac{g}{k^2}-\frac{g}{k^2a}x, |x|\leqslant a.$

391. A particle of mass m is repelled from a fixed centre O by a force which changes in accordance with the law: $F=k^2mr$, where r is a radius vector of the particle. Initially the particle was located at $M_0(a, 0)$ and had a velocity v_0 directed parallel to the axis y. Find the path of the particle.

Ans. $\left(\frac{x}{a}\right)^2-\left(\frac{ky}{v_0}\right)^2=1$ (hyperbola).

392. A flexible thread, fixed at the point A, passes through a smooth fixed ring O (Fig. 260). A small ball of mass m grams is attached to a free end of the thread. The natural length of the thread is $l=AO$. A force equal to k^2m dynes must be applied to elongate the thread 1 cm. When the thread is stretched along the straight line AB until its length is doubled, then the ball is given a velocity v_0, perpendicular to AB. Find the path of the ball. Neglect the effect of gravity and assume that the tension in the thread is proportional to its extension.

Ans. The ellipse: $\frac{k^2x^2}{v_0^2}+\frac{y^2}{l^2}=1.$

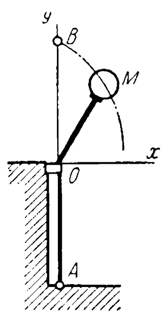


Fig. 260

393. A particle M is attracted to two centres C_1 and C_2 by forces proportional to the first power of the distances: $km\times\overline{MC}_1$ and $km\times\overline{MC}_2$. The centre C_1 is fixed at the origin but the centre C_2 moves uniformly along the axis Ox, thus $x_2=2(a+bt)$. Find the path of the particle M, assuming that, when $t=0$, the particle M is in the plane xy, its coordinates being $x=y=a$, and the projections of the velocity are: $\dot{x}=\dot{z}=b$, $\dot{y}=0$.

Ans. The helix is located on an elliptic cylinder with axis Ox, and equation is $\frac{y^2}{a^2}+\frac{2kz^2}{b^2}=1$, the screw pitch being $\pi b\sqrt{\frac{2}{k}}$.

394. *Deviation of cathode beams in an electric field.* A particle of mass m, carrying a negative electric charge e, moves with velocity v_0 in a uniform electric field of intensity E, and in the direction perpendicular to the field intensity. Determine the path of the further motion of the particle, assuming that in the electric field it is under the action of a force $F=eE$ which is directed to the side opposite to the intensity E. Neglect the force of gravity.

Ans. It is a parabola whose parameter is $\frac{mv_0^2}{eE}$.

395. *Deviation of cathode beams in a magnetic field.* A particle of mass m, carrying a negative electric charge e, enters into a uniform magnetic field of intensity H with a velocity v_0, perpendicular to the direction of the field intensity (Fig. 261). Find the path of the further motion of the particle, assuming that it is under the action of a force $\mathbf{F}=-e(\mathbf{v}\times\mathbf{H})$.

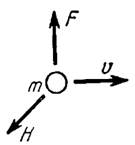


Fig. 261

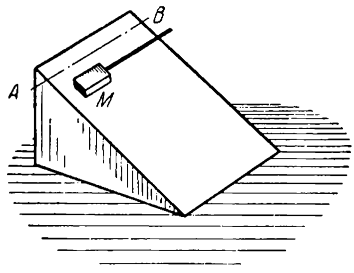


Fig. 262

H i n t. When solving this problem it is advisable to use the equation of motion of a particle as the projections on the tangent and on the main normal to the path.

Ans. A circumference of radius $\frac{mv_0}{eH}$.

396. Fig. 262 shows a heavy body M moving on a smoothless inclined plane and constantly stretched horizontally by a thread parallel to a straight line AB. At a certain instant the motion of the body becomes rectilinear and uniform while one of two mutually perpendicular components of the velocity, parallel to AB, equals 12 cm/sec. Determine the second component v_1 of the velocity and the tension T in the thread. The other data are as follows: the inclination of the surface is $\tan\alpha=\frac{1}{30}$, the coefficient of friction is $f=0.1$, the weight of the body is 0.3 kgf.

Ans. $v_1=4.24$ cm/sec; $T=0.0283$ kgf.

28. Theorem on Change of Momentum of a Particle. Theorem on Change of Angular Momentum of a Particle. Motion under the Action of Central Forces

397. A train runs on a horizontal and rectilinear track. When the brakes are applied the resistance equals 0.1 of the train weight. At the instant when braking starts the train is running at a speed of 72 km/h. How long and how far will the train travel after the brakes are applied?

Ans. 20.4 sec; 204 m.

398. A heavy body slides down a rough plane inclined at an angle $\alpha=30°$ to the horizontal. The body started its motion without the initial velocity. Find the time T taken to slide a distance $l=39.2$ m, if the coefficient of friction is $f=0.2$.

Ans. $T=5$ sec.

399 A weight M is tied to the end of an inextensible thread MOA, as shown in Fig. 263. A part OA of the thread passes through a vertical tube. The weight rotates about an axis of the tube along the circular path of radius $MC=R$, making 120 rpm. When the thread OA is slowly pulled into the tube, then the external part of the thread is shortened to length OM_1 and the weight describes the circular path of radius $\frac{1}{2}R$. Find the speed of rotation along this circular path.

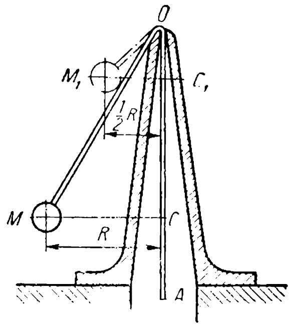


Fig. 263

Ans. 480 rpm.

400. A dynamometer is installed between the locomotive and the railway cars to determine the weight of a loaded train. After 2 min of motion the average reading of the dynamometer is 100,800 kgf and the train has reached a speed $v=57.6$ km/h (initially the train was at rest). The coefficient of friction is $f=0.02$. Determine the weight of the train.

Ans. The weight of the train is 3,000,000 kgf.

401. What should be the coefficient of friction f between the automobile tires and the road, if at the speed of 72 km/h the automobile is brought to rest in 6 sec after the brakes are applied.

Ans. $f=0.34$.

402. A particle M rotates about a fixed centre under the action of an attractive force directed towards this centre (Fig. 264).

Assuming that the velocity at the position nearest to the centre is $v_1=30$ cm/sec and r_2 is five times that of r_1, determine the velocity v_2 at the point on the path furthest away from the centre.

Ans. $v_2=6$ cm/sec.

403. Two meteorites M_1 and M_2 describe the same elliptical orbit with the sun at one focus S, as shown in Fig. 265. The distance between the meteorites is so small that the arc M_1M_2 of the ellipse can be considered as a straight line. It is assumed that the distance

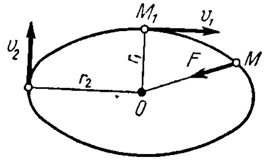


Fig. 264

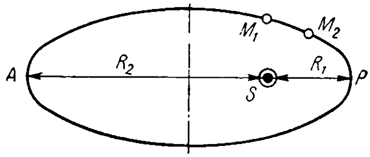


Fig. 265

M_1M_2 is a when its central point is at perihelion P. Provided that both meteorites have the same areal velocities, determine the distance M_1M_2 when its central point is at aphelion A. It is known that $SP=R_1$ and $SA=R_2$.

Ans. $M_1M_2=\dfrac{R_1}{R_2}\,a.$

404. A small ball of weight p knotted to an inelastic string slides along a smooth horizontal surface. A free end of the string is pulled through a hole in the plane with a constant velocity a. At the origin the string is straight, the distance between the hole and the ball is R, and the projection of the initial velocity of the ball on the direction normal to the string is v_0. Determine the motion of the ball and the tension T in the string.

Ans. Considering the hole as the origin of polar coordinates and an angle $\varphi_0=0$, the motion of the ball is given by

$$r=R-at;\quad \varphi=\frac{v_0t}{R-at}\qquad T=\frac{pv_0^2R^2}{g\,(R-at)^3}$$

405. The following data are given: the radius of the earth is $R=637\times10^6$ cm, its average density is 5.5; a major semi-axis of the terrestrial orbit a is 149×10^{11} cm, the period of rotation of the earth about the sun is $T=365.25$ days. It is assumed that the celestial force of gravitation between two masses each of 1 gram at 1-cm distance equals $\dfrac{gR^2}{m}$, where m is the mass of the earth. According to Kepler's law, the value of the force of gravitation between the earth and the sun is $\dfrac{4\pi^2a^3}{T^2}\times\dfrac{m}{r^2}$, where r is the dis-

tance between the earth and the sun. Determine the mass M of the sun.

Ans. $M = 197 \times 10^{31}$ g.

406. A particle of mass m moves under the action of a central force F It describes a lemniscate: $r^2 = a \cos 2\varphi$, where a is a constant, and r is the distance between the particle and the centre of attraction. At the initial moment $r = r_0$ the velocity of the particle is v_0, and it forms an angle α with the straight line connecting the particle and the centre of attraction. Determine the magnitude of the force F as a function of the distance r

Hint. In accordance with Binet's formula

$$F = -\frac{mc^2}{r^2}\left(\frac{d^2 \frac{1}{r}}{d\varphi^2} + \frac{1}{r}\right),$$

where c is a double areal velocity of the particle.

Ans. The force of attraction is $F = \frac{3ma^2}{r^7} r_0^2 v_0^2 \sin^2\alpha$.

407. A particle M of mass m moves about a fixed centre O under the action of a force F which is originated from this centre and depends only on the distance $MO = r$ The velocity of the particle is $v = \frac{a}{r}$, where a is a constant. Determine the magnitude of the force F and the path of the particle.

Ans. The force of attraction is $F = \frac{ma^2}{r^3}$ and the path is a logarithmic spiral.

408. Determine the motion of a particle of mass 1 g acted on by a central force of attraction proportional to the inversed cube of the distance between the particle and the centre of force. Other data are as follows: at a distance of 1 cm the force value is 1 dyne, at the initial moment the distance between the particle and the centre is $r_0 = 2$ cm, the velocity $v_0 = 0.5$ cm/sec, and it makes an angle of 45° with the straight line, drawn from the centre to the particle.

Ans. $r = 2e^{\varphi}$; $r^2 = 4 + t\sqrt{2}$.

409. A particle of mass m moves under the action of a central force along the path, defined by polar coordinates as: $r = \frac{p}{1 + e\cos\varphi}$, where p and e are constant coefficients, and the centre of attraction is located at the pole of the coordinate system. Find the value of the force which acts on the particle.

Ans. $F_r = -m\frac{h^2}{pr^2}$, $F_{\varphi} = 0$, where $h = r^2\frac{d\varphi}{dt} = \text{const}$.

410. A particle M of mass 1 g is attracted to a fixed centre O by a force inversely proportional to the fifth power of the distance. At 1-cm distance the force equals 8 dynes. Initially the particle is at distance $OM_0=2$ cm, and its velocity is perpendicular to OM_0 and equal to $v_0=0.5$ cm/sec. Determine the path of the particle.

Ans. The circumference of a circle of 1-cm radius.

411. A particle of mass 20 g is attracted by a force to a fixed centre in accordance with Newton's law of gravitation. During 50 sec the particle completely traces out an ellipse with semi-axes 10 and 8 cm long, respectively. Determine the maximum and minimum magnitudes of the force of gravitation F

Ans. $F_{max}=19.7$ dynes; $F_{min}=1.2$ dynes.

29. Work and Power

412. A weight of 2000 kgf is moved up a plane inclined at an angle of 30° to the horizontal. The coefficient of friction is 0.5. Determine the minimum work which should be done to lift the weight to a height of 5 m.

Ans. 18,660 kgfm = 183 kilojoules.

413. A pump driven by 2-hp engine rises 5000 m³ of water to a height of 3 m. The efficiency of the pump is 0.8. How long does it take for the pump to do the work?

Hint. The efficiency of the engine is called the ratio of the useful work done (in this case the work done to lift the water) to the work expended. The latter should be of greater magnitude than that of the useful work due to resistance effects.

Ans. 34 h 43 min 20 sec.

414. Find the power in hp and kw of a machine which lifts a 200-kgf hammer to a height of 0.75 m 84 times per minute. The efficiency of the machine is 0.7.

Ans. 4 hp = 2.94 kw.

415. Compute the power of a turbo-generator at a tram power station if the number of tram cars is 45, each weighing 10,000 kgf. The frictional resistance is 0.02 of the car weight, and the average speed of each car is 12 km/h. The line losses equal 5 per cent.

Ans. 421 hp = 309 kw.

416. A weight of 20 kgf is to be lifted up an inclined plane through a distance of 6 m. The plane makes an angle of 30° to the horizontal and the coefficient of friction is 0.01. Compute the work done to lift the weight.

Ans. 61.04 kgfm = 598 joules.

417. When a ship travels at a speed of 15 knots its engine develops power equal to 5144 hp. Determine the value of water resistance to the motion of the ship, if the overall efficiency of the engine and the propeller is 0.4 and 1 knot$=$0.5144 m/sec.

Ans. 20,000 kgf.

418. In a steam engine an average vapour pressure on 1 cm^2 of a piston area equals 5 kgf for one complete stroke. The length of a piston stroke is 40 cm, the piston area is 300 cm^2. The engine makes 120 working strokes per minute, and its overall efficiency is 0.9. Determine the power of the engine in hp and kw.

Ans. 14.4 hp$=$10.6 kw.

419. A grinding wheel of 60-cm diameter makes 120 rpm. Its power consumption is 1.6 hp. The coefficient of friction between the grinding wheel and the work piece is 0.2. Determine the force which is exerted by the grinding wheel on the machined piece.

Ans. 160 kgf.

420. Determine the power of the electric motor of a planer if the length of its working stroke is 2 m with the time period of 10 sec. The cutting force equals 1200 kgf, and the efficiency of the machine is 0.8. The motion is assumed to be uniform.

Ans. 4 hp.

421. To determine the power of an electric motor, a pulley A of diameter d=63.6 cm is wedged to the shaft, as shown in Fig. 266. A band passes over the pulley. The right side BC of the band is held by a spring scales Q, and the left DE is pulled down by the 1-kgf weight. The speed of the electric motor is 120 rpm, and at this instant the reading on the spring scales shows a tension of 4 kgf in the right side. Compute the power of the motor.

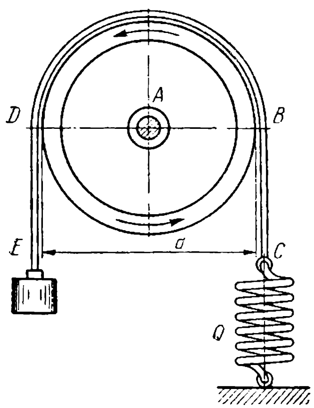


Fig. 266

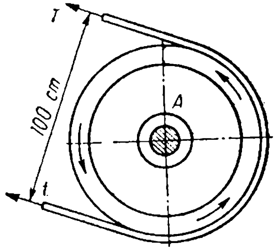


Fig. 267

Hint. The difference between the tension of the side BC and that of the side DE equals the force resisting the motion of the pulley. The work done by this force is assumed to be done for 1 sec.

Ans. 0.16 hp = 117.8 w.

422. Fig. 267 shows a belt which is used to transmit a power of 20 hp. The radius of the belt pulley is 50 cm and its velocity equals 150 rpm. Assuming that the tension T of the tight side of the belt is twice the tension t of the slack side, determine the tensions T and t.

Ans. $T = 382$ kgf; $t = 191$ kgf.

30. Theorem on Change of Kinetic Energy of a Particle

423. A heavy body initially at rest slides down an inclined plane which makes an angle of 30° with the horizontal. The coefficient of friction is 0.1. What is the velocity of the particle after passing 2 m from the start?

Ans. 4.02 m/sec.

424. A 3-kgf particle moves to the left on a horizontal line with the velocity of 5 m/sec. A constant force, directed to the right, is applied to the particle. In 30 sec the force value is reduced to 0 and then the velocity of the particle becomes 55 m/sec towards the right. Determine the value of the force and the work it has done.

Ans. 0.612 kgf; 459 kgfm = 4.5 kilojoules.

425. A train is ascending a slope of inclination $\alpha = 0.008$ radian at the speed of 36 km/h. At a certain instant an engineer sees a danger and applies the brakes. The friction resistance in the axles due to braking equals 0.1 of the train weight. How far does the train travel before coming to rest, and how long does it take? It is assumed that $\sin\alpha = \alpha$.

Ans. 55.3 m; 11.06 sec.

426. A nail is driven into a wall whose resistance is $R = 70$ kgf. With each blow of the hammer the nail is driven $l = 0.15$ cm deeper into the wall. If the velocity of the hammer with each blow equals $v = 1.25$ m/sec, find its weight.

Ans. $P = 1.37$ kgf.

427. In 1751 a meteorite weighing 39 kgf fell to the earth. On reaching the earth it penetrated into the ground to a depth of $l = 1.875$ m. The investigations in the vicinity of the collision showed that the resisting force of the ground was $F = 50.000$ kgf. Find the velocity with which the meteorite reached the surface of

the earth. Determine the height from which the meteorite started its motion and attained the above-mentioned velocity before striking the ground. The force of gravitation is assumed to be constant and air resistance may be neglected.

Ans. $v=217$ m/sec; $H=2390$ m.

428. A train weighing $P=500{,}000$ kgf runs with its engine shut off. The brakes are not applied. The train is under the action of the resistance to motion $R=(765+51\,v)$ kgf, where v is the velocity in m/sec. The initial velocity of the train is $v_0=15$ m/sec. Determine the distance the train travels before coming to rest.

Ans. $s=4.6$ km.

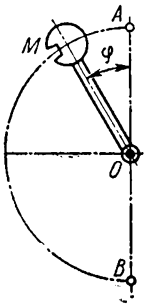


Fig. 268

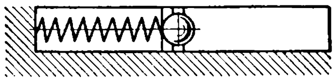


Fig. 269

429. A heavy steel casting M forms the main part of a device designed for testing different materials by impact. M is connected with a rod which is free to rotate without friction about a fixed horizontal axle O, as shown in Fig. 268. Neglecting the mass of the rod and considering the steel casting M as a particle for which OM equals 0.981 m, determine the velocity v of this particle at its lowest position B, when it falls from rest from the highest position A.

Ans. $v=6.2$ m/sec.

430. Write down the expression for the potential energy of an elastic spring deflected 1 cm due to a 400-kgf load. The deflection x increases proportionally to the pull exerted on the spring.

Ans. $V=0.2x^2+\text{const}$.

431. A spring of the mechanism, shown in Fig. 269, has a free length of 20 cm. To compress the spring for 1 cm a 0.2-kgf force is required. Determine the velocity v with which the 0.03-kgf ball will leave the muzzle, if the spring is compressed to a length of 10 cm and the system is held in horizontal position.

Ans. $v=8.1$ m/sec.

432. The static deflection of a beam, loaded by a weight Q at its centre, is 2 mm. Neglecting the mass of the beam, determine its maximum deflection under two conditions: (1) when the weight

Q is put on the undeflected beam and dropped without any initial velocity; (2) when the weight Q falls from rest on the centre of the undeflected beam from a height of 10 cm.

Hint. It should be taken into consideration that the force, which acts on the weight of the beam, is proportional to the deflection of the beam.

Ans. (1) 4 mm; (2) 22.1 mm.

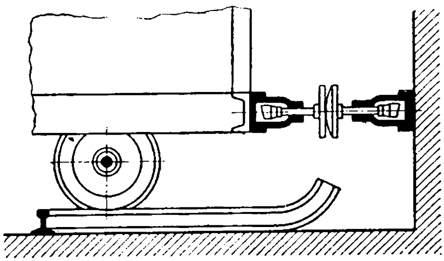


Fig. 270

433. A 16,000-kgf railway car strikes two fixed spring buffers with the velocity of 2 m/sec (Fig. 270). Determine the maximum compression of the buffer springs under the impact of the car, assuming that the car and buffer springs are identical and when acted on by a 5000-kgf force they are compressed a distance of 1 cm.

Ans. 5.7 cm.

434. Two unstrained springs AC and BC lie along the horizontal line Ax. They are hinged at fixed points A and B. At the point C the free ends of both springs are attached to a 1.962-kgf weight. The spring AC is compressed to a length of 1 cm by a force of 2 kgf, and the spring CB is stretched out to a length of 1 cm by a 4-kgf force. $AC=BC=10$ cm. The weight is constrained to move with the velocity $v_0=2$ m/sec in such a direction that it subsequently passes through the point D with the coordinates: $x_0=8$ cm, $y_0=$ $=2$ cm. The point A is the origin of coordinates, which are shown in Fig. 271. Determine the velocity of the weight at the instant when the latter passes through D located on the vertical plane xy.

Ans. $v=1.78$ m/sec.

435. Fig. 272 shows a load M weighing P, which is suspended from a point O, by a weightless unstretched thread of length l. The load M is released from rest and moves in vertical plane

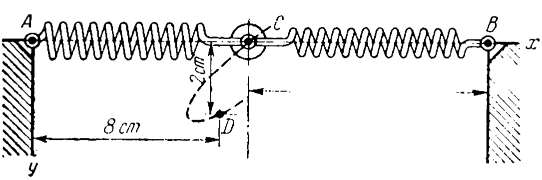


Fig. 271

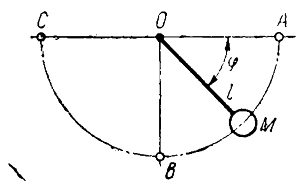


Fig. 272

starting from A. Being free of any resistance effect, the load M reaches the point C where its velocity is zero. Assuming that the potential energy due to gravity of the load M at the point B is zero, plot the graphs of the changes of kinetic and potential energy as well as their total values as function of the angle φ.

Ans. Two sinusoids and one straight line, defined by the equations: $T = Pl \sin\varphi$, $V = Pl(1 - \sin\varphi)$, $T + V = Pl$.

436. A particle falls to the earth from the height equal to the radius of the earth. Find the value of the constant vertical force which should be applied to the particle to make it move at the speed equal to that of the gravitational force of the earth. The force is inversely proportional to the square of the distance between the particle and the centre of the earth.

Ans. $\frac{P}{2}$, where P is the weight of the particle at the surface of the earth.

437. A horizontal spring with a particle attached to its end is compressed by a force P and is held at rest. Suddenly the force P changes its direction opposite to the previous one. How many times is the maximum extension l_2 of the spring longer compared to its initial compression l_1? Neglect the mass of the spring.

Ans. $\frac{l_2}{l_1} = 3$.

438. A body is projected vertically upwards from the surface of the earth with an initial velocity v_0. Determine the height H which the body will reach taking into consideration that the force of gravitation is inversely proportional to the square of the distance from the centre of the earth. The radius of the earth is $R = 6370$ km, $v_0 = 1$ km/sec. Neglect the effect of air resistance.

Ans. $H = \dfrac{Rv_0^2}{2gR - v_0^2} = 51$ km.

439. Express the speed v_0 which is required for a rocket to be projected from the surface of the earth in the direction of the moon. The rocket is intended to reach the point, where the gravitational forces of the earth and the moon are equal. Neglect the effects of motion of the earth and the moon as well as the air resistance. The acceleration due to gravity at the surface of the earth is $g = 9.8$ m/sec^2. The ratio of the mass of the moon to that of the earth is $m : M = 1$ 80, and the distance between them is $d = 60R$, where $R = 6000$ km (the radius of the earth).

The coefficient f for the gravitational force is defined by the equation

$$m_1 g = m_1 f \left[\frac{M}{R^2} - \frac{m}{(d-R)^2}\right].$$

Ans. $$v_0^2 = \frac{2gR\,(d-R)}{d}\,\frac{\sqrt{\frac{M}{m}}\,(d-R)-R}{\sqrt{\frac{M}{m}}\,(d-R)+R} = \frac{59}{30}\,\frac{1-\alpha}{1+\alpha}\,gR,$$

where $\alpha = \frac{1}{59\sqrt{80}}$, or $v_0 = 10.75$ km/sec.

31. Review Problems

440. A 1-kgf weight is suspended by a string 50 cm long to a fixed point O. Initially the weight is displaced at an angle of 60° to the vertical and is given a velocity $v_0 = 210$ cm/sec in the vertical plane along the perpendicular to the string directed downwards. Determine: (1) the tension in the string in the lowest position; (2) the height at which the weight is lifted above the lowest position, counted along the vertical; (3) the value of the velocity v_0, required for the weight to move on the circumference of a circle.

Ans. (1) 2.9 kgf; (2) 47.5 cm; (3) $v_0 > 443$ cm/sec.

441. A trolley weighing P runs down an inclined track AB which then forms a circular loop of radius a (Fig. 273). Determine the height h from which the trolley should start from rest in order to run round the circular loop without leaving the surface of the latter. Find also the value of the pressure N of the trolley on the loop at a point M, where the angle MOB equals φ.

Ans. $h \geqslant 2.5a$; $N = P\left(\frac{2h}{a} - 2 + 3\cos\varphi\right)$.

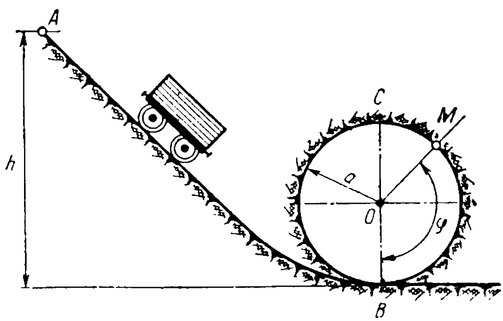


Fig. 273

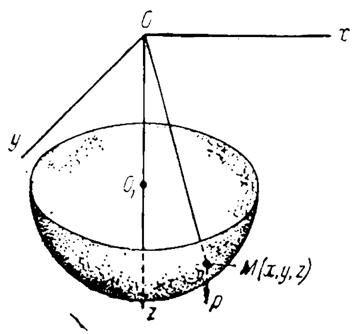


Fig. 274

442. A parachute jumper weighing 70 kgf jumps from a plane, and after falling down 100 m he opens his parachute. If for the first 5 sec after opening the canapy the falling speed diminishes to 4.3 m/sec, find the tension force in the parachute shroud lines. The force resisting the motion is assumed to be constant. The air resistance to motion may be neglected.

Ans. 127.4 kgf.

443. Fig. 274 represents a spherical pendulum that has a thread OM of length l which is fastened at one end to a fixed point O and at the other end to a heavy particle M of weight P The particle M is displaced from the position of equilibrium so that its coordinates are: $x=x_0$, $y=0$ at time $t=0$ and its initial velocity is: $\dot{x}_0=0$, $\dot{y}_0=v_0$, $z_0=0$. Find the initial data required for which the particle M will trace a circular path, and the time necessary for the particle to go around the circumference of this path.

Ans. $v_0=x_0\sqrt{\frac{g}{z_0}}$; $T=2\pi\sqrt{\frac{z_0}{g}}$

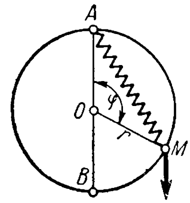


Fig. 275

444. A weight $M=5$ kgf is suspended from a spring at an upper point A on a circular ring located in a vertical plane. The weight slides down the ring without friction (Fig. 275) What must be the stiffness of the spring to make the pressure of the weight on the ring at its bottom point B equal to 0, if the other data are as follows: the radius of the ring is 20 cm, M weighs 5 kgf, the distance AM is 20 cm initially, and the spring has its natural length. The initial velocity of the weight is zero. Neglect the weight of the spring.

Ans. The spring must be stretched by 1 cm under a force of 0.5 kgf.

445. Fig. 276 shows a weight of 1 kgf which is suspended by a thread 50 cm long to a fixed point O. In the initial position M_0 the weight is deflected from the vertical at an angle of 60°, and here it is given a velocity $v_0=350$ cm/sec in the vertical plane downwards and perpendicular to the thread.

(1) Find the position M of the weight when the tension in the thread is zero, and also the velocity v_1 at this position.

(2) Determine the path of the subsequent motion of the weight up to the time when the thread is again in tension. Also find the time taken to cover this path.

Ans. (1) M is located above the horizontal through the point O at the distance $MD=25$ cm; $v_1=157$ cm/sec.

(2) The path of the weight after it passes the position M is a parabola $MABC$ defined by axes Mx and My and expressed by the equation $y=x\sqrt{3}-0.08x^2$; the time taken is 0.55 sec.

446. A weight M of mass m is suspended by a thread OM of length l to a fixed point O (Fig. 277). When $t=0$, the thread OM makes an angle α with the vertical, and the velocity of the weight

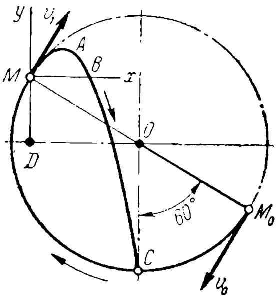


Fig. 276

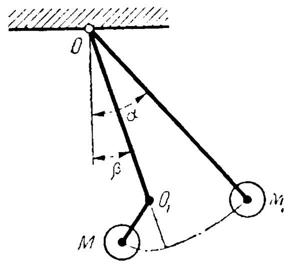


Fig. 277

M is zero. During the subsequent motion the thread strikes a thin wire O_1 perpendicularly to the plane of motion of the weight, and the position of the wire O_1 is defined by polar coordinates: $h=OO_1$ and β. Determine the minimum value of the angle α for which the thread OM will wind around the wire when striking it. Also determine the change in tension of the thread at the moment when the latter strikes the wire. Neglect the gauge of the wire.

Ans. $\alpha=\arccos\left[\frac{h}{l}\left(\frac{3}{2}+\cos\beta\right)-\frac{3}{2}\right]$;
the tension in the thread increases by the magnitude $2mg\,\frac{h}{l}\left(\frac{3}{2}+\cos\beta\right)$.

447. A particle of mass m moves on the inside surface of a circular cylinder of radius r (Fig. 278). The surface is assumed to be absolutely smooth. The axis of the cylinder is vertical. Taking into consideration the effect of gravity, determine the pressure of the particle on the cylinder surface. The initial velocity of the particle is v_0, and makes an angle α with the horizontal. If, at starting, the particle was on the axis x, set up the equation of motion of the particle.

Ans. $N=\frac{mv_0^2\cos^2\alpha}{r}$; $x=r\cos\left[\frac{v_0\cos\alpha}{r}t\right]$; $y=r\sin\left[\frac{v_0\cos\alpha}{r}t\right]$;
$z=v_0t\sin\alpha+\frac{gt^2}{2}$.

448. A stone M rests on the top of a smooth hemispherical dome of radius R, as shown in Fig. 279. It is given an initial horizontal velocity v_0. At which point will the stone leave the surface of the dome? At what values of v_0 will the stone leave the surface of the dome when starting its motion? Neglect the force resisting the motion of the stone on the surface of the dome

Ans. $\varphi = \arccos\left(\frac{2}{3} + \frac{v_0^2}{3gR}\right)$; $\quad v_0 \geqslant \sqrt{gR}$.

449. A particle of mass m moves on a catenary

$$y = \frac{a}{2}\left(e^{\frac{x}{a}} + e^{-\frac{x}{a}}\right) = a \operatorname{ch} \frac{x}{a}$$

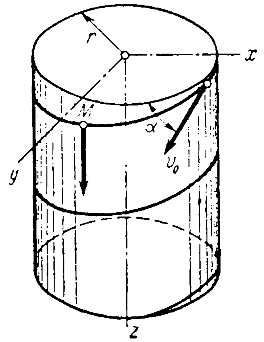


Fig. 278

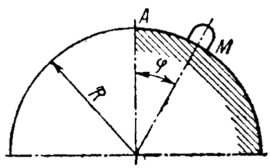


Fig. 279

under the action of a force of repulsion which is parallel to the axis Oy, is directed from the axis Ox and equal to kmy (Fig. 280). When $t=0$, $x=1$ m and $\dot{x}=1$ m/sec. Determine the force N exerted by the particle on the curve, and the motion of the particle, when $k=1$ sec^{-2} and $a=1$ m. The radius of the curve of the catenary is $\frac{y^2}{a}$. Neglect the effect of gravity.

Ans. $N=0$; $x=(1+t)$ m.

450. What shape should be given to a pipe so that a small ball located in any part of the pipe is in equilibrium, if the pipe rotates with a constant angular velocity ω about the axis Oy (Fig. 281)?

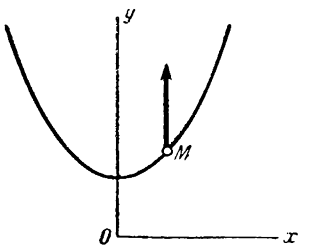


Fig. 280

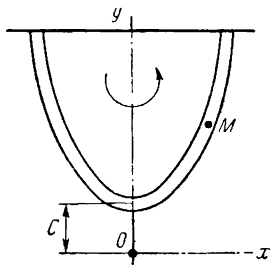


Fig. 281

Ans. A bent pipe is in the shape of the parabola

$$y=\frac{1}{2}\frac{\omega^2}{g}x^2+c.$$

451. A particle M of mass $m=1$ g is on a smooth surface of a circular cone, with semi-angle α given by $2\alpha=90°$ (Fig. 282). It moves from the vertex O under the action of a force of repulsion which is proportional to the first power of the distance: $F=c\times OM$ dynes, where $c=1$ dyne/cm. When $t=0$, the particle M is at A, and the distance OA equals $a=2$ cm. The initial velocity $v_0=2$ cm/sec is directed parallel to the base of the cone. Neglecting gravity, determine the motion of the particle M.

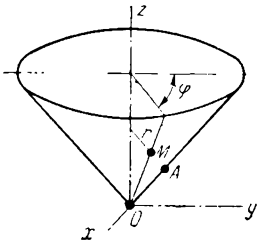


Fig. 282

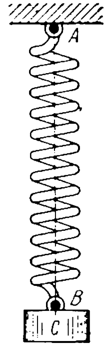


Fig. 283

Hint. The position of the particle M is defined by a coordinate z and polar coordinates r and φ in the plane perpendicular to the axis Oz; the equation of the surface of the cone is $r^2-z^2=0$.

Ans. $r^2=e^{2t}+e^{-2t}$; $\tan\left(\frac{\varphi}{\sqrt{2}}+\frac{\pi}{4}\right)=e^{2t}$.

452. The data given are the same as in the preceding problem, except that the axis of the cone is directed vertically upwards. Taking into consideration the effect of gravity, determine the value of the pressure of the particle on the surface of the cone.

Ans. $N=m\sin\alpha\left[g+\frac{a^2v_0^2\sin 2\alpha}{2r^3}\right]$.

32. Oscillations

453. A spring AB is attached with one end to the point A, as shown in Fig. 283. To stretch the spring by 1 cm a force of 0.02 kgf should be applied at the point B under a static load. At a certain moment a 0.1-kgf weight C is fastened to the free end of the unstrained spring, and then the weight is dropped from rest. Neglecting the mass of the spring, derive the equation for the

subsequent motion of the weight and also find the amplitude and the period of oscillations of the spring. The motion of the weight is directed along the axis vertically downwards from the position of static equilibrium.

Ans. $x=-5\cos 14t$ cm; $a=5$ cm; $T=0.45$ sec.

454. A weight Q falling from a height $h=1$ m from rest strikes the centre of a flexible horizontal beam. Both ends of the beam are fixed. Write down the equation for the subsequent motion of the weight on the beam. This motion is determined relative to the axis directed vertically downwards from the position of static equilibrium. The static deflection of the beam at its centre due to the load is 0.5 cm. Neglect the mass of the beam.

Ans. $x=(-0.5\cos 44.3t+10\sin 44.3t)$ cm.

455. Each spring of a railway car carries the load of P kgf. When in equilibrium, the spring is deflected 5 cm due to the load applied. Determine the period T of natural vibrations of the car on the springs. The stiffness of the spring is proportional to the deflection sag of the spring.

Ans. $T=0.45$ sec.

456. Determine the period of free vibrations of a machine foundation mounted on a resilient ground. The overall weight of the foundation and the machine is $Q=90{,}000$ kgf and the area of the basement of the foundation is $S=15$ m^2. The coefficient of stiffness of the ground is $c=\lambda S$, where $\lambda=3$ kgf/cm^3 is a so called specific stiffness of the ground.

Ans. $T=0.09$ sec.

457. Determine the period of free oscillations of a weight Q attached to two parallel springs and also find the coefficient of stiffness of the spring equivalent to that of the given double spring, if the weight is attached in such a way that the elongations of both springs, having set coefficients of stiffness c_1 and c_2, are the same (Fig. 284).

Ans. $T=2\pi\sqrt{\frac{Q}{g(c_1+c_2)}}$; $c=c_1+c_2$;

the location of the weight is such that $\frac{a_1}{a_2}=\frac{c_2}{c_1}$.

458. Determine the period of free oscillations of a weight Q sqeezed between two springs with different coefficients of stiffness c_1 and c_2 (Fig. 285).

Ans. $T=2\pi\sqrt{\frac{Q}{g(c_1+c_2)}}$.

459. Determine the coefficient of stiffness c of the spring, which is equivalent to a double spring consisting of two successively connected springs with different coefficients of stiffness c_1 and c_2 (Fig. 286). Also find the period of oscillation of the weight Q which is attached to the double spring.

Ans. $c=\dfrac{c_1c_2}{c_1+c_2}$; $\quad T=2\pi\sqrt{\dfrac{Q(c_1+c_2)}{gc_1c_2}}$.

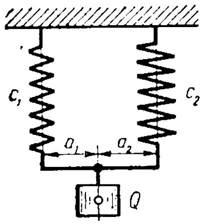


Fig. 284

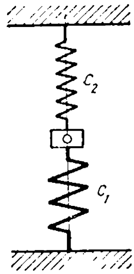


Fig. 285

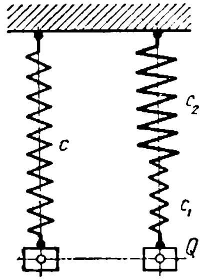


Fig. 286

460. A body weighing P gf is suspended by a thread to a fixed point. It starts vibrating from the position of equilibrium. During the motion the tension in the thread is proportional to its elongation and its free length is l. Due to the action of a static load equal to q gf, the thread is stretched by 1 cm. The initial velocity of the body is zero. Determine the length of the thread x as a function of time and also find the characteristic of the initial length of the thread x_0 required to keep the thread stretched when the motion of the body occurs.

Ans. $x=l+\dfrac{P}{q}+\left(x_0-l-\dfrac{P}{q}\right)\cos\left(\sqrt{\dfrac{qg}{P}}\,t\right)$; $\quad l\leqslant x_0\leqslant l+\dfrac{2P}{q}$.

461. A uniform rod rests on two cylindrical pulleys of equal radii rotating in opposite directions, as shown in Fig. 287 The centres of both pulleys O_1 and O_2 lie on a horizontal line O_1O_2 of length $2l$. The rod is set in motion by frictional forces developed at the points of contact with the pulleys. These forces are proportional to the pressure of the rod on the pulleys, and the coefficient of proportionality (coefficient of friction) is f.

(1) Determine the motion of the rod after its displacement from the position of symmetry at x_0, when $v_0=0$.

(2) Find the coefficient of friction f when $l=25$ cm assuming that the period of vibrations T of the rod is 2 sec.

Ans. (1) $x=x_0\cos\left(\sqrt{\dfrac{fg}{l}}\,t\right)$;

(2) $f=\dfrac{4\pi^2l}{gT^2}=0.25.$

462. A body of weight $Q=12$ kgf is attached to the end of a spring. It undergoes harmonic vibrations. The reading on the stop-watch shows that the body performed 100 complete vibrations per 45 sec. After this an additional weight $Q_1=6$ kgf is hung on the spring. Determine the period of vibrations when both weights are suspended from the spring.

Ans. $T_1=T\sqrt{\dfrac{Q+Q_1}{Q}}=0.55$ sec.

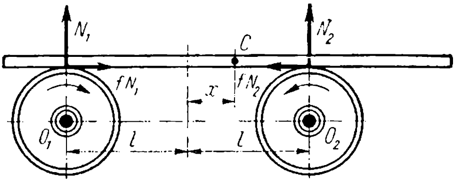


Fig. 287

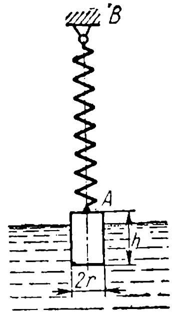


Fig. 288

463. To define the acceleration due to gravity at a certain place on the earth two experiments are conducted: a weight P_1 is suspended from the end of the spring, and the static elongation l_1 of the latter is measured. Then an additional weight P_2 is attached to the end of the same spring and the static elongation l_2 is registered again. Both experiments are repeated twice, and the periods of free vibrations T_1 and T_2, performed by one weight after the other, are measured. The second experiment is carried out to find out the effect of the mass of the spring itself, assuming that when the motion of the weight is performed, this effect is equivalent to the increase of some additional mass to the vibrating one.

Derive the formula for the acceleration due to gravity according to these experimental data.

Ans. $g=\dfrac{4\pi^2(l_1-l_2)}{T_1^2-T_2^2}$.

464. Fig. 288 shows a cylinder of radius r, height h and weight P It is suspended from a spring AB which is rigidly fixed at the end B. While being in the position of equilibrium the cylinder is immersed in water to a depth equal to $\dfrac{1}{2}h$. At the beginning the cylinder was immersed to $\dfrac{2}{3}$ of its height and then it was set in motion from rest directed along the vertical line. Assuming that the stiffness of the spring is c and the action of water is considered as an additional force of Archimedes, determine the motion of the cylinder with respect to the position of equilibrium. The specific weight of the water is γ.

Ans. $x=\dfrac{1}{6}h\cos kt$, where $k^2=\dfrac{g}{P}(c+\pi\gamma r^2)$.

465. A body A of weight 0.5 kgf rests on a rough horizontal plane and is connected at a fixed point B by a spring with horizontal axis BC (Fig. 289) The coefficient of friction between the plane and the body is 0.2. A force of 0.25 kgf is required to stretch the spring by 1 cm. The body A is moved from the point B in such a way that the spring is extended 3 cm and then it is released from rest. Determine: (1) the number of swings performed by the body A; (2) the magnitude of the swings; and (3) the duration T of each swing.

Hint. A body will be brought to rest when, the velocity being zero, the elastic force of the spring is either equal or less than the frictional force.

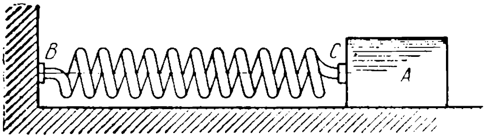


Fig. 289

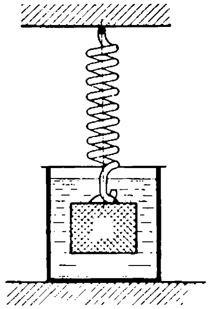


Fig. 290

Ans. (1) 4 swings;
(2) 5.2 cm; 3.6 cm, 2 cm, 0.4 cm;
(3) $T=0.141$ sec.

466. To determine the viscosity of a liquid Coulomb conducted the following experiment: a thin lamina A suspended from a spring was forced to oscillate first in the air and then in the liquid (Fig. 290). He found out that the duration of the swing is T_1 in the first case, and T_2 in the second one. The force of friction between the lamina and the liquid can be expressed by the formula $2Skv$, where $2S$ is the surface of the lamina, v is its velocity, and k is the coefficient of viscosity. Neglecting friction between the lamina and the air, determine the coefficient k as a function of T_1 and T_2, if the weight of the lamina is P

Ans. $k=\dfrac{\pi P}{gST_1T_2}\sqrt{T_2^2-T_1^2}$.

467. A 5-kgf body is suspended by a spring whose coefficient of stiffness is 2 kgf/cm. The resistance of the medium is proportional to the velocity. After four oscillations the amplitude decreases 12 times. Determine the period of oscillations and the logarithmic decrement of damping.

Ans. $T=0.319$ sec; $\dfrac{nT}{2}=0.311$.

468. A body of 5.88-kgf weight is suspended by a spring and oscillates with the period $T=0.4\ \pi$ sec, but, when a force of resistance which is proportional to the first power of the velocity acts on the body, the period becomes $T_1=0.5\ \pi$ sec. Find the force of resistance k when the velocity is 1 cm/sec, and also determine the motion if initially the spring was stretched 4 cm from equilibrium and then left alone.

Ans. $k=0.036$; $x=5e^{-3t}\sin\left(4t+\arctan\frac{4}{3}\right)$.

469. A body of weight 1.96 kgf is suspended by a spring which is stretched 20 cm by a force of 1 kgf. The body is acted upon by a force of resistance proportional to the first power of the velocity, and when the velocity is 1 cm/sec the force equals 0.02 kgf. Initially, the spring was stretched 5 cm from equilibrium, and the body started its motion from rest. Determine the motion of the body.

Ans. $x=5e^{-5t}(5t+1)$ cm.

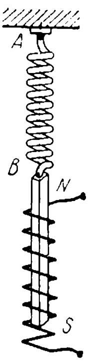


Fig. 291

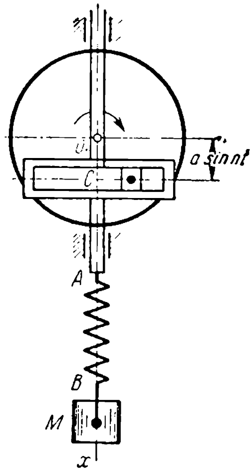


Fig. 292

470. A magnetic rod of weight 0.1 kgf is suspended from a spring of stiffness $c=0.02$ kgf/cm. The other end of the rod (Fig. 291) passes through a coil with an alternating current $i=20\sin 8\pi t$ amperes flowing through it. At the starting the rod hangs in the position of static equilibrium and then, when $t=0$, the current starts to flow causing a force that pulls the rod into the solenoid. The force of interacting between the magnet and the coil is defined by the equation $F=16\pi i$ dynes. Determine the forced oscillations of the magnet.

Ans. $x=-0.023\sin 8\pi t$ cm.

471. Fig. 292 shows a weight M suspended from a spring AB. The upper end of the spring performs harmonic oscillations along the vertical with an amplitude a and a frequency n: $O_1C=a\sin nt$ cm. Determine the forced oscillations of the weight M when the following data are given: M weighs 0.4 kgf, the spring is stretched 1 cm under the action of the force 0.04 kgf, $a=2$ cm, and $n=7$ sec^{-1}.

Ans. $x=4\sin 7t$ cm.

472. Referring to the previous problem, determine the motion of the weight M which is suspended from the spring AB. The upper end of the spring A performs harmonic oscillations along the vertical with an amplitude a and a frequency k. Under the action of the weight the static extention of the spring equals δ. Initially, the point A occupies its central position while the weight M is in equilibrium. The initial position of the weight is assumed to be the origin and the axis Ox is directed vertically downwards.

Ans. $x=\frac{ag}{k^2\delta-g}\left[k\sqrt{\frac{\delta}{g}}\sin\sqrt{\frac{g}{\delta}}\,t-\sin kt\right]$, when $k \gtrless \sqrt{\frac{g}{\delta}}$;

$x=\frac{a}{2}\left[\sin\sqrt{\frac{g}{\delta}}\,t-\sqrt{\frac{g}{\delta}}\,t\cos kt\right]$, when $k=\sqrt{\frac{g}{\delta}}$.

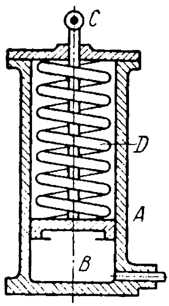


Fig. 293

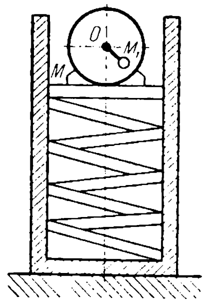


Fig. 294

473. The static deflection of the springs of a loaded railway car is $\Delta l_{static}=5$ cm. Determine the critical speed of the car at which the "galloping" begins, if at the rail-joints the car is subjected to a shaking force which causes forced oscillations of the car on the springs. The length of rails is $L=12$ m.

Ans. $v=96$ km/h.

474. Fig. 293 represents the indicator of a steam engine which consists of a cylinder A, whose piston B exerts a thrust on the spring D. The piston B is connected with a rod BC which operates as a recording pen C. The vapour pressure p on the piston B is measured in kgf/cm^2, and its change is defined by the equation $p=4+3\sin\frac{2\pi}{T}t$, where T is the duration of one revolution of the shaft. The latter makes 3 rps; the other data are as follows: the area of the piston is $\sigma=4$ cm^2, the weight of the moving part of the indicator is $Q=1$ kgf, the spring is compressed 1 cm by a force of 3 kgf. Determine the amplitude of the forced oscillations of the recording pen C.

Ans. $a=4.5$ cm.

475. An electric motor is mounted on a platform M which rests on a helical spring, as shown in Fig. 294. The spring can be compressed 1 cm by a force of 30 kgf. The total weight of the platform and the motor is 32.7 kgf. The shaft of the motor carries a load M_1, weighing 0.2 kgf, at the distance of 1.3 cm from the axis O of the shaft. The angular velocity of the motor is 30 sec^{-1}. Derive the equation of vibration of the platform, assuming that it was displaced from the position of static equilibrium. Let $g=981$ cm/sec^2.

Ans. $x=0.12t \sin (30t)$ cm.

476. A particle suspended from a spring weighs $p=3$ gf. The coefficient of stiffness of the spring is $c=12$ gf/cm. The disturbing force acting on the particle is $F=H \sin (62.6t+\beta)$ gf, and the force of resistance is proportional to the first power of the velocity and equal to $R=\alpha\dot{x}$ gf. Define the decrease of the amplitude of forced vibrations, if the force of resistance increases three times.

Ans. The amplitude of forced vibrations will decay three times.

33. Relative Motion

477. A 2.5-kgf weight C is attached to the end A of a flexible vertical rod AB, as shown in Fig. 295. When displaced from the position of equilibrium, the weight C oscillates harmonically under the action of a force proportional to the distance from the initial position. A force of 0.1 kgf deflects the end of the rod by 1 cm. Determine the amplitude of forced oscillations of the weight C in case when the particle B performs harmonic oscillations with amplitude 1 mm and period 1.1 sec along the horizontal.

Ans. 5.9 mm.

478. In a mathematical pendulum of a length l the particle suspended moves along the vertical with a uniform acceleration. Determine the period T of small oscillations of the pendulum under two conditions: (1) when the acceleration of the particle is directed upwards and has any value p; (2) when this acceleration is directed downwards and its value is $p<g$.

Ans. (1) $T=2\pi\sqrt{\dfrac{l}{p+g}}$ (2) $T=2\pi\sqrt{\dfrac{l}{g-p}}$

479. A mathematical pendulum OM of a length l is initially deflected through an angle α from its position of equilibrium. Its velocity is zero. At this instant the velocity of the point of suspension of the pendulum is also zero, and then it drops down with a uniform acceleration $p\geqslant g$. Determine the length s of the circumference of the circle described by the particle M while in a relative motion about the point O (Fig. 296).

Ans. (1) When $p=g$, $s=0$;
(2) when $p>g$, $s=2l(\pi-\alpha)$.

480. A particle falls freely from a height of 500 m to the earth in the northern hemisphere. Taking into consideration the rotation of the earth about its axis and neglecting the air resistance, determine the magnitude of the deviation of the falling particle in the east direction before it strikes the ground. The geographical latitude of the place is 60°

Ans. The deviation is 12 cm.

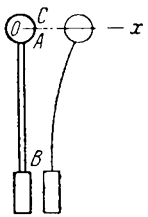


Fig. 295

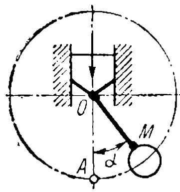


Fig. 296

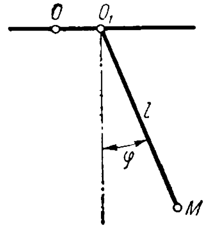


Fig. 297

481. The car runs along a straight horizontal track. A pendulum which is installed in a railway car performs small harmonic oscillations. Its central position is deviated 6° from the vertical. (1) Determine the acceleration w of the car; (2) find the difference between two oscillation periods of the pendulum: T, when the car is at rest, and T_1, for the present case.

Ans. (1) $w = 103$ cm/sec^2; (2) $T - T_1 = 0.0028T$

482. The point of suspension of a pendulum of a length l performs rectilinear horizontal harmonic oscillations about a fixed point O. $OO_1 = a \sin pt$. Determine the small oscillations of the pendulum, assuming that, when $t=0$, the pendulum is at rest (Fig. 297).

Ans. $\varphi = \dfrac{ap^2}{l(k^2-p^2)}\left(\sin pt - \dfrac{p}{k}\sin kt\right), \quad k = \sqrt{\dfrac{g}{l}}.$

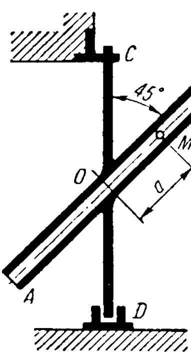


Fig. 298

483. A ring moves on a smooth rod AB which rotates uniformly about a vertical axis in a horizontal plane. The axis passes through the end A, and the rod makes one revolution per second. The length of the rod is 1 m. When $t=0$, the ring is at 60-cm distance from the end A, and its velocity is zero. Determine the instant t_1 when the ring leaves the rod.

Ans. $t_1 = \dfrac{1}{2\pi} \ln 3 = 0.175$ sec.

484. Fig. 298 shows a pipe AB which rotates about a vertical axis CD with a constant angular

velocity ω. The angle between AB and CD is always 45°. A small heavy ball is placed in the pipe. Determine the motion of the ball, assuming that its initial velocity is zero and the initial distance between the ball and a point O equals a. Neglect friction.

Ans. $OM = \frac{1}{2}\left(a - \frac{g\sqrt{2}}{\omega^2}\right)\left(e^{+0.5\omega t\sqrt{2}} + e^{-0.5\omega t\sqrt{2}}\right) + \frac{g\sqrt{2}}{\omega^2}$.

485. Determine how the acceleration due to gravity changes in relation to the latitude of the place φ, considering that the earth rotates about its axis. The radius of the earth is $R=6370$ km.

Ans. If we neglect the term in ω^4 due to its smallness then

$$g_1 = g\left(1 - \frac{\cos^2\varphi}{289}\right),$$

where g is the acceleration of gravity at the pole, φ is the geographical latitude of the place.

486. How many times should the angular velocity of rotation of the earth about its axis be increased to make a heavy particle at the surface of the earth at the equator completely weightless? The radius of the earth is $R=6370$ km.

Ans. 17 times.

487. A pendulum, suspended from a long thread, is given a small initial velocity in the north-south plane. Assuming that the deviation of the pendulum is negligible compared to the length of the thread and, taking into consideration the earth's rotation about its axis, determine the time elapsed when the plane of pendulum rotations coincides with that of west-east. The pendulum is located in latitude 60° north.

Ans. $T=13.86\ (0.5+K)$ hours, where $K=0, 1, 2, 3, \ldots$

IX. DYNAMICS OF A SYSTEM*

34. Principles of Kinetics and Statics

488. A locomotive runs along a straight horizontal track with a speed of $v=72$ km/h. A parallel rod ABC (Fig. 299) weighs 200 kgf, and its mass is assumed to be uniformly distributed along its length. The length of the crank is $r=0.3$ m, the radius of each wheel is $R=1$ m. The wheels roll without slipping. Determine the supplementary pressure on the rails exerted by the force of inertia of the parallel rod ABC in the lowest position.

Ans. 2450 kgf.

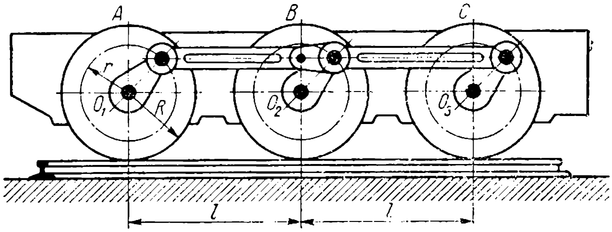


Fig. 299

489. A locomotive runs with uniform acceleration along a straight horizontal track and in 20 seconds after starting it attains a speed of 72 km/h. Determine the position of the free water surface in the tender-tank.

Hint. When the water is in equilibrium position with respect to the tender, the resultant force, which acts upon any liquid particle on a free surface, is directed along the normal to this surface.

Ans. The plane is inclined at an angle $\alpha=\arctan 0.102=5°50'$ to the horizontal.

490. A rod AB of length l is dragged behind a truck, as shown in Fig. 300. The other end B is fastened at D to the truck by a rope

* All problems marked by an asterisk can be solved easily by applying Lagrange's equations.

BD of length a. The truck moves with uniform acceleration along a straight horizontal road. $CD=h$. Neglecting the cross-sectional dimensions of the rod, determine the acceleration w of the truck when the rope and the rod form one straight line.

Ans. $w=\frac{g}{h}\sqrt{(l+a)^2-h^2}$.

491. A prism accelerates along a horizontal plane. Its side edge makes an angle α with the horizontal. What should the acceleration of the prism be to ensure that a weight, resting on the side edge, does not move with respect to the prism?

Ans. $w=g\tan\alpha$.

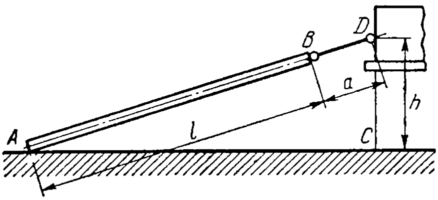


Fig. 300

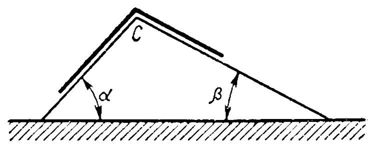


Fig. 301

492. A homogeneous chain rests on a three-edged prism whose side edges make angles α and β with the horizontal (Fig. 301). The central part of the chain lies on the upper rib C of the prism. With what acceleration should the prism be pulled to the left along the horizontal plane in order to prevent the displacement of the chain with respect to the prism?

Ans. $w=g\tan\frac{\alpha-\beta}{2}$.

493. A metal bar A is subjected to a test in order to investigate the effect of rapidly alternating tensile and compressive forces. For this purpose the upper end of the sample A is fixed to a slider B of the slider-crank mechanism BCO while a weight Q is applied to the lower end (Fig. 302). Determine the magnitude of the tensile force under the condition when the crank OC rotates about its axle O with a constant angular velocity ω.

Hint. The expression $\sqrt{1-\left(\frac{r}{l}\right)^2\sin^2\varphi}$ should be expanded in a series and all the terms in $\frac{r}{l}$ higher than the second power should be excluded.

Ans. $Q+\frac{Q}{g}r\omega^2\left(\cos\omega t+\frac{r}{l}\cos 2\omega t\right)$.

494. A winch lifts a weight B of 2000 kgf. It is mounted on a beam which rests on two supports C and D (Fig. 303). $CD=8$ m, $AC=3$ m. The weight is lifted with uniform acceleration of 0.5 m/sec^2. Determine the supplementary pressure on the supports C and D exerted by the forces of inertia of the weight.

Ans. 63.75 kgf; 38.25 kgf.

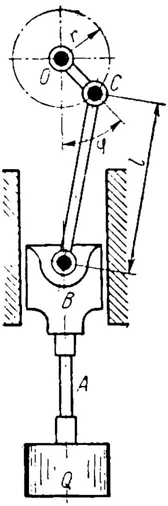


Fig. 302

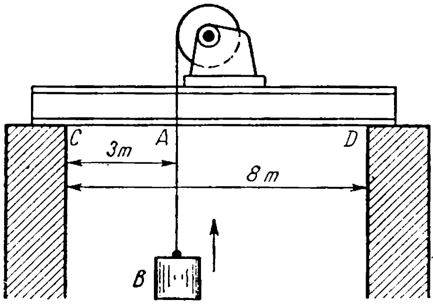


Fig. 303

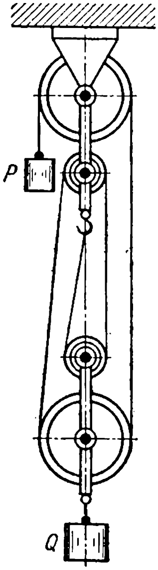


Fig. 304

495. An automobile of weight P runs with acceleration w along a straight road. Determine the vertical pressure of the front and rear wheels of the car if the centre of gravity C of the automobile lies at height h above the road. The distances of the front and rear axles from the vertical passing through the centre of gravity are a and b, respectively. The moments of forces of inertia of the rotating wheels are to be neglected. What should the motion of the automobile be to have pressures of the front and rear wheels equal?

Ans. $N_1=\dfrac{P(gb-wh)}{g(a+b)}$; $N_2=\dfrac{P(ga+wh)}{g(a+b)}$;

when the slowing down due to braking takes place, $w=g\dfrac{a-b}{2h}$.

496.* With what acceleration w does the weight P descend while lifting the weight Q by means of a polyspast, shown in Fig. 304? Under what conditions will the weight P have uniform motion? The masses of the pullyes and the cable may be neglected.

Hint. The acceleration of the weight Q is four times less than that of the weight P.

Ans. $w=4g\,\dfrac{4P-Q}{16P+Q}$; $\quad \dfrac{P}{Q}=\dfrac{1}{4}$.

497.* Two laminae each of weight P_1 rest in equilibrium on a smooth horizontal table. A smooth wedge of the weight P and semi-angle α moves the two laminae apart, as shown in Fig. 305. Derive the equation of motion of the wedge and the laminae and also determine the pressure exerted by the wedge on each lamina.

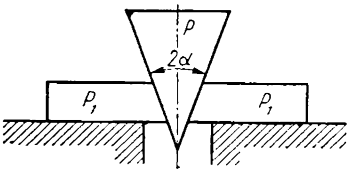


Fig. 305

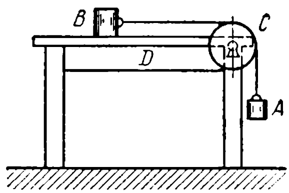


Fig. 306

Ans. The equation of motion of the wedge is $S=\dfrac{wt^2}{2}$, where $w=g\,\dfrac{P\cot\alpha}{P\cot\alpha+2P_1\tan\alpha}$; the equation of motion of each lamina is $s_1=\dfrac{w_1t^2}{2}$, where $w_1=g\,\dfrac{P}{P\cot\alpha+2P_1\tan\alpha}$; the pressure is $N=\dfrac{PP_1}{(P\cot\alpha+2P_1\tan\alpha)\cos\alpha}$.

498. A body A of weight P_1 sets a body B weighing P_2 in motion while descending by means of a weightless and inextensible thread which runs over a fixed pulley C, as shown in Fig. 306. Determine the pressure with which the table D acts on the floor. The weight of the table is P_3.

Ans. $N=P_1+P_2+P_3-\dfrac{P_1^2}{P_1+P_2}$.

499. A body A weighing P_1 descends down an inclined plane D, which makes an angle α with the horizontal, and pulls a load B that weighs P_2 by means of a weightless and inextensible thread passing over a fixed pulley C, as shown in Fig. 307 Determine the horizontal component of the pressure with which the inclined plane D acts on the floor rib E.

Ans. $N=P_1\,\dfrac{P_1\sin\alpha-P_2}{P_1+P_2}\cos\alpha$.

500. Fig. 308 represents a regulator which consists of two cylindrical disks each weighing P_1. Both disks are suspended eccentrically at a distance a from the axle of the regulator. The latter rotates with constant angular velocity ω about a fixed vertical axle; l is the distance between the centres of the disks and the points of suspension. The controller capsule A of weight P_2 rests on these disks, and it is connected with the regulator. Determine the relation between the angular velocity ω and the angle φ of deflection of the disks from the vertical. Neglect friction.

Ans. $\omega^2 = g \dfrac{2P_1+P_2}{2P_1(a+l\sin\varphi)} \tan\varphi.$

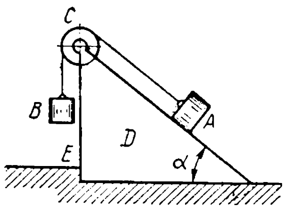


Fig. 307

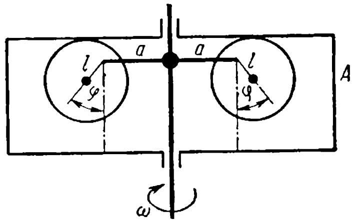


Fig. 308

501. Three stabilizers are installed in a ship to calm its rolling. The main part of each stabilizer is a flywheel weighing 110,000 kgf. When the stabilizers are in operation, the speed of their flywheels is 910 rpm. Calculate the additional side pressure on the bearings of the shaft, caused by a 1.08-mm displacement of the centre of gravity of the flywheel with respect to its axis of rotation. The displacement occurs due to the heterogeneity of the metal and to inaccuracy of machining the flywheel.

Ans. The pressure $N=109{,}700$ kgf, and it is directed along a straight line passing through the axis of rotation and the centre of gravity.

502. A uniform rod of weight P and length l oscillates with constant angular velocity ω about a fixed vertical axis perpendicular to the rod and passing through the end of the latter. Determine the magnitude of the stretching force in the cross-section of the rod which is at a distance a from the axis of rotation.

Ans. $F = \dfrac{P(l^2-a^2)\omega^2}{2gl}.$

503. Fig. 309 represents a uniform rectangular plate of weight P which rotates uniformly with angular velocity ω about a vertical axis. Determine the disruptive force, which acts on the

plate in a direction perpendicular to the axis of rotation, at the cross-section passing through the axis of rotation.

Ans. $\frac{Pa\omega^2}{4g}$.

504. A thin straight uniform rod of weight P and length l rotates about an axis which is perpendicular to the rod and passes through the end of the latter. Its motion is expressed in the form: $\varphi = at^2$. Find the magnitudes, directions and points of application of the resultant forces J_n and J_τ, of the centrifugal and tangential forces of inertia of the particles of the rod.

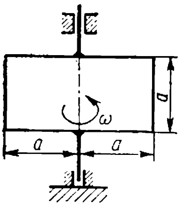


Fig. 309

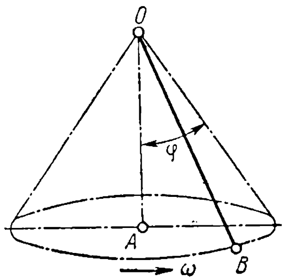


Fig. 310

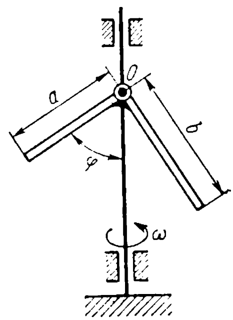


Fig. 311

Ans. The resultant of the tangential forces of inertia is $J_\tau = \frac{Pal}{g}$, directed perpendicular to the rod, it is applied at a point which is located at the distance $\frac{2}{3}l$ from the axis of rotation. The resultant of the centrifugal forces of inertia is $J_n = \frac{2Pa^2lt^2}{g}$, directed along the rod from the axis of rotation.

505. The data are the same as in Problem 494, except that the mass of the winch drum is to be taken into consideration and its radius is $r = 50$ cm. The moment of inertia of the winch about the axis of rotation equals 0.8 kgfmsec^2.

Ans. 63.85 kgf; 38.15 kgf.

506. A thin straight uniform rod of length l and weight P rotates with constant angular velocity ω about a fixed point O (a ball joint). The rod describes a conical surface with axis OA and vertex at the point O, as shown in Fig. 310. Calculate the angle of deflection of the rod from the vertical, and also determine the magnitude N which denotes the pressure acting on the joint O.

Ans. $\varphi = \arccos\frac{3g}{2l\omega^2}$; $\quad N = \frac{1}{2}\frac{P}{g}l\omega^2\sqrt{1 + \frac{7g^2}{4l^2\omega^4}}$.

507. Two thin straight uniform arms of the lengths a and b, respectively, are rigidly connected at 90°, as shown in Fig. 311. The vertex O of the angle is hinged to the vertical shaft. The entire system rotates with constant angular velocity ω. Find the relation between ω and the angle of deflection φ, formed by the arm of the length a and the vertical.

Ans. $\omega^2=3g\,\dfrac{b^2\cos\varphi-a^2\sin\varphi}{(b^3-a^3)\sin 2\varphi}$.

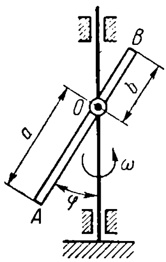


Fig. 312

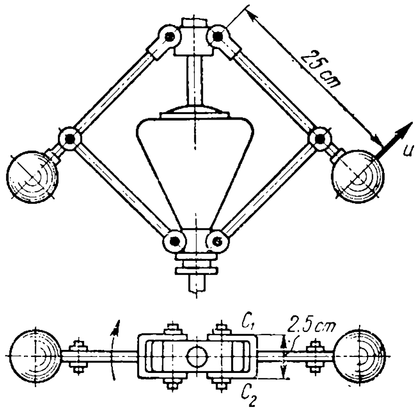


Fig. 313

508. A thin straight uniform rod AB is hinged to a vertical shaft at a point O (Fig. 312). The shaft rotates with constant angular velocity ω. Determine the angle of deflection φ between the rod and the vertical. $OA=a$, and $OB=b$.

Ans. $\cos\varphi=\dfrac{3}{2}\,\dfrac{g}{\omega^2}\,\dfrac{a-b}{a^2-ab+b^2}$

509. A pendulum of a centrifugal governor (Fig. 313) runs uniformly at a speed of 180 rpm. A change in the load speeds up the engine and sets the governor in motion moving the balls outwards with a relative velocity $u=0.2$ m/sec. Each ball weighs 10 kgf. Neglecting the weight of the arms, calculate the additional pressure on the bearings C_1 and C_2 caused by the Coriolis acceleration. It is assumed that the angle between the arm linkage and the axis of the governor is 45°, and that the number of revolutions is constant. The dimensions of the linkage are shown in Fig. 313, which represents the upper and side views of the governor.

Ans. The pressures on the bearings act in opposite directions and it is equal 54.2 kgf each.

510. A weight Q is lifted by means of a screw jack which is set in motion by a handle $OA = 0.6$ m. A force $P = 16$ kgf is applied perpendicularly to the handle, as shown in Fig. 314. Determine the magnitude of the weight Q, if the pitch of the screw thread is $h = 12$ mm.

Ans. $Q = 5020$ kgf.

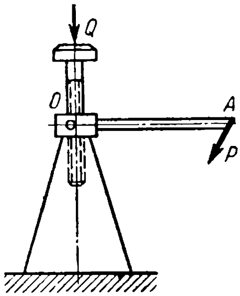


Fig. 314

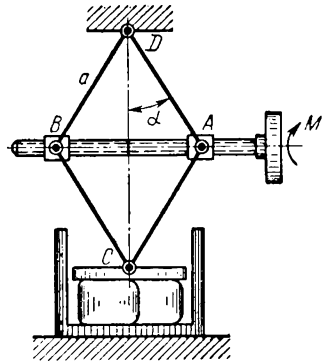


Fig. 315

511. The hand wheel of a crank press is acted on by a couple of moment M. Each end of the axle of the hand wheel has a screw thread, of pitch h with opposite directions, which passes through two nuts hinged to two vertices of a rod rhomb, as shown in Fig. 315. Each side of the rhomb equals a. The upper vertex of the rhomb is stationary while the lower one is fixed to the horizontal plate of the press. Determine the value of the pressure P with which the press acts on a body being compressed at the instant, when the angle at the vertex of the rhomb equals 2α.

Ans. $P = \pi \dfrac{M}{h} \cot \alpha$.

512. Determine the dependence between forces P and Q in a wedge press if a force P, applied to the end of the handle of length a, is perpendicular to the axis of the screw and the handle. The pitch of the screw thread is h. The angle at the vertex of the wedge is α. (See Fig. 316.)

Ans. $Q = P \dfrac{2\pi a}{h \tan \alpha}$.

513. A lever ABC with unequal arms fixed at B carries a platform $CFDE$ with a hinge at C, as shown in Fig. 317 The platform is hinged at D to a rod OD, which is free to rotate about the fixed point O. What weight p should be suspended to the point A

to balance the weight $P=100$ kgf, placed on the platform, assuming that CD is vertical and the rods OD and BC are equal and parallel? $BC=0.1\ AB$.

Ans. $p=10$ kgf.

514. Fig. 318 represents a diagram of a testing machine for the destructive test of samples. Determine the dependence between the force X in a sample K and the distance x of the weight P from

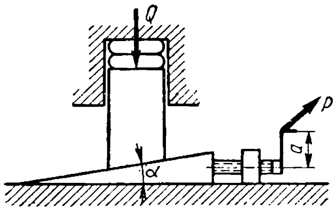


Fig. 316

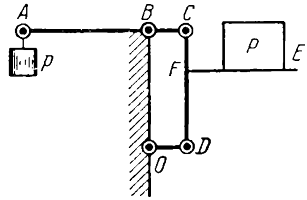


Fig. 317

its zero position O, considering that the machine is balanced by the weight Q in such a way that all levers are horizontal, when the weight P is in zero position and the force X is absent. Distances l_1, l_2 and e are available.

Ans. $X=P\,\dfrac{xl_1}{el_2}$.

515. Two loads K and L are connected by a system of levers, as shown in Fig. 319. The system is in equilibrium. Determine the correlation of the loads, assuming that: $\dfrac{BC}{AC}=\dfrac{1}{10}$, $\dfrac{ON}{OM}=\dfrac{1}{3}$, $\dfrac{DE}{DF}=\dfrac{1}{10}$.

Ans. $P_L=\dfrac{BC}{AC}\cdot\dfrac{ON}{OM}\cdot\dfrac{DE}{DF}P_K=\dfrac{1}{300}\,P_K$.

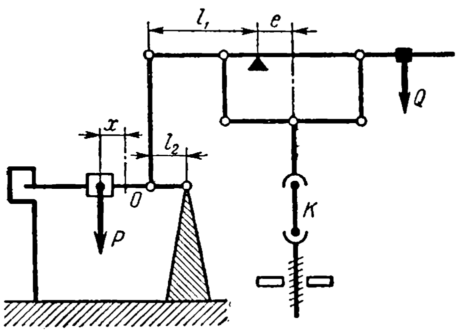


Fig. 318

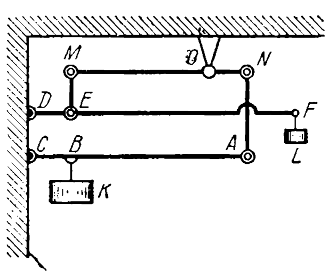


Fig. 319

516. Fig. 320 represents a scale consisting of a system of levers. A load P is placed on the platform at a point F $AB=a$; $BC=b$; $CD=c$; $IK=d$; the length of the platform is $EG=L$. Determine the relation between the lengths b, c, d and l, when the load p balances the load P and does not depend on its position on the platform. What should the weight of the load p be in this case?

Ans. $\frac{b+c}{b}=\frac{l}{d}$; $p=\frac{b}{a}P$

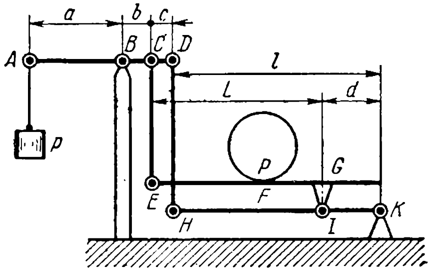


Fig. 320

517. A hinged four-link system $O_1A_1A_2O_2$ is shown in Fig. 321. Forces F_1 and F_2, respectively, are applied

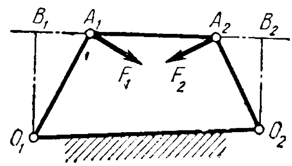


Fig. 321

to joints A_1 and A_2. The force F_1 is perpendicular to a rod O_1A_1 and the force F_2 is perpendicular to a rod O_2A_2. The system is in equilibrium. Find the ratio between the moments of the forces F_1 and F_2 and the shortest distances O_1B_1 and O_2B_2, respectively, from the axes of rotation O_1 and O_2 to the rod A_1A_2.

Ans. $\frac{O_1A_1 \cdot F_1}{O_2A_2 \cdot F_2}=\frac{O_1B_1}{O_2B_2}$.

518. Fig. 322 represents an ellipsograph with a slider crank. A force P is applied along the guide of the slider in the direction of the axis of rotation O of the crank OC. Considering that the crank OC and the slider guide B make an angle φ, what is the value of a torque which should be applied to the crank OC to make the mechanism assume the position of equilibrium? The mechanism is located in the horizontal plane and $OC=AC=CB=l$.

Ans. $M=2Pl\cos\varphi$.

519. Find the correlation for the press between the forces P and Q, shown in Fig. 323.

Ans. $Q=P\cot\alpha\cot\beta$.

520. Fig. 324 shows a rocker mechanism where $OC=R$ and $OK=l$. The crank OC swings about a horizontal axis O. The slider A moves along the crank OC and drives a rod AB, which is forced to move along the vertical guides K. Determine the value of the

force Q which should be applied perpendicular to OC at the point C to balance the force P directed upwards along the rod AB.

Ans. $Q=\dfrac{Pl}{R\cos^2\varphi}$

521. A rack-jack mechanism, shown in Fig. 325, has a handle A of length R. The handle sets in motion gears *1*, *2*, *3*, *4* and *5* which drive a gear rack B. Determine the magnitude of the force which should be applied perpendicular to the end of the handle to make the cup C exert a pressure of 480 kgf in equilibrium. The radii of the gears are: $r_1=3$ cm, $r_2=12$ cm, $r_3=4$ cm; $r_4=16$ cm, $r_5=3$ cm, and the radius of the handle is $R=18$ cm.

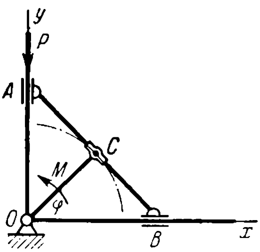


Fig. 322

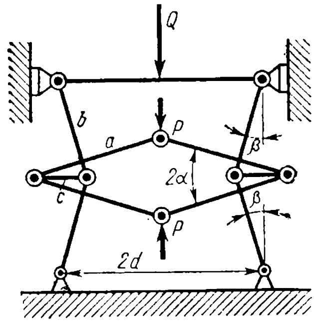


Fig. 323

Ans. $P=Q\,\dfrac{r_1r_3r_5}{r_2r_4R}=5$ kgf.

522. Two rigidly connected shafts A and B of a differential winch are driven by a handle C of length R. A load D of weight Q kgf is fixed to a moving pulley E driven by a rope (Fig. 326). When the handle C starts rotating, the left side of the rope uncoils from the shaft A of a radius r_1,

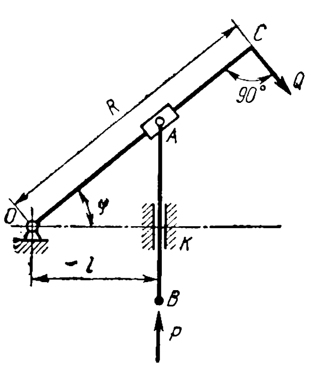


Fig. 324

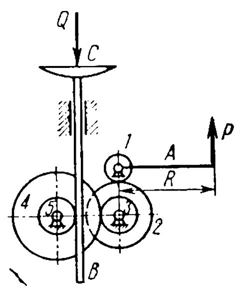


Fig. 325

while the right side coils on the shaft B of a radius $r_2 (r_2 > r_1)$. If $Q = 720$ kgf, $r_2 = 12$ cm, $r_1 = 10$ cm, and $R = 60$ cm find the magnitude of the force P which should be applied perpendicularly to the end of the handle to balance the weight D.

Ans. $P = Q\frac{r_2 - r_1}{2R} = 12$ kgf.

523. A mechanism, shown in Fig. 327, consists of two levers: a straight lever AB and an angle lever CD. Both rotate about fixed

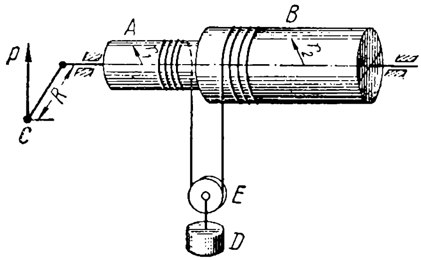


Fig. 326

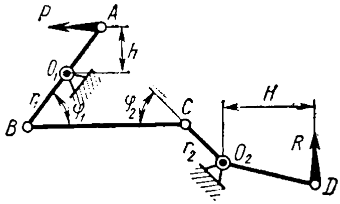


Fig. 327

hinges O_1 and O_2. The ends B and C of the levers are hinged to a connecting rod BC which makes angles φ_1 and φ_2 with them. The distance of elevation of the point A over the axis O_1 is h, and the horizontal distance between the point D and the axis O_2 is H. A horizontal force P is applied to the point A, and a vertical force R to the point D. $O_1B = r_1$, $O_2C = r_2$. Find the correlation between the forces P and R when the system remains in equilibrium.

H i n t. The projections of the virtual displacements of the ends B and C of the connecting rod on its line of direction are equal.

Ans. $P = R\frac{Hr_1 \sin\varphi_1}{hr_2 \sin\varphi_2}$.

524. A lifting mechanism consists of a four-link system $OCBO_1$. A vertical force R is applied to the hinge B. The link BC is rigidly fixed to a disk of centre B. A horizontal force S is applied tangentially to the disk at the point A. The lengths of the links are:

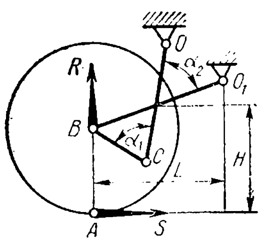


Fig. 328

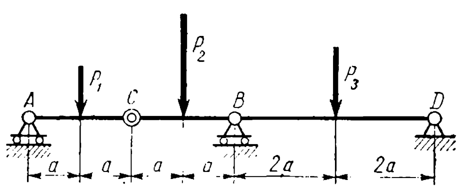


Fig. 329

$BC=l_1$, $BO_1=l_2$. All other data are given in Fig. 328 (H, L, α_1, α_2). Determine the correlation between R and S in the position of equilibrium, as shown in the figure. Neglect the weights of the links and disk, and also the friction in the hinges.

Ans. $S=R\dfrac{Ll_1 \sin \alpha_1}{Hl_2 \sin \alpha_2}$.

525. A sectional beam AD rests on three supports. It consists of two units hinged at C. The beam is under the action of three vertical forces: $P_1=2000$ kgf, $P_2=6000$ kgf and $P_3=3000$ kgf. Dimensions are as shown in Fig. 329. Find the reactions at the supports A, B and D.

Ans. $R_A=1000$ kgf; $R_B=10{,}500$ kgf; $R_D=-500$ kgf.

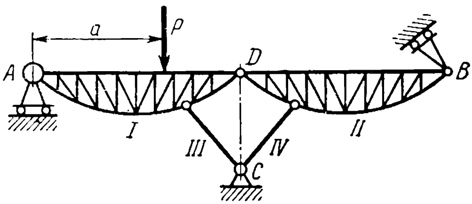


Fig. 330

526. Two arched trusses I and II are joined together by a hinge D (Fig. 330). They are supported by two bars III and IV hinged at a fixed support C. At A and B the trusses rest on rollers. The truss I carries a load P which acts at distance a from the roller support A. Find the reaction of the roller support B.

Hint. Determine first of all the positions of the absolute instantaneous centres C_1 and C_2 of the trusses I and II.

Ans. $R_B=P\dfrac{a}{b}\dfrac{DC_2}{DC_1}$, where b is the arm of the reaction R_B relative to the instantaneous centre C_2. The reaction R_B is directed perpendicular to the plane of sliding of the roller B downwards from the left to the right.

36. General Equation of Dynamics

527. Three loads of identical mass M are connected by a weightless, inextensible thread, running over a weightless pulley A. Two loads rest on the smooth horizontal plane while the third one is suspended vertically, as shown in Fig. 331. Determine the acceleration of the system and the tension in the cross-section ab of the thread.

Ans. $w=\dfrac{1}{3}g$; $T=\dfrac{1}{3}Mg$.

528. A flexible inextensible thread passes over a pulley and carries two loads at its ends: a load M_1 of weight P_1, and a load M_2 of weight P_2 (Fig. 332); $P_2 > P_1$. Find the value of the acceleration w of the loads and the tension T in the thread. Neglect the mass of the pulley and the thread.

Ans. $w = g\dfrac{P_2 - P_1}{P_1 + P_2}$ $T = \dfrac{2P_1P_2}{P_1 + P_2}$

529. Determine the acceleration w of the loads and the tensions T_1 and T_2 in the threads in the previous problem, taking into consideration the weight of the pulley, which equals P, and assuming that its mass is distributed uniformly on the rim.

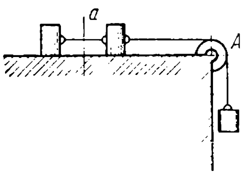


Fig. 331

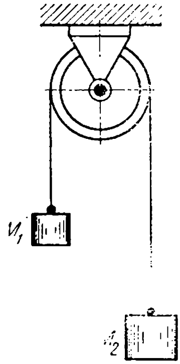


Fig. 332

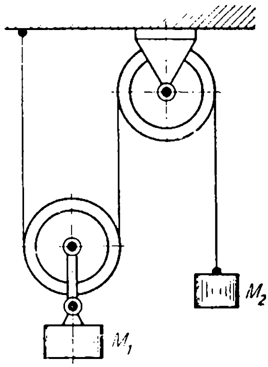


Fig. 333

Ans. $w = g\dfrac{P_2 - P_1}{P_1 + P_2 + P}$; $T_1 = \dfrac{P_1(2P_2 + P)}{P_1 + P_2 + P}$; $T_2 = \dfrac{P_2(2P_1 + P)}{P_1 + P_2 + P}$

530. The system of pulleys, shown in Fig. 333, carries two weights: $M_1 = 10$ kgf and $M_2 = 8$ kgf. Determine the acceleration w_2 of the weight M_2 and the tension T in the thread. Neglect the masses of the pulleys.

Hint. The acceleration of M_1 is equal to half of the acceleration of M_2.

Ans. $w_2 = 2.8$ m/sec^2; $T = 5.7$ kgf.

531. Two pulleys of radii R_1 and R_2 and of weights P_1 and P_2, respectively, are connected by an endless belt and rotate about parallel fixed axles, as shown in Fig. 334. Find the angular acceleration of the first pulley which is under the action of a torque M. The masses of the pulleys are assumed to be uniformly distributed along their rims. Neglect the friction on the axles, any sliding effect and the mass of the belt.

Ans. $\varepsilon_1 = \dfrac{Mg}{(P_1 + P_2)R_1^2}$.

532. A lower pulley C of a lift is acted on by a torque M (Fig. 335). The pulleys C and D of radius r and weight Q each are homogeneous cylinders. If the weight of a counterbalance B equals P_2, determine the acceleration of the load A of weight P_1 moving upwards. Neglect the mass of the belt.

Ans. $w = g\dfrac{M+(P_2-P_1)r}{(P_1+P_2+Q)r}$.

533. A shaft of a capstan of radius r is set in motion by a constant torque M applied to the handle AB

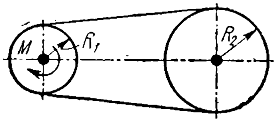


Fig. 334

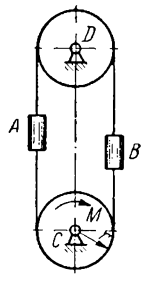


Fig. 335

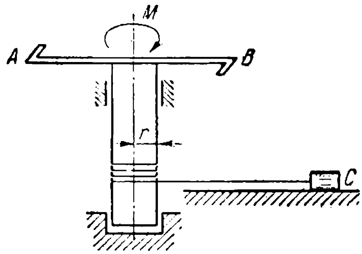


Fig. 336

(Fig. 336). The coefficient of sliding friction between the load and the horizontal surface is f. If the masses of the rope and capstan are neglected, determine the acceleration of the load C of the weight P.

Ans. $w = g\dfrac{M-fPr}{Pr}$.

534. A load A of weight P moves down a smooth plane inclined at an angle α to the horizontal, as shown in Fig. 337. While moving the load drives a drum B, of weight Q and radius r, by means of a weightless inextensible thread. Assuming that the drum is a round homogeneous cylinder, find its angular acceleration. Neglect the mass of the stationary fixed pulley C.

Ans. $\varepsilon = \dfrac{2Pg\sin\alpha}{r(2P+Q)}$.

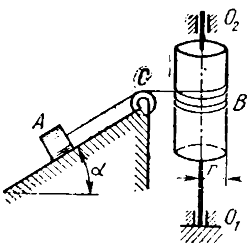


Fig. 337

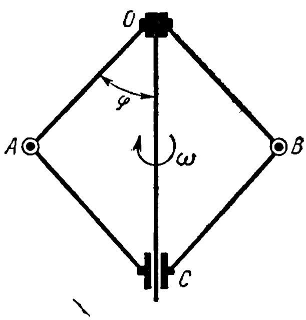


Fig. 338

535. A centrifugal-type governor rotates with a constant angular velocity ω about a vertical axis. All the links have the same length l (Fig. 338) Determine the angle between the arms OA and OB and the vertical, taking into consideration only the weight p of each ball and the weight p_1 of the sleeve C.

Ans. $\cos\varphi = \dfrac{(p+p_1)g}{pl\omega^2}$.

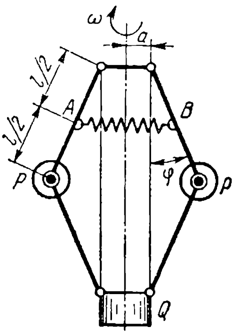


Fig. 339

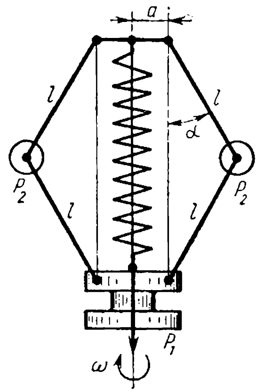


Fig. 340

536. Fig. 339 represents a governor with the following dimensions: all links are of the same length l, the weight of each ball is P, the weight of the sleeve is Q, the stiffness factor of a spring is c, and the spring is not stretched, when $\varphi=0$. The spring is fixed at points A and B, as shown in Fig. 339. The axes of suspension of the links are at a distance a from the axis of the governor. Find the connection between the angular velocity of the governor and the angle φ, when the system is in equilibrium.

Ans. $\omega^2 = g\dfrac{\left(P+Q+\dfrac{cl}{2}\cos\varphi\right)\tan\varphi}{P(a+l\sin\varphi)}$.

537. A centrifugal-type governor rotates with a constant angular velocity ω. Its sleeve, which weight is P_1, is pressed down by a spring with stiffness factor c. When $\alpha=0$, the spring is not deformed and its upper end is fixed to the axis of the governor. Both balls have equal weights P_2, and the length of the links is l. The axes of suspension are at a distance a from the axis of the governor. Find the dependence between the angular velocity of the governor and the angle α formed by the arms and the vertical. Neglect the weight of the links and the spring (Fig. 340).

Ans. $\omega^2 = g\dfrac{P_1+P_2+2lc(1-\cos\alpha)}{P_2(a+l\sin\alpha)}\tan\alpha$.

538. Fig. 341 shows a centrifugal spring governor with two loads A and B of weight $P_A=P_B=15$ kgf, mounted on a smooth horizontal bar which is connected to a spindle and with a sleeve C weighing $P_C=10$ kgf. The latter is tied to A and B by two links of length $l=25$ cm. Two identical springs with stiffness factor $c=15$ kgf/cm are fixed at the ends of the bar, and their inner ends are attached to the loads A and B. The springs press the loads A and B out to the axis of rotation.

The distance between the axis of the spindle and the hinges of the links is $e=3$ cm.

Determine the speed of rotation of the governor when the opening angle is $\alpha=60°$ if at normal angular velocity the angle α_0 equals 30° and the springs are not compressed. Neglect the weight of the links and any forces of friction.

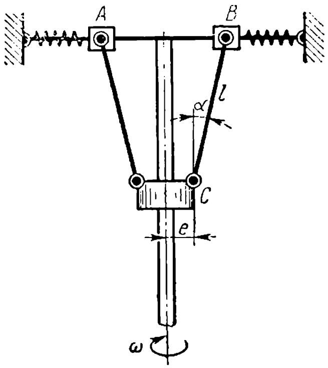


Fig. 341

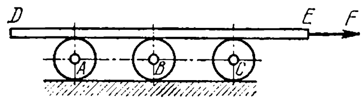


Fig. 342

Ans. $n=188$ rpm.

539. A rod DE of a weight Q rests on three rollers A, B and C each weighing P, as shown in Fig. 342. A horizontal force F acts on the rod to the right thus setting the rod and the rollers in motion. No sliding takes place between the rod and the rollers and between the rollers and the horizontal plane. Find the acceleration of the rod DE. The rollers are assumed to be homogeneous and of round cylindrical form.

Ans. $w=\dfrac{8gF}{8Q+9P}$.

540. A roller A of weight Q moves without sliding down an inclined plane. While moving down, it lifts a load C of weight P by means of a weightless inextensible thread which runs over a pulley B, as shown in Fig. 343. The pulley rotates about a fixed axis O perpendicular to its plane. The roller A and the pulley B are homogeneous round disks of equal weight and radius. The inclined plane forms an angle α with the horizontal. Determine the acceleration of the axis of the roller

Ans. $w=g\dfrac{Q\sin\alpha-P}{2Q+P}$.

541. A load A of weight P moves down on a weightless inextensible thread which first runs over a fixed weightless pulley D and then coils on a sheave B, thus forcing the shaft C to roll without sliding along a horizontal rail, as shown in Fig. 344. The sheave B of radius R is mounted tightly on the shaft C of radius r; the total weight of the sheave B and the shaft C is Q. The axis O

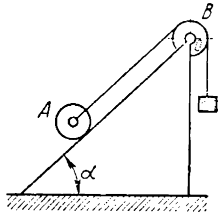


Fig. 343

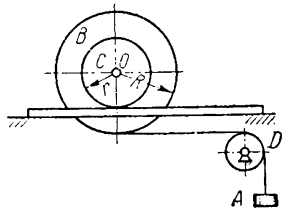


Fig. 344

is perpendicular to the plane of the drawing and the radius of gyration relative to O is ρ. Find the acceleration of the load A.

Ans. $w=g\,\dfrac{P(R-r)^2}{P(R-r)^2+Q(\rho^2+r^2)}$.

542. A prism A of weight P slides down the smooth side of another prism B of weight Q, as shown in Fig. 345. The edge makes an angle α with the horizontal. Determine the acceleration of the prism B. Neglect friction between the prism B and the horizontal plane.

Ans. $w=g\,\dfrac{P\sin 2\alpha}{2(Q+P\sin^2\alpha)}$.

543.* A cord, which runs over two pulleys A and B with fixed axles, carries a movable pulley C connected to the weight $p=4$ kgf (Fig. 346) Weights $p_1=2$ kgf and $p_2=3$ kgf are attached to both ends of the cord. The portions of the cord, which are not in contact with the pulleys, are vertical. Determine the accelerations of all the three weights neglecting the masses of the pulleys and the cord and the effect of friction on the axles.

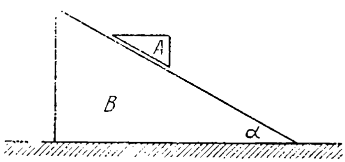


Fig. 345

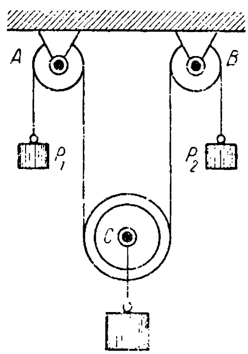


Fig. 346

Ans. $w=\frac{1}{11}g$ (upwards); $w_1=\frac{1}{11}g$ (upwards);

$w_2=\frac{3}{11}g$ (downwards).

544.* Fig. 347 shows an arrangement used for hoisting loads M_1 and M_2 of the same weight p. The string, which connects both loads, proceeds from the load M_1 over a pulley O, which rotates about a horizontal axis, and then runs over a moving pulley Q, to which a load M of weight P is attached. From here the string passes over a pulley O_1, which has a common axle with the pulley O, and then it is attached to the load M_2. Determine the acceleration w of the load M. Neglect friction, the masses of both pulleys and the string.

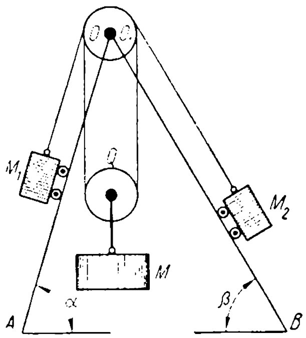


Fig. 347

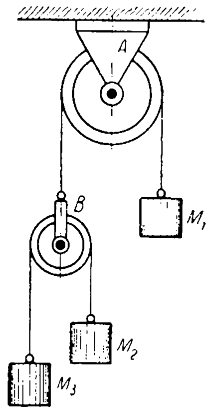


Fig. 348

Ans. Considering that the acceleration w is positive when directed downwards, and negative when directed upwards, we find:

$$w=g\,\frac{P-p(\sin\alpha+\sin\beta)}{P+2p}.$$

545.* Fig. 348 shows a system which consists of two pulleys: a fixed pulley A and a moving pulley B. The pulleys carry three weights M_1, M_2 and M_3, suspended by inextensible strings in the way, shown in the figure. The masses of the weights are equal to m_1, m_2 and m_3, respectively; $m_1<m_2+m_3$, and $m_2\gtreqless m_3$. Neglect the masses of the pulleys. If the initial velocities of all the weights are zero, find the relation between the masses m_1, m_2 and m_3 for which the weight M_1 will sink downwards.

Ans. The relation is $m_1>\frac{4m_2m_3}{m_2+m_3}$.

37. Theorem on the Motion of the Centre of Mass of a System

546. A homogeneous prism B rests on a homogeneous prism A which is placed on a horizontal plane, as shown in Fig. 349. The cross-sections of the prisms are right triangles. The weight of the prism A is three times that of the prism B. Assuming that the prisms and the plane are perfectly smooth, determine the length l through which the prism A moves when B, which is sliding down along A, touches the plane.

Ans. $l=\frac{a-b}{4}$

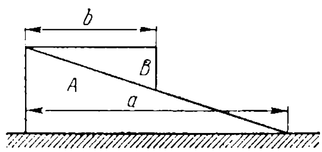


Fig. 349

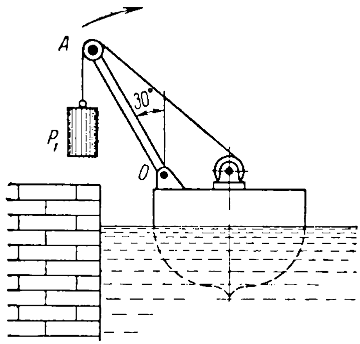


Fig. 350

547. A floating crane lifts a weight $P_1=2000$ kgf, the jib being turned through an angle of 30° to the vertical (Fig. 350). The crane weighs $P_2=20{,}000$ kgf. The length of the jib is $OA=8$ m. Determine the displacement of the floating crane. The resistance of the water and the weight of the jib should be neglected.

Ans. The crane moves a distance of 0.36 m to the left.

548. Two loads P_1 and P_2, connected by an inextensible cord which runs over a pulley B, slide down the smooth side edges of a rectangular body. The latter rests with its base BC on a smooth horizontal plane (Fig. 351). Find the displacement of the rectangular body along the horizontal plane, if the load P_1 descends a distance $h=10$ cm. The weight of the rectangular body is $P=4P_1=16P_2$. Neglect the mass of the cord and the pulley.

Ans. The rectangular body will move a distance 3.77 cm to the right.

549. A horizontal engine has a crank mechanism inside. The weight of its bedding is $P_1=14{,}000$ kgf, and the weight of the unbalanced parts, which perform reciprocal motions with the slider is $P_2=1000$ kgf. The length of the crank is $r=45$ cm, and the shaft makes 240 rpm. Determine the displacement of the bedding on the horizontal foundation, considering that it is not fastened down by bolts and the reciprocal motions of the unbalanced

parts are performed (approximately) in accordance with the law of harmonic vibrations. Neglect any friction between the bedding and the foundation.

Ans. $s=3 \sin 8\pi t$ cm.

550. A tram car performs vertical harmonic oscillations on its springs with an amplitude of 2.5 cm and period $T=0.5$ sec. The weight of the loaded tram car is 10,000 kgf, and the total weight of the bogie and wheels is 1000 kgf. Determine the value of the pressure exerted by the tram car on the rails.

Ans. The pressure changes from 7000 to 15,000 kgf.

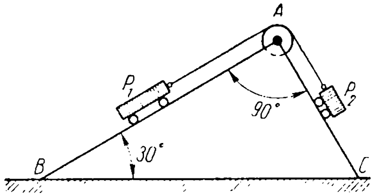


Fig. 351

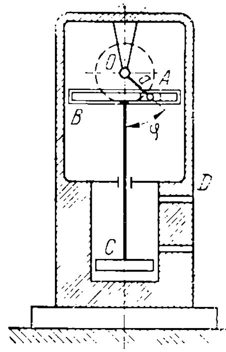


Fig. 352

551. Determine the pressure which a water pump exerts on the ground when running idle (Fig. 352) The other data are as follows: the weight of stationary parts of the body D and the bedding E is P_1, the crank $OA=a$ weighs P_2, and the rocker B and piston C weigh P_3. The crank OA rotates uniformly with angular velocity ω, and it is assumed to be uniform.

Ans. $N=P_1+P_2+P_3+\frac{a\omega^2}{2g}(P_2+2P_3)\cos \omega t.$

552. A vertical gas engine, shown in Fig. 353, has a cylinder, a frame and bearings which together weigh 10,000 kgf. The piston weighs 981 kgf and its centre of gravity is at B in the cross-head. The stroke of the piston is 60 cm, and the crank makes 300 rpm. The ratio of the length of the crank r to the length of the connecting rod l is $^1/_6$. The engine is fastened by bolts to the bedding C, and their tension is assumed to be zero when the engine is not running. Determine the maximum pressure N exerted by the engine on the bedding and the maximum tension T in the bolts (see the Hint to Problem 493). Neglect the mass of the crank and that of the connecting rod.

Ans. $N=35{,}600$ kgf; $T=23{,}500$ kgf.

553. A metal-cutting tool consists of a crank mechanism OAB, as shown in Fig. 354. A movable knife is attached to a slider B while another one is rigidly fixed to the base C. The length of the crank is r and its weight is P_1, the length of the connecting rod is l, the weight of the slider B and the movable knife is P_2, the weight of the base C and frame D equals P_3. Neglect the mass of the crank. The crank OA rotates uniformly with angular velocity ω, and it is assumed to be uniform. (See the Hint to Problem 493.) Determine the value of the pressure exerted by the base on the ground.

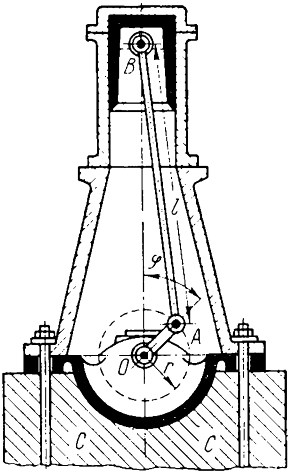


Fig. 353

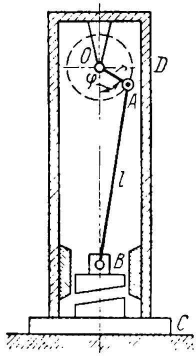


Fig. 354

$$Ans.\ N = P_1 + P_2 + P_3 + \frac{r\omega^2}{2g}\left[(P_1 + 2P_2)\cos\omega t + 2P_2\frac{r}{l}\cos 2\omega t\right].$$

554. An electric motor of weight P is mounted without being fixed to a smooth horizontal foundation (Fig. 355). A uniform rod of length $2l$ and weight p is fixed at one end to the shaft

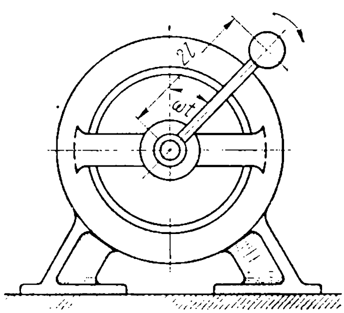


Fig. 355

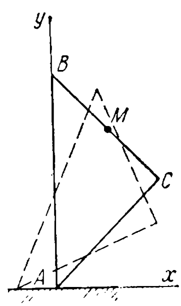


Fig. 356

of the motor at an angle of 90°. A point load Q is attached to the other end of the rod. The shaft rotates with an angular velocity ω.

Determine (1) the horizontal motions of the motor; (2) the maximum horizontal shearing stress R acting on the bolts in case they are used to fasten the housing of the electric motor to its foundation.

Ans. (1) Harmonic vibrations with an amplitude of $\frac{l(p+2Q)}{p+P+Q}$ and a period of $\frac{2\pi}{\omega}$; (2) $R=\frac{P+2Q}{g}l\omega^2$.

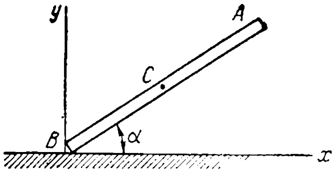


Fig. 357

555. Using the data of the previous problem, calculate the angular velocity ω of the shaft for the electric motor to hop over if it is not fastened down by bolts.

Ans. $\omega > \sqrt{\frac{(p+P+Q)g}{(p+2Q)l}}$

556. A uniform lamina ABC of the shape of an isosceles right-angled triangle, with hypotenuse AB 12 cm long, is placed with its vertex A on a smooth horizontal plane in such a way that AB becomes vertical (Fig. 356). The lamina falls down under the action of gravity. Determine the path which a point M will trace if it is at the middle of BC.

Hint. When the lamina moves, the vertex A remains on the horizontal plane.

Ans. The arc of an ellipse $9(x-2)^2+y^2=90$.

557. A uniform rod of length $2l$ rests with its end B on a smooth plane at an angle α to the horizontal, as shown in Fig. 357 Then, the rod is released. Find the equation of the path of the point A on the rod.

Ans. The ellipse $(x-l\cos\alpha)^2+\frac{y^2}{4}=l^2$.

38. Theorem on the Change of Linear Momentum of a System

558. Calculate the linear momentum of a belt drive, shown in Fig. 358, if the masses of both pulleys and the belt are taken into consideration. The centre of gravity of each pulley is located on the axis of rotation.

Ans. The linear momentum equals zero.

559. A wheel of radius $r=50$ cm and weight $P=100$ kgf rolls without sliding along a rail. It makes 60 rpm. Calculate the angular momentum of the wheel.

Ans. 10.2π kgfsec.

560. Determine the magnitude and direction of the linear momentum of an ellipsograph, considering that the weight of the crank is P_1, and the weight of the rule AB is $2P_1$ (Fig. 359). The sockets A and B each weighs P_2. $OC=AC=CB=l$. The centres of gravity of the crank and the rule are located at their mid-points. The crank rotates with an angular velocity ω.

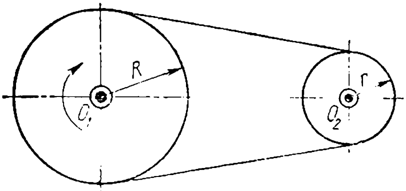


Fig. 358

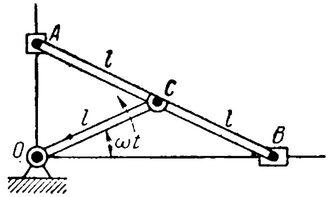


Fig. 359

Ans. The vector magnitude $Q=\frac{\omega l}{2g}(5P_1+4P_2)$; the direction of the vector is perpendicular to the crank.

561. A wheel of radius r and a weight p rolls on the inside of the circumference of another wheel, as shown in Fig. 360. The small wheel has its centre of gravity at a point O_1. The straight bar AB weighs k times the rolling wheel and its centre of gravity is at its mid-point. The crank OO_1 rotates about the axis O with a constant angular velocity ω. Calculate the linear momentum of the system. Neglect the mass of the crank.

Ans. The projections of the linear momentum on the coordinate axes are: (1) on the axis Ox: $-\frac{p}{g}r\omega\cos\omega t$;

(2) on the axis Oy: $\frac{p}{g}r\omega(1+2k)\sin\omega t$.

562. A tug of weight 600,000 kgf is towing a barge of weight 400,000 kgf. When the tug attains a speed of 1.5 m/sec, the rope cable is stretched, and the barge starts moving behind the tug. Calculate the total speed of the tug and the barge, assuming that the tractive force and the water resistance balance each other.

Ans. 0.9 m/sec.

563. A carriage of weight 240 kgf runs straight with a speed of 3.6 km/h. A man, weighing 50 kgf, jumps on the steps in the

direction perpendicular to the motion of the carriage. Determine the speed of the carriage with the man on it.

Ans. 2.98 km/h.

564. The nozzle of a fire hose discharges water at an angle $\alpha=30°$ to the horizontal with a velocity of 8 m/sec (Fig. 361). Determine the value of the pressure with which the water jet strikes the wall. Neglect the effect of gravity on the shape of the jet. It is assumed that on reaching the wall, all the water particles gain velocities directed along the wall.

Ans. 9.05 kgf.

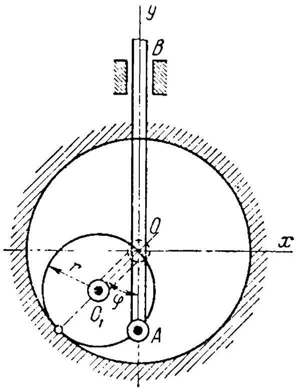


Fig. 360

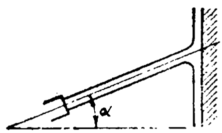


Fig. 361

565. Water is flowing through a pipe elbow of $d=300$ m diameter with the velocity $v=2$ m/sec (Fig. 362). Determine the horizontal component N of the additional pressure exerted on the supporting structure of the pipe elbow.

Ans. $N=28.9$ kgf.

566. Water flows through a channel of alternating cross-section. It is located symmetrically with respect to the vertical plane, as shown in Fig. 363. The water enters the channel with the initial velocity $v_0=2$ m/sec at an angle $\alpha_0=90°$ to the horizontal. At the inlet the cross-section of the

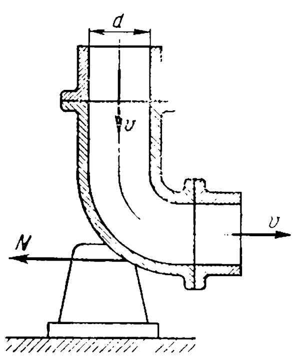


Fig. 362

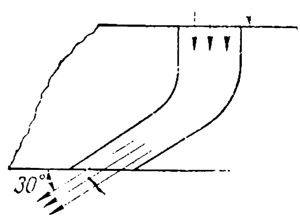


Fig. 363

channel is 0.02 m² At the outlet the velocity of the water is $v_1=4$ m/sec and directed at $\alpha_1=30°$ to the horizontal. Determine the horizontal component of the force with which the water acts on the walls of the channel.

Ans. 14.1 kgf.

567. The axis of a pipe 20 cm in diameter lies in the horizontal plane. Fig. 364 shows the top view. Water flows through the pipe with the velocity of 4 m/sec. At the exit the angle between the inlet flow and outlet flow of water is 60°

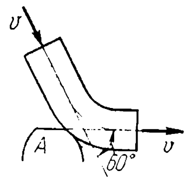


Fig. 364

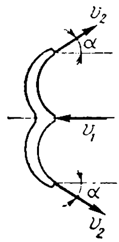


Fig. 365

Determine the value of the additional force acting on the supporting structure A of the pipe elbow.

Ans. 51.2 kgf.

568. Determine the horizontal component of the pressure with which a jet of water acts against a fixed turbine blade (Fig. 365), if the following data are given: the volume of water discharged is Q, a specific weight is γ, the velocity v_1 of the water that strikes against the blade is horizontal, and the velocity v_2 of the water that leaves the blade makes an angle α with the horizontal.

Ans. $N=\frac{\gamma}{g}Q(v_1+v_2\cos\alpha)$.

39. Theorem on the Change of Principal Angular Momentum of a System. Differential Equation of Rotation of a Rigid Body about a Fixed Axis. Elementary Theory on Gyroscopes

569. A homogeneous circular disk of weight $P=50$ kgf and radius $R=30$ cm rolls without sliding along a horizontal plane at 60 rpm. Calculate the principal angular momentum of the disk with respect to two axes: one that passes through the centre of the disk perpendicular to the plane, and the other relative to the instantaneous axis.

Ans. (1) 1.44 kgfmsec; (2) 4.32 kgfmsec.

570. Fig. 366 shows a pulley with a rope running over it. A man

holds one side of the rope at A while a load, equal to the weight of the man, is attached to the other end of the rope at B. What would happen to the load if the man starts climbing up the rope with the velocity a relative to the rope? Neglect the weight of the pulley.

Ans. The load will ascend together with the rope with the velocity $\frac{a}{2}$.

571. Solve the previous problem, taking into consideration the weight of the pulley which is one quarter that of the weight of the man.

When calculating the moment of inertia of the pulley, assume that its mass is distributed uniformly along its rim.

Ans. The load will ascend with velocity $\frac{4}{9}a$.

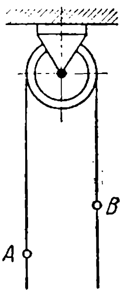


Fig. 366

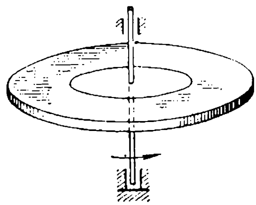


Fig. 367

572. A circular horizontal platform rotates without friction about a vertical axis OZ which passes through its centre O, as shown in Fig. 367 A man whose weight is p walks around on the platform at a constant distance r from the axis Oz with a uniform relative velocity u. Determine the angular velocity ω of the platform, rotating about its axis, if its weight P is assumed to be distributed uniformly over the circular surface of radius R. Initially the velocity of the man and the platform were zero.

Ans. $\omega = \frac{2pr}{PR^2+2pr^2}u.$

573. The platform of Zhukovsky* on which a man stands with his arms stretched out, is given an initial velocity corresponding to

* N. E. Zhukovsky (1847-1921) is a celebrated Russian scientist and inventor. Zhukovsky's special contribution was in the field of applying methods of mechanics to the solution of actual engineering problems. The platform of Zhukovsky, mentioned in Problem 573, is a circular horizontal disk supported by step ball bearings so that it is free to rotate with negligible friction. It is a simple instrument which is used to demonstrate visually the law of conservation of angular momentum.

15 rpm. At this instant the moment of inertia of the man and the system relative to the axis of rotation is 0.8 kgfmsec^2. When the man drops his arms down, the moment of inertia is decreased to 0.12 kgfmsec^2. Determine the speed of rotation of the system with the man standing on it (Fig. 367).

Ans. 100 rpm.

574. Two rigid bodies rotate independently about a common fixed axis with constant angular velocities ω_1 and ω_2. Their moments of inertia with respect to this axis are J_1 and J_2, respectively. What must the angular velocity of both bodies be, if they are connected while rotating?

Ans. $\omega = \dfrac{J_1\omega_1 + J_2\omega_2}{J_1 + J_2}$.

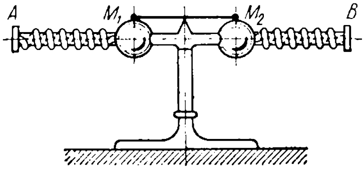


Fig. 368

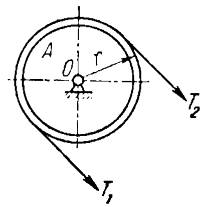


Fig. 369

575. A uniform rod AB of length $2L = 180$ cm and weight $Q = 2$ kgf is suspended at equilibrium from a knife edge in such a way that its axis becomes horizontal. Two balls M_1 and M_2, each weighing $P = 5$ kgf, are free to slide along the rod. They are placed symmetrically with respect to the axis of rotation, and their centres are held apart at a distance $2l_1 = 72$ cm by means of a thread. Two identical springs are fixed at the ends of the rod, and at their other ends they are connected to the balls in the way, shown in Fig. 368. The rod is set spinning about its vertical axis with the velocity $n_1 = 64$ rpm. Then the thread is cut, and, after some rotation, the balls assume the position of equilibrium under the action of the springs and friction. The distance between the balls becomes $2l_2 = 108$ cm. Considering the balls as particles, and neglecting the masses of the springs, determine the speed of rotation per minute n_2 of the rod in its new position.

Ans. $n_2 = \dfrac{6Pl_1^2 + QL^2}{6Pl_2^2 + QL^2} n_1 = 34$ rpm.

576. A belt drives a pulley A of radius $r = 20$ cm and weight $P = 3.27$ kgf (Fig. 369). The tensions in its tight and slack sides are $T_1 = 10.1$ kgf and $T_2 = 5.05$ kgf, respectively. What must the moment

of forces of resistance be to make the pulley rotate with an angular acceleration $\varepsilon = 1.5\ \mathrm{sec}^{-2}$, if the disk is considered to be homogeneous?

Ans. 1 kgfm.

577. A special device for studying the moment of force of friction in axle journals consists of a flywheel mounted on a shaft. The flywheel weighs 500 kgf and its radius of gyration is $\rho = 1.5$ m. The flywheel is given the velocity $n_0 = 240$ rpm, then permitted to coast, and, at the end of 10 min, it stops. Determine the moment of friction, assuming it to be constant.

Ans. 4.8 kgfm.

578. An electrical brake is used for the rapid braking of flywheels of large diameters. It consists of two diametrically opposite poles with a constant current winding. The flywheel, while rotating, induces electric currents, which produce a braking moment proportional to the velocity v of the rim of the flywheel. $M_1 = kv$, where k is a coefficient which depends on the magnetic flux and the dimensions of the flywheel. The moment of force of friction M_2 in the bearings is assumed to be constant and it depends on the diameter of the flywheel D and its moment of inertia relative to the axis of rotation J If the flywheel initially rotates with angular velocity ω_0, how long does it take to come to rest?

Ans. $T = \dfrac{2J}{kD} \ln \left(1 + \dfrac{kD\omega_0}{2M_2}\right).$

579. A rigid body in equilibrium is set spinning about a stationary vertical axis by a constant moment of force M. This motion causes a moment of the force of resistance M_1, which is proportional to the square of the angular velocity of the rigid body: $M_1 = \alpha\omega^2$ If the moment of inertia of the rigid body with respect to the axis of rotation is J, state the law of change of angular velocity.

Ans. $\omega = \sqrt{\dfrac{M}{\alpha}}\,\dfrac{e^{\beta t} - 1}{e^{\beta t} + 1}$ where $\beta = \dfrac{2}{J}\sqrt{\alpha M}.$

580. Use the data of the previous problem, except that the moment of the forces of resistance M_1 is proportional to the angular velocity of rotation of the rigid body: $M_1 = \alpha\omega$.

Ans. $\omega = \dfrac{M}{\alpha}\left(1 - e^{-\frac{\alpha}{J}t}\right).$

581. A shaft of radius r rotates about a horizontal axis. It is set in motion by a rope to which a weight is attached. The shaft is connected with n identical laminae to make the value of angular velocity of the shaft to be almost constant some time after the

start. The air resistance exerted on the lamina is reduced to the force, applied normally to the lamina at a distance R from the axis of the shaft, and is proportional to the square of its angular velocity. The coefficient of proportionality is k. The mass of the weight is m. The moment of inertia of all the rotating parts about the axis of rotation is J Determine the angular velocity ω of the shaft, considering that it was zero initially. Neglect the mass of the rope and the friction in the supports.

Ans. $\omega=\sqrt{\frac{mgr}{knR}}\,\frac{e^{at}-1}{e^{at}+1}$, where $\alpha=\frac{2}{J+mr^2}\sqrt{mgnkrR}$; for large t the angular velocity ω is close to the constant value $\sqrt{\frac{mgr}{knR}}$.

582. Referring to the data of the previous problem, state the law of rotation of the shaft, assuming that without the weight the initial angular velocity of the shaft is ω_0 and the initial turning angle equals zero.

Ans. $\varphi=\frac{J}{nkR}\ln\left(1+\frac{nkR\omega_0}{J}t\right)$.

583. Referring to Problem 581, state the law of rotation of the shaft, if the force of resistance is proportional to the angular velocity of the shaft. The initial turning angle equals zero.

Ans. $\varphi=\sigma\left[t+\frac{1}{\gamma}\left(e^{-\gamma t}-1\right)\right]$, where $\sigma=\frac{mgr}{nkR}$; $\gamma=\frac{nkR}{J+mr^2}$.

584.* A transmission shaft is driven by an engine. The shafts of the transmission and the engine are connected by an endless belt which passes over two pulleys mounted on these shafts, as shown in Fig. 370. A torque M is applied to the engine shaft. The moments of inertia of the transmission and engine shafts together with their pulleys are denoted by J_1 and J_2, respectively. The radius of the pulley of the engine is r_1. The ratio between the angular velocity of the transmission and that of the engine is k. The endless belt is of weight p. Neglecting friction in the bearings of the shafts, find the angular acceleration of the engine shaft.

Ans. $\varepsilon_1=\frac{Mg}{(J_1+k^2J_2)g+pr_1^2}$.

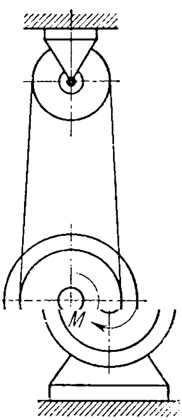


Fig. 370

585.* An electric winch lifts a weight P, as shown in Fig. 371. Its driving shaft has a constant torque M. The moments of inertia of the driving and driven shafts, with gears and other parts mounted

on them, equal J_1 and J_2, respectively. The transmission ratio is $\frac{z_2}{z_1}=k$. The radius of the drum, on which the string winds, is R. Neglect friction in the bearing and the mass of the cable. Determine the acceleration of the weight.

$$Ans.\quad w=g\,\frac{(Mk-PR)R}{PR^2+(J_1k^2+J_2)g}.$$

586. A special system, shown in Fig. 372, is used to determine the moment of inertia J of a flywheel A with respect to the axis passing through the centre of gravity. The flywheel has a radius $R=50$ cm. A load B of weight $p_1=8$ kgf is attached to the free end of a thin wire which passes over the rim of the flywheel. The time $T=16$ sec, required for the load to descend from the height $h=2$ m, is carefully measured. To exclude the effect of friction in the bearings a second test is carried out. The load of weight $p_2=4$ kgf is taken and the duration of descent for this load is $T_2=25$ sec, while the height remains the same. Assuming that the moment of the forces of friction is constant and it does not depend on the weight of the load, calculate the moment of inertia J.

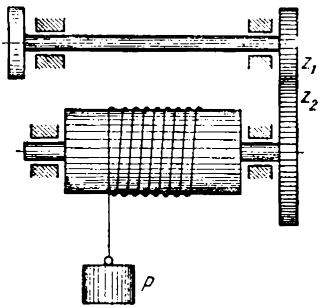


Fig. 371

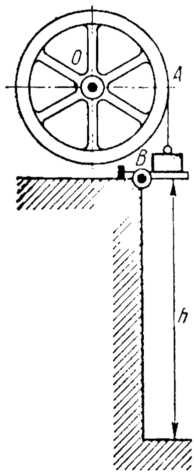


Fig. 372

Fig. 373

$$Ans.\quad J=R^2\,\frac{\frac{1}{2h}(p_1-p_2)-\frac{1}{g}\left(\frac{p_1}{T_1^2}-\frac{p_2}{T_2^2}\right)}{\frac{1}{T_1^2}-\frac{1}{T_2^2}}=1061\ \text{kgfmsec}^2.$$

587. Fig. 373 shows a reversible pendulum which is used for determining the acceleration due to gravity. It consists of a rod with two three-edged knives A and B on it. One knife is stationary

while the other one is free to slide along the rod. When the rod is suspended first from one knife and then from the other, the distance AB between the knives may be varied to obtain equal periods of oscillations about each knife. What must the acceleration due to gravity be, if for equal periods of oscillation the distance between the knives AB is l and the period of oscillation is T?

Ans. $g=\frac{4\pi^2 l}{T^2}$

588. Two rigid bodies are free to swing about the same horizontal axis either separately or braced together. Determine the reduced length of the compound pendulum, if p_1 and p_2, respectively, are the weights of the bodies, the distances between their centres of gravity and the common axis of oscillation are a_1 and a_2, and the reduced lengths at separate swings are l_1 and l_2.

Ans. $l_{\text{reduced}}=\frac{p_1a_1l_1+p_2a_2l_2}{p_1a_1+p_2a_2}$

589. In order to adjust the frequency of vibration of a clock pendulum an additional weight p is attached to it at distance x from the axis of suspension. The clock pendulum has weight P with the reduced length l, and a is the distance between the centre of gravity of the pendulum and the axis of suspension. Assuming the additional weight as a material particle, determine the change Δl in the reduced length of the pendulum for given values of p and x. Find also the value of $x=x_1$, for which a given change Δl is attained by means of the additional weight of the negligible mass.

Ans. The length of the equivalent pendulum should be reduced by $\Delta l_{\text{reduced}}=\frac{px(x-l)}{Pa+px}$; $x_1=\frac{1}{2}(l+\Delta l)$

590. To determine the moment of inertia J of a given body about an axis AB, which passes through the centre of gravity G of the body, the latter is suspended by means of two fixed rods AD and BE, as shown in Fig. 374. Both rods are loosely fit on a horizontal fixed axle DE in such a way that AB is parallel to DE. Then the body is set in motion, and the period T of one swing is determined. If the weight of the body is p and the distance between the axes AB and DE is h, what must the value of the moment of inertia J be? Neglect the masses of the rods.

Ans. $J=hp\left(\frac{T^2}{\pi^2}-\frac{h}{g}\right)$.

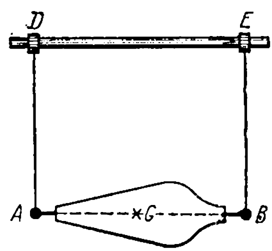


Fig. 374

591. The system, shown in Fig. 375, is used to determine the moment of inertia

of the connecting rod. A thin cylindrical rod is passed through the bushing of the small end of the connecting rod. The system is set to swing about a horizontal axis. The duration of one hundred swings is $100\,T = 100$ sec, where T is half the period. In order to find the distance $AC = h$ between the centre of gravity C and the centre A of the bushing, the connecting rod is placed horizontally on the platform of a decimal scale in such a way that, at B, it rests on the platform while, at A, it suspended

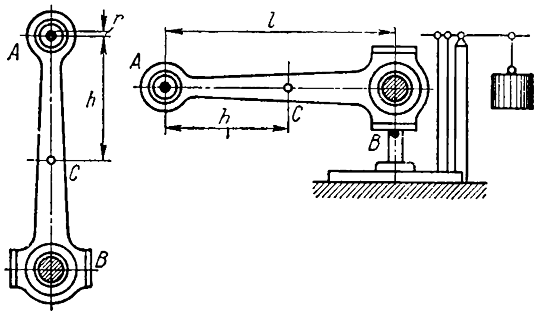


Fig. 375

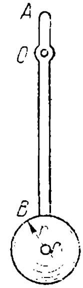


Fig. 376

from a rope tackle block. The reading on the scales shows the weight $P = 50$ kgf. The other data are as follows: the weight of the connecting rod is $Q = 80$ kgf, the distance between the verticals passing through the points A and B is $l = 1$ m; the radius of the bushing of the small end of the connecting rod is $r = 4$ cm. Determine the central moment of inertia of the connecting rod about the axis perpendicular to the plane of the sketch.

Ans. $J = \frac{Pl + Qr}{g}\left(\frac{g}{\pi^2}T^2 - \frac{P}{Q}\,l - r\right) = 1.77$ kgfmsec^2.

592. A pendulum consists of a rod AB and a ball of mass m and radius r (Fig. 376) The centre of the ball C is on the line of direction of the rod. Determine at which point O on the rod the axis of suspension should be located, so that the duration of one small oscillation is equal to the given value T Neglect the mass of the rod.

Hint. A moment of inertia of the ball of mass m and radius r about the axis, passing through the centre, is $\frac{2}{5}mr^2$

Ans. $OC = \frac{1}{2\pi^2}\left(gT^2 + \sqrt{g^2T^4 - 1.6\pi^4 r^2}\right)$.

Since OC should be $\geqslant r$, then the solution is only possible, if $T^2 \geqslant 1.4\frac{\pi^2}{g}r$; the solution with the minus sign in front of the root is impossible.

593. At what distance from the centre of gravity should a physical pendulum be suspended to give the minimum period of oscillations?

Ans. It should be suspended at the distance which is equal to the radius of inertia of the pendulum about the axis, which passes through its centre of gravity perpendicular to the plane of oscillation.

594. A pendulum consists of two weights fastened on the ends of a rod. The distance between the weights is l. The upper weight has a mass m_1 and the lower one m_2. Determine at what distance x from the lower weight should an axis of suspension be placed to give the minimum period of small oscillations. Neglect the mass of the rod and consider the weights to be particles.

Ans. $x=l\sqrt{m_1}\,\dfrac{\sqrt{m_1}+\sqrt{m_2}}{m_1+m_2}$.

595. At what distance from the axis of suspension should an additional weight be attached to a physical pendulum to make the frequency of oscillation the same?

Ans. It should be placed at the distance of the reduced length of physical pendulum.

596. A circular cylinder of mass M, length $2l$ and radius $r=\dfrac{l}{6}$ swings about an axis O, which is perpendicular to the plane of the sketch (Fig. 377). What change will occur in the period of swinging of the cylinder, if a point mass m is attached to it at the distance $OK=\dfrac{85}{72}\,l$?

Ans. The period of swinging will not change as the point mass m is applied at the centre of swinging of the cylinder.

597. A seismograph is used to record the vibration of the earth. In a seismograph a physical pendulum is used, whose axis of suspension makes an angle α with the vertical. The distance between the axis of suspension and the centre of gravity of the pendulum is a, the moment of inertia of the pendulum about the axis, passing through its centre of gravity parallel to the axis of suspension, equals J_c. The pendulum weighs P

Determine the period of oscillation of the pendulum.

Ans. $T=2\pi\sqrt{\dfrac{gJ_c-Pa^2}{Pga\sin\alpha}}$.

598. A homogeneous ball of radius r and mass m is attached to an elastic wire which is twisted through an angle φ_0 and is then released. The moment of the couple required to twist the wire for

one radian is c. Neglecting the effect of air resistance, determine the motion of the ball. It is assumed that moment of the elastic forces of the wire is proportional to the angle of twist φ.

Ans. $\varphi=\varphi_0 \cos \sqrt{\frac{5c}{2mr^2}}\, t$.

599. To adjust the hairspring of a watch, special balance wheels are used, such as shown in Fig. 378. The balance wheel A is free to rotate about the axis perpendicular to its plane which passes through the centre of gravity O and has the moment of inertia J with respect to this axis. The balance wheel is set in motion by a helical spring, one end of which is attached to the wheel while the other end is fixed to the watch frame. When the balance wheel starts to rotate, the moment of the elastic forces of the spring is not zero, it is proportional to the angle of twist. To coil the spring for one radian a moment c is required. Initially, when the moment of the elastic forces was zero, the balance wheel was given an initial angular velocity ω_0. State the law of motion of the balance wheel.

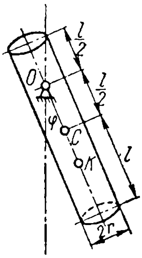


Fig. 377

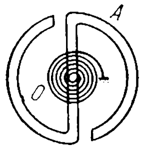


Fig. 378

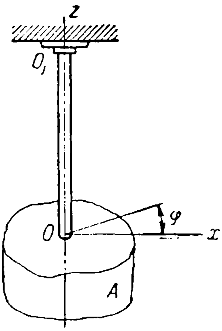


Fig. 379

Ans. $\varphi=\omega_0 \sqrt{\frac{J}{c}} \sin \sqrt{\frac{c}{J}}\, t$.

600. A body A is attached to an elastic vertical rod OO_1 to determine the moment of inertia J_z of the body about its vertical axis Oz. The rod is twisted by turning the body A around its axis Oz through a small angle φ_0 and it is released then. The time for 100 swings is $100\,T_1=2$ min, where T_1 is one half of a period. The oscillations are found to be harmonic, as the moment of elastic forces of the rod, proportional to the angle of twist, is $c\varphi$. To determine the coefficient c a second experiment is carried out: a homogeneous circular disk of weight $P=1.6$ kgf and radius $r=15$ cm is mounted on the rod at O. The duration of one swing becomes $T_2=1.5$ sec. Determine the moment of inertia J_z of the body A. (See Fig. 379.)

Ans. $J_z = \frac{Pr^2}{2g}\left(\frac{T_1}{T_2}\right)^2 = 0.117$ kgfcmsec^2.

601. The data given are the same as in the previous problem, except that to determine the coefficient c another experiment is carried out: a homogeneous circular disk of weight P and radius r is mounted on the body, whose moment of inertia is to be determined. If the period of oscillations of the body without the disk is τ_1 and with the disk is τ_2, find the moment of inertia J_z of the body.

Ans. $J_z = \frac{Pr^2}{2g}\frac{\tau_1^2}{\tau_2^2 - \tau_1^2}$.

602. A disk suspended from the end of an elastic wire performs torsion oscillations in a liquid. The moment of inertia of the disk about the axis of the wire is J. The moment of the couple required to twist the wire through one radian equals c. The moment of resistance to motion is $\alpha S \omega$, where α is the coefficient of viscosity of the liquid, S is the sum of areas of the top and bottom bases of the disk, and ω is the angular velocity of the disk.

Determine the period of oscillation of the disk in the liquid.

Ans. $T = \frac{4\pi J}{\sqrt{4cJ - \alpha^2 S^2}}$.

603. Referring to the previous problem, derive the law of decrease of the amplitude of the disk oscillations.

Ans. The decrease of the amplitude of the disk oscillations is a geometric progression with a denominator $e^{-\frac{\alpha\pi S}{\sqrt{4cJ - \alpha^2 S^2}}}$

604. A disk, suspended from a flexible wire, oscillates in a liquid. This experiment is carried out to determine the coefficient of viscosity of the liquid. The moment of force equal to $M_0 \sin pt$ ($M_0 = \text{const}$) is applied to the disk from outside and thus resonance occurs. The moment of the force resisting the motion is $\alpha S \omega$, where α is a coefficient of viscosity, S is the sum of areas of the top and bottom bases of the disk, and ω is the angular velocity of the disk. Determine the coefficient of viscosity α, if the amplitude of forced oscillations of the disk at resonance is φ_0.

Ans. $\alpha = \frac{M_0}{\varphi_0 S p}$

605. Fig. 380 represents a vibrograph used to record horizontal vibrations of a machine foundation. It has a pendulum OA consisting of a lever with a weight attached to its end, and it is free to rotate about its horizontal axle O, being held in vertical equilibrium by means of its own mass and a helical spring. The maximum static moment of the weight of the pendulum with respect to its

axis of rotation is $Qh=4.5$ kgfcm, the moment of inertia with respect to the same axis is $J=0.03$ kgfcmsec^2, and the stiffness factor of the spring, which is proportional to the angle of twist, is $c=0.1$ kgfcm. In equilibrium the spring is unstretched. Determine the period of natural oscillations of the pendulum for small angles of deviation. Neglect the effect of resistance.

Ans. $T=0.5$ sec.

606. The data given are the same as in the previous problem, except that the vibrograph is fixed to a foundation which performs harmonic horizontal vibrations according to the law: $x=a\sin 60t$ cm. Determine the amplitude a of vibrations of the foundation, if the amplitude of forced oscillations of the pendulum occurs at 6°

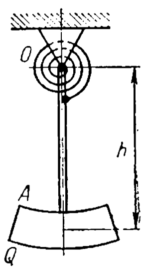


Fig. 380

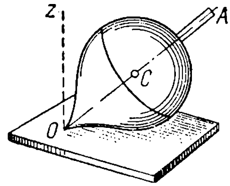


Fig. 381

Ans. $a=6.5$ mm.

607. A top spins about its axis OA in a clockwise direction with constant angular velocity $\omega=600$ sec^{-1} (Fig. 381). The axis OA is inclined to the vertical, and the lower end of the axis O is fixed. The centre of gravity C of the top lies on the axis OA at a distance $OC=30$ cm from the point O. The radius of inertia of the top about the axis OA equals 10 cm. Determine the motion of the axis of the top, assuming that for high values of the angular velocity ω the principal angular momentum of the top is directed along the axis OA and equals $J\omega$.

Ans. The axis rotates with constant angular velocity ω_1 about the vertical Oz in a clockwise direction and describes a circular cone; $\omega_1=0.49$ sec^{-1}

608. A turbine has a speed of 1500 rpm. Its shaft is parallel to the longitudinal axis of the ship. The weight of all the rotating parts is 6000 kgf, and the radius of inertia is $\rho=0.7$ m. Determine the gyroscopic pressure on the bearings, if the ship performs a gyration about its vertical axis, turning through 10° per second. The distance between the bearings is $l=2.7$ m.

Ans. 3090 kgf.

609. Determine the maximum gyroscopic pressure on the bearings of a high-speed turbine mounted on a ship (Fig. 382) The ship is under the action of pitching with an amplitude of 9° and a period of 15 sec about its axis perpendicular to that of the rotor The latter weighs 3500 kgf, its radius of inertia is 0.6 m and it makes 3000 rpm. The distance between the bearings is 2 m.

Ans. 1320 kgf.

610. The moment of inertia of a propeller and revolving parts of a rotary engine of an aircraft, with respect to a longitudinal axis

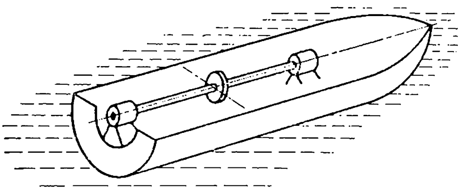


Fig. 382

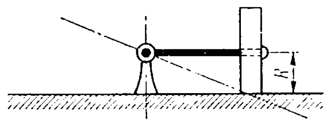


Fig. 383

of the plane, is $J=0.8$ kgfmsec², and its speed is $n=1200$ rpm.

Determine the gyroscopic moment of these parts which acts on the aircraft at the instant when the latter starts to turn sideways at a speed $v=40$ m/sec, making an arc of radius $R=25$ m.

Ans. $M=160$ kgfm. The aircraft is diving or rising depending on the direction of turn.

611. A locomotive is set in motion by a turbine whose axle is parallel to the axle of the wheels, and it rotates in the same direction at a speed of 1500 rpm. The moment of inertia of the moving parts of the turbine, with respect to the axis of rotation, is $J_z=$ $=20$ kgfmsec² If the locomotive runs on a curved track of radius 250 m at a speed of 15 m/sec, determine the value of the additional pressure on the rails. The track gauge is 1.5 m.

Ans. The pressure exerted on one rail is 126 kgf downwards and on the other one is 126 kgf upwards.

612. Each roller of a crusher weighs $P=1200$ kgf, and the radius of inertia about its axis is $\rho=0.4$ m, its radius being $R=0.5$ m. The instantaneous axis of rotation of the roller passes through the middle of the line of contact of the roller with the bottom part of the crusher (Fig. 383) If the transport velocity of rotation of the roller about the vertical axis is $n=60$ rpm, determine the pressure of the roller on the bottom horizontal part of the crusher.

Ans. $N=2740$ kgf.

613. Fig. 384 represents the ring of an aircraft tachometer used to determine the frequency of vibration in airplanes. A ring AB is free to rotate about the axle A which coincides with one of its diameters, and is fixed perpendicular to the axis of spin of the tachometer. By means of a pull rod BC, the ring is connected with a heavy sleeve, and is held by a helical spring at the initial position, corresponding to the angle φ_0, between the longitudinal axis of

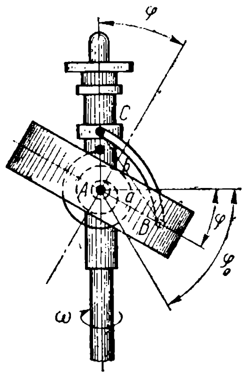


Fig. 384

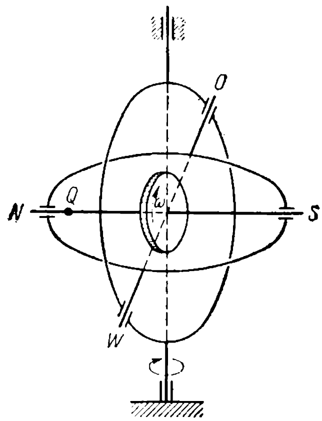


Fig. 385

the ring and the axis of spin of the tachometer when the spring is not compressed. A sleeve is connected with the pen of the recording chart. Determine the dependence between the angular velocity ω of the tachometer and the angle of deviation φ of the axis of the ring from the axis of the tachometer under present conditions. The equatorial and polar moments of inertia of the ring are denoted by A and C, the weight of the sleeve is Q, $AB=a$, the chord $BC=b$, and the spring stiffness is denoted by c kgfcm. The moment of force of resistance of the spring is proportional to the angle of twist. Neglect the weight of the pull rod BC and any effects of friction. The calculations should be carried out to the second power of the expansion in terms of $\frac{a}{b}$.

Ans. $$\omega^2 = \frac{c(\varphi_0-\varphi)+Qa\left(1-\frac{a}{b}\sin\varphi\right)\cos\varphi}{(C-A)\sin\varphi\cos\varphi}.$$

614. A gyroscope in equilibrium on a suspension frame system is placed at latitude φ in the northern hemisphere of the earth (Fig. 385). The rotor axis of the gyroscope lies in the plane of a meridian along the horizontal at this locality. A counterweight is fixed to the inner ring of the suspension system, in the direction

of the rotor axle, in such a way that the latter remains in the plane of the meridian while rotating together with the earth. The angular velocity of spin of the rotor about its axis is ω, the moment of inertia of the rotor with respect to this axis is J, the radius of the inner ring of the suspension system is a, and the angular velocity of rotation of the earth is ω_1. Neglecting friction and the mass of the rings, determine the weight Q of the counterweight.

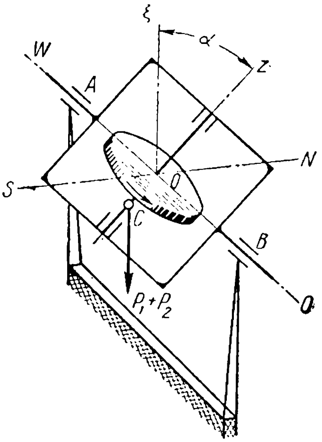


Fig. 386

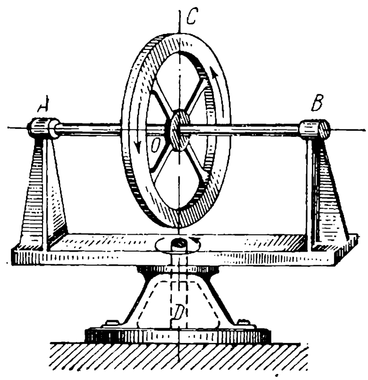


Fig. 387

Ans. $Q = \dfrac{J\omega\omega_1 \sin\varphi}{a}$

615. A gyroscope frame with two degrees of freedom has an axle AB which is mounted horizontally along the line O-W at latitude $\varphi = 30°$ The rotor of the gyroscope of weight $p_1 = 2$ kgf and radius $r = 4$ cm spins with constant angular velocity $\omega = 3000$ sec^{-1}. The rotor and the frame have a common centre of gravity C, which is located on the axis Oz of the rotor, at a distance $OC = h$ from the axis AB. The static moment of the gyroscope is $H = (p_1 + p_2)h = 1.3$ gfcm. Determine the position of the frame at equilibrium, i. e. the angle α of deviation of the rotor axle Oz in the plane of the meridian from the vertical $O\xi$ of the place. The rotor is assumed to be a homogeneous disk (Fig. 386)

Ans. $\alpha = 45°$

616. A wheel of radius a and weight $2p$ rotates about its horizontal axis AB with the constant angular velocity ω_1 (Fig. 387). The axle AB also rotates about a vertical axis CD, passing through the centre of the wheel, with the constant angular velocity ω_2. The directions of rotations are indicated by arrows.

$AO = OB = h$. It is assumed that the mass of the wheel is distributed uniformly along its rim. Find the pressures N_A and N_B on the bearings A and B.

$$Ans.\ N_A = p\left(1+\frac{a^2\omega_1\omega_2}{gh}\right); \quad N_B = p\left(1-\frac{a^2\omega_1\omega_2}{gh}\right).$$

40. Theorem on the Change of Kinetic Energy of a System

617. The larger pulley of a chain transmission rotates with angular velocity ω (Fig. 388) The radius of the pulley is R and the moment of inertia about the axis of rotation is J_1. The smaller pulley has radius r and moment of inertia J_2 about its axis of rotation. A chain of weight Q runs round both pulleys. Compute the kinetic energy of the whole system.

$$Ans.\ T = \frac{\omega^2}{2}\left[J_1 + \left(\frac{R}{r}\right)^2 J_2 + \frac{Q}{g} R^2\right].$$

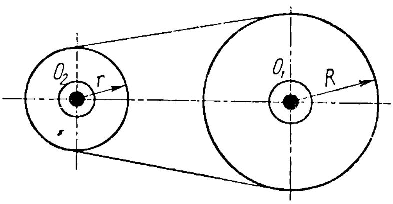


Fig. 388

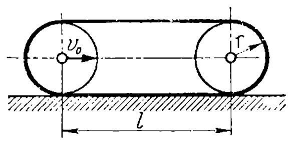


Fig. 389

618. The speed of translatory motion of a body, which weighs 920 kgf, is 900 m/sec, and its angular velocity corresponds to 45 revolutions per second. The moment of inertia about its axis of rotation passing through its centre of gravity is 2 kgfmsec^2 Determine the ratio in per cent between the kinetic energy of rotation and the kinetic energy of translation of the body.

Ans. 0.2 per cent.

619. Compute the kinetic energy of the caterpillar of a tractor which moves with the velocity v_0. The distance between the wheel axles is l, the radius of each wheel is r, and the weight of one metre of the caterpillar is γ (Fig. 389)

$$Ans.\ T = 2\frac{\gamma}{g}(l+\pi r)v_0^2.$$

620. Fig. 390 represents a crank mechanism, where the connecting rod is of the slotted bar type. J_0 a moment of inertia of the crank OA about its axis of rotation perpendicular to the plane of the sketch. The length of the crank is a and the mass of the

slotted bar is m. Neglect the mass of the slide block. OA rotates with angular velocity ω. At what positions of the mechanism will the kinetic energy attain its maximum and minimum values?

Ans. $T=\frac{1}{2}(J_0+ma^2\sin^2\varphi)\,\omega^2$

The minimum value of the kinetic energy is reached at the extreme positions of the slotted bar while the maximum is reached when the slotted bar is in the middle position.

621. Compute the value of the kinetic energy of the slider-crank mechanism, shown in Fig. 391, if the following data are given: the mass of the crank is m_1, its length is r, the mass of the slider is m_2 and the length of the connecting rod is l. The crank is considered to be a uniform rod, and its angular velocity is ω. Neglect the mass of the connecting rod.

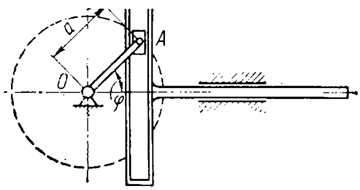


Fig. 390

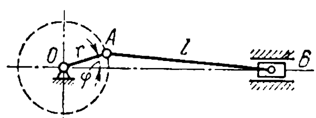


Fig. 391

$$Ans.\ T=\frac{1}{2}\left\{\frac{1}{3}m_1+m_2\left[\sin\varphi+\frac{r}{2l}\frac{\sin 2\varphi}{\sqrt{1-\left(\frac{r}{l}\right)^2\sin^2\varphi}}\right]^2\right\}r^2\omega^2.$$

622. The problem is the same as the previous one, except that the mass m_3 of the connecting rod should be taken into consideration at the position when the crank OA is perpendicular to the guide line of the slider.

Ans. $T=\frac{1}{2}\left(\frac{1}{3}m_1+m_2+m_3\right)r^2\omega^2.$

623. A planetary mechanism lying in the horizontal plane is driven by a crank OA, which connects the axles of three identical wheels I, II and III (Fig. 392) The wheel I is rigidly fixed, and the crank of weight Q rotates with angular velocity ω. Each wheel has radius r and weight P Compute the kinetic energy of the mechanism assuming that the wheels are homogeneous disks and the crank is a uniform rod. What work will be done by the couple applied to wheel III?

Ans. $T=\frac{r^2\omega^2}{3g}(33P+8Q)$.

The work of the couple is zero.

624. Fig. 393 represents mill-rollers A and B each weighing 200 kgf with identical diameters of 1 m. The distance between them CD is 1 m. Both rollers are mounted on the horizontal axle CD which rotates about the vertical axis EF Find the kinetic energy of one roller when the axle CD makes 20 rpm. When calculating the moment of inertia, the rollers are assumed to be thin homogeneous disks.

Ans. 39 kgfm.

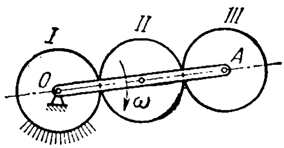


Fig. 392

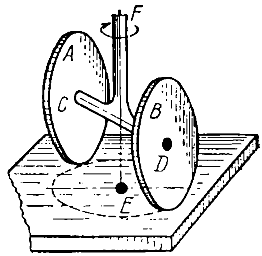


Fig. 393

625. A rocker mechanism consists of a crank OC of length R and mass m_1, a slider A of mass m_2 and a rod AB of mass m_3 (Fig. 394). The crank OC swings about an axle O perpendicular to the plane of the sketch. The slider A moves along the crank OC and sets the rod AB in motion. The latter moves in vertical guides K. The crank is assumed to be uniform, and $OK=l$. Express the kinetic energy of the rocker as a function of the angular velocity ω and angle of rotation φ of the crank OC. Consider the slider to be a point mass.

Ans. $T=\frac{\omega^2}{6\cos^4\varphi}[m_1R^2\cos^4\varphi+3l^2(m_2+m_3)]$.

626. An aircraft weighing $P=3000$ kgf rests on three supports on the ground. The tail support carries 10 per cent of the total weight of the aircraft. Both front wheels are placed on the platforms of scales. When the propeller makes $n=1432$ rpm, the readings on the scales are $N_1=1100$ kgf and $N_2=1600$ kgf. Determine the horse-power developed by the engine, if the coefficient of efficiency of the propeller is $\eta=0.8$ and the distance between the front wheels is $l=2$ m.

Ans. 1250 hp.

627. A dynamometer, shown in Fig. 395, is used to measure the power capacity of a motor. It consists of a band, with two vertical parts AC and DB, running under the pulley E of the mo-

tor being tested. The dynamometer also has a lever BF supported at O by a three-cornered prism. Raising or lowering of the support O changes the tension in the band and so alters the friction between the band and the pulley. A weight P is suspended from the lever so that BF is in equilibrium in the horizontal position. Determine the power of the motor, when it makes 240 rpm and the weight P is 3 kgf. The arm of the lever l is 50 cm long.

Ans. 0.5 hp=0.368 kw.

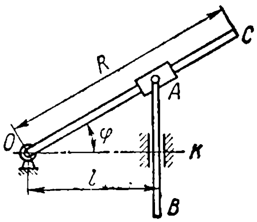


Fig. 394

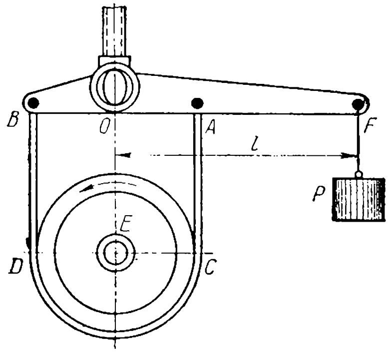


Fig. 395

628. A shaft of diameter 60 mm carries a flywheel of diameter 50 cm which makes 180 rpm. Determine the coefficient of sliding friction f between the shaft and the bearings if, after switching off the engine, the flywheel makes 90 revolutions before coming to rest. The mass of the flywheel is assumed to be distributed uniformly over its rim, and mass of the shaft should be neglected.

Ans. $f=0.07$.

629. A cylindrical shaft of diameter 10 cm and weight 500 kgf carries a flywheel 2 m in diameter which weighs 3000 kgf. At a given instant the flywheel rotates with the velocity of 60 rpm and then it is left alone. If the coefficient of friction in the bearings is 0.05, how many revolutions will the shaft make before coming to rest? When solving the problem assume that the mass of the flywheel is uniformly distributed over its rim.

Ans. 109.8 revolutions.

630. Fig. 396 shows a heavy uniform rod OA of length $l=3.27$ m, which is kept in static equilibrium with one of its ends O mounted on an axle so that it is free to rotate in the vertical plane. Find the initial velocity which must be given to the other end of the rod A so that it makes one quarter of a revolution.

Ans. $v=\sqrt{3gl}=9.81$ m/sec.

631. Two equal weights P and P_1 are connected by a flexible inextensible cord running over a small pulley A, as shown in Fig. 397 The weight P_1 is free to slide along a smooth vertical rod CD which is located at distance a from the axis of the pulley. Initially the centre of gravity of the weight P_1 is at the same level as the axis of the pulley, and then the weight P_1 starts to descend from rest under the action of gravity. Determine the relation between the velocity of the weight P_1 and the depth of descent h.

Ans. $v^2 = 2g(a^2+h^2)\,\dfrac{P_1 h - P(\sqrt{a^2+h^2}-a)}{P_1(a^2+h^2)+Ph^2}$.

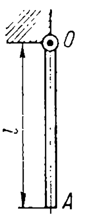


Fig. 396

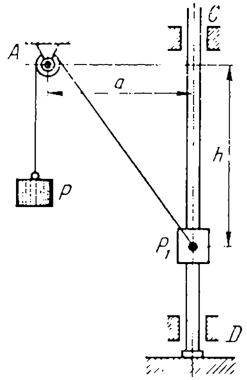


Fig. 397

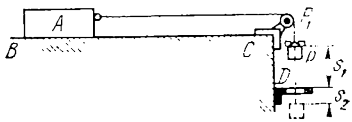


Fig. 398

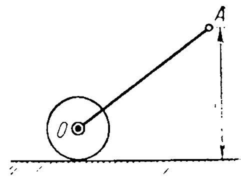


Fig. 399

632. A weight P which carries the load P_1 hangs on an inextensible string which passes over a pulley, as shown in Fig. 398. A body A of weight Q which rests in equilibrium on a rough horizontal plane BC is set in motion by the weight and the load. After dropping a distance s_1, the weight P passes through a ring D, which catches the load, and P continues downwards a distance s_2 before it comes to rest. The other data given are as follows: $Q=0.8$ kgf; $P=0.1$ kgf; $P_1=0.1$ kgf; $s_1=50$ cm, and $s_2=30$ cm. Neglecting the masses of the string and the pulley and the effect of friction in the pulley, determine the coefficient of dynamic friction f between the body A and the plane BC.

Ans. $f = \dfrac{s_1(P_1+P)(P+Q)+s_2P(P+P_1+Q)}{Q[s_1(P+Q)+s_2(P+P_1+Q)]} = 0.2.$

633. A homogeneous thread of length L lies on a horizontal table with a part over the edge thus setting the thread in motion. Determine the time T required for the thread to leave the table, if we know that initially the part hanging over the edge was of length l and the velocity was zero.

Ans. $T = \sqrt{\dfrac{L}{g}}\ln\left(\dfrac{L+\sqrt{L^2-l^2}}{l}\right)$.

634. A heavy homogeneous thread of length $2a$ hangs in equilibrium on a smooth pivot. Then it starts to move with initial velocity v_0. Determine the velocity of the thread when it just leaves the pivot.

Ans. $v=\sqrt{ag+v_0^2}$.

635. A cylindrical roller, 60 cm in diameter and weighing 392 kgf, is pushed by a constant force P in the direction AO (Fig. 399). The handle AO is 1.5 m long, and the point A is 1.2 m above the floor. Neglecting the effect of friction in the bearings, determine the value of the force P, if the roller has moved a distance of 2 m and the axis of the roller has a velocity of 80 cm/sec ($g=980$ cm/sec^2).

Ans. $P=12$ kgf.

636. A wheel of radius r rolls, without slipping, up a plane inclined at an angle α to the horizontal. If the coefficient of rolling friction is k and the wheel is assumed to be a homogeneous disk, determine the initial velocity, parallel to the line of slope of the inclined plane, which must be given to the axle of the wheel to make it ascend to a height h.

Ans. $v=\frac{2}{3}\sqrt{3gh\left(1+\frac{k}{r}\cot\alpha\right)}$.

637. The moments of inertia of two shafts I and II with pulleys and gears mounted on them are $J_1=500$ kgfcmsec^2 and $J_2=400$ kgfcmsec^2 (Fig. 400). The gear ratio of the transmission drive is $k_{12}=2/3$. If the system is set in motion from rest by a torque moment $M_1=50$ kgfm applied to the shaft I, how many revolutions will the shaft II make before it rotates at 120 rpm? Neglect the friction in the bearings.

Ans. After 2.34 revolutions.

638. A belt conveyer, shown in Fig. 401, is set in motion from rest by a shaft drive connected to the bottom pulley B. A constant torque of moment M is transmitted by a shaft drive to the pulley B. If the weight of the load A is P and the pulleys B and C of

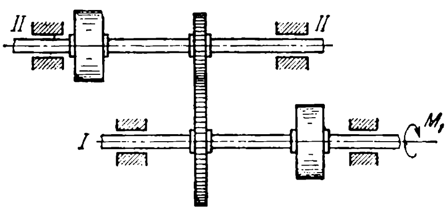


Fig. 400

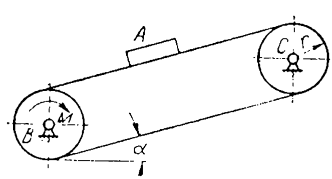


Fig. 401

radius r and weight Q are each assumed to be homogeneous disks, determine the velocity v of the conveyer belt as a function of its displacement s. The belt makes an angle α with the horizontal and it does not slide on the pulleys. Neglect the mass of the belt.

Ans. $v=\sqrt{\frac{2g(M-Pr\sin\alpha)}{r(P+Q)}s}$.

639. One end of a cable is wound on a cylindrical drum, with horizontal axis of rotation, while the other end carries a load of weight P. Neglecting the effect of friction in the bearings of the drum axle as well as the masses of the cable and shaft, determine the velocity of the load after it has travelled a distance h from rest. The weight of the drum is Q, and it is assumed to be a homogeneous, solid cylinder.

Ans. $v=2\sqrt{gh\frac{P}{2P+Q}}$.

640. Fig. 402 shows an epicyclic gear train lying in a horizontal plane. The system is set in motion by a constant torque of moment M applied to the crank OA. Knowing that the fixed gear I has a radius r_1 and the movable gear II of weight P has a radius r_2, determine the angular velocity of the crank as a function of its angle of rotation. OA weighs Q. The gear II is assumed to be a homogeneous disk and the crank is a uniform rod.

Ans. $\omega=\frac{2}{r_1+r_2}\sqrt{\frac{3gM}{9P+2Q}\varphi}$.

641. Fig. 403 represents an eccentric mechanism lying in a horizontal plane. The eccentric A sets the roller B and the rod D in a reciprocating motion. A spring E, that is connected with the rod, provides a constant contact between the roller and the eccentric. The weight of the eccentric is p and the eccentricity e equals half of its radius. The coefficient of stiffness of the spring is c. At the extreme left position of the rod the spring is not compressed. What angular velocity is required for the eccentric to move the rod D from the extreme left to the extreme right position? Neglect the masses of the roller, the rod and the spring. The eccentric is assumed to be a homogeneous disk.

Ans. $\omega=2\sqrt{\frac{cg}{3p}}$.

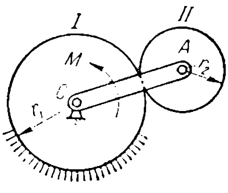


Fig. 402

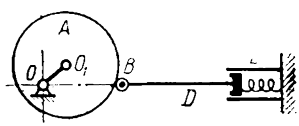


Fig. 403

642. An aircraft has landed at a speed of 10 m/sec. The resistance of the air to the plane's motion is 60 kgf, each wheel of the aircraft of radius 0.5 m weighs 100 kgf and the coefficient of rolling friction between the wheels and the ground is 1 cm. The weight of the aircraft, excluding that of the wheels, is 1100 kgf. The wheels are assumed to be homogeneous disks. Compute the distance travelled by the aircraft before coming to rest.

Ans. 89.5 m.

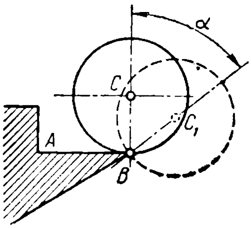


Fig. 404

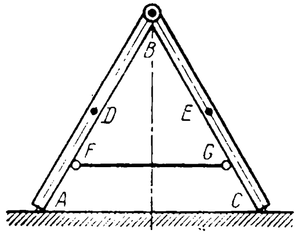


Fig. 405

643. A heavy homogeneous cylinder is given a negligible initial velocity and rolls without sliding along a horizontal platform AB with a sharp edge B, as shown in Fig. 404. The direction of the sharp edge is parallel to the generating line of the cylinder. The radius of the cylinder base is r At the instant when the cylinder leaves the platform, the plane, passing through the axis of the cylinder and the edge B, makes an angle $CBC_1=\alpha$ with the vertical.

Determine the angular velocity of the cylinder at the moment when it leaves the platform and the magnitude of the angle α. Neglect the effects of rolling friction and air resistance.

Hint. At the instant when the cylinder leaves the platform, the component of the weight directed along the line C_1B is equal in magnitude to the centrifugal force of rotation about the platform edge $\frac{Q}{g}r\omega^2$, where Q is the weight of the cylinder.

Ans. $\omega=2\sqrt{\frac{g}{7r}}$; $\alpha=\arccos\frac{4}{7}=55.1^\circ$

644. A step ladder ABC, hinged at B, rests on a smooth horizontal floor, as shown in Fig. 405. $AB=BC=2l$. The centres of gravity are at the mid-points D and E of the rods. The radius of gyration of each part of the ladder about the axis passing through the centre of gravity is ρ. The distance between B and the floor is h. At a certain moment the ladder collapses due to the rupture of a link FG between the two halves of the ladder. Neglecting the effect of friction in the hinge, determine: (1) the velocity v of the point B at the moment when it hits the floor, (2) the velocity of

the point B at the moment when it is at distance $\frac{1}{2}h$ from the floor.

Ans. (1) $v=2l\sqrt{\frac{gh}{l^2+\rho^2}}$; (2) $v=\frac{1}{2}\sqrt{gh\frac{16l^2-h^2}{2(l^2+\rho^2)}}$.

645. A vertical rod AB of length $2a$ rests in equilibrium. At a certain instant the rod falls down while its end A slides along a smooth horizontal floor. Determine the velocity of the centre of gravity of the rod as a function of its height h above the floor (Fig. 406).

Ans. $v=(a-h)\sqrt{\frac{6g(a+h)}{4a^2-3h^2}}$.

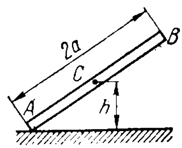


Fig. 406

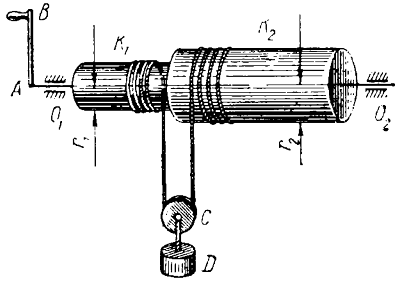


Fig. 407

646. Fig. 407 represents a windlass which consists of two rigidly fixed shafts K_1 and K_2 of different radii r_1 and r_2, respectively. Their moments of inertia about the axis O_1O_2 are J_1 and J_2, respectively. Both shafts are set in motion by a handle AB. A load D of weight P is attached to the movable pulley C, and a weightless unstretched string, which supports this load, is wound with its left part on the shaft K_1 and its right part on the shaft K_2. The pulley C is thus hanging in the loop between the two parts of the string. On turning the handle AB, the left part of the string unwinds from the shaft K_1 and the right part winds up on the shaft K_2. The handle is under the action of a constant torque of moment M. Find the angular velocity of rotation of the handle at the instant, when the load D is raised to a maximum height s. Initially the system was in equilibrium. Neglect the masses of the handle and the pulley.

Ans. $\omega=2\sqrt{2gs\,\frac{2M-P(r_2-r_1)}{(r_2-r_1)[P(r_2-r_1)^2+4g(J_1+J_2)]}}$

647. A windlass is driven from rest by a belt transmission which connects a pulley II, mounted on the shaft of the windlass, with a pulley I, mounted on the shaft of the motor (Fig. 408). The pulley I of weight P_1 and radius r is acted on by a constant torque

of moment M. The pulley II weighs P_2 and its radius is R. The drum of the windlass weighs P_3 and its radius is r. The weight of the load is P_4. Find the velocity of the load P_4 at the moment when it has risen a distance h. Neglect the masses of the belt and string and any effects of friction in the bearings. Both the pulleys and the drum are considered to be homogeneous circular cylinders.

Ans. $v=2\sqrt{\dfrac{gh\left(M\dfrac{R}{r^2}-P_4\right)}{P_1\left(\dfrac{R}{r}\right)^2+P_2\left(\dfrac{R}{r}\right)^2+P_3+2P_4}}.$

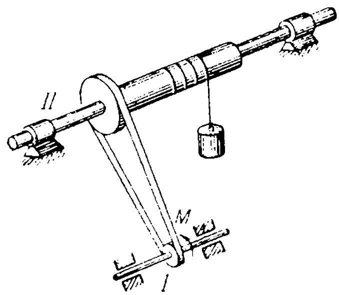


Fig. 408

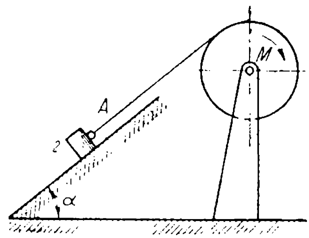


Fig. 409

648. The problem is the same as the preceding one, except that the mass of the string should be taken into consideration. The length of the string is l and its weight per unit length is p. Initially a part of the string of $2h$ length hung down from the drum of the windlass.

Ans. $v=2\sqrt{\dfrac{gh\left(M\dfrac{R}{r^2}-P_4-\dfrac{3}{2}ph\right)}{P_1\left(\dfrac{R}{r}\right)^2+P_2\left(\dfrac{R}{r}\right)^2+P_3+2P_4+2pl}}.$

649. A constant torque of moment M is applied to the drum of a windlass of radius r and weight P_1 (Fig. 409). A cable has one of its ends A attached to the weight P_2 while the other is wound on the drum. The weight is pulled up a plane by the cable, the plane being inclined at an angle α to the horizontal. What will be the angular velocity of the drum after it has turned through an angle φ? The coefficient of rolling friction between the weight and the plane is f. Neglect the mass of the cable. The drum is assumed to be a homogeneous circular cylinder which was initially in equilibrium.

Ans. $\omega=\dfrac{2}{r}\sqrt{g\dfrac{M-P_2r(\sin\alpha+f\cos\alpha)}{P_1+2P_2}\varphi}.$

650. The problem is the same as the preceding one, except that the mass of the cable should be taken into consideration. The length of the cable is l and its weight per unit length is p. Initially, a part of the cable of length a hung down from the drum of the windlass. Neglect the change in the potential energy of the cable, wound on the drum.

$$Ans.\ \omega = \frac{1}{r}\sqrt{2g\,\frac{2M-2P_2r(\sin\alpha+f\cos\alpha)-pr(2a-r\Delta\varphi)\sin\alpha}{P_1+2P_2+2pl}\,\Delta\varphi}.$$

651. A wheel A is connected to a second wheel B by means of inextensible string, passing over a pulley C, which rotates about

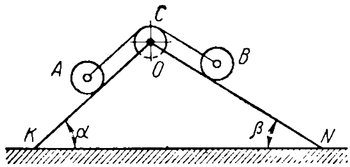


Fig. 410

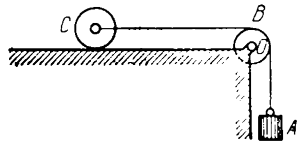


Fig. 411

a fixed horizontal axle O, as shown in Fig. 410. The wheel A rolls down the inclined plane OK thus pulling up the wheel B which rolls along the inclined plane ON. Determine the velocity of the axle of the wheel A, when it has travelled a distance s down the slope. Initially the system was in equilibrium. Both wheels and the pulley are assumed to be homogeneous disks of identical weight and radius. Neglect the weight of the string.

$$Ans.\ v = 2\sqrt{\frac{1}{7}gs(\sin\alpha-\sin\beta)}.$$

652. The problem is the same as the preceding one, except that the rolling friction between the wheels and the inclined planes should be taken into account. The coefficient of rolling friction is k and each wheel has a radius r

$$Ans.\ v = 2\sqrt{\frac{1}{7}gs\left[\sin\alpha-\sin\beta-\frac{k}{r}(\cos\alpha+\cos\beta)\right]}.$$

653. A homogeneous cable of length L and weight Q is fastened at one end to a load A of weight P_1 and the other end runs over a pulley B and is attached to the axle of a roller, which moves without slipping along a fixed plane, as shown in Fig. 411. The pulley B is free to rotate about the axle O perpendicular to the plane of the sketch. The pulley and the roller are homogeneous circular disks each of radius r and weight P_2. The coefficient of rolling friction between the roller C and the horizontal plane is k. Initially, when the system was in equilibrium, a part l of the

cable hung down from the pulley. Determine the velocity of the load A as a function of its vertical height h.

$$Ans.\ v=\sqrt{\frac{2gh\left[P_1+\frac{Q}{2L}(2l+h)-P_2\frac{k}{r}\right]}{P_1+2P_2+Q}}$$

41. Plane Motion of a Rigid Body

654. A heavy body consists of a rod AB of length 80 cm and weight 1 kgf, and a disk of radius 20 cm and weight 2 kgf attached to it, as shown in Fig. 412. Initially the rod is vertical and the body is moving so that the centre of gravity M_1 of the rod has zero velocity and the centre of gravity M_2 of the disk has velocity 360 cm/sec, directed horizontally to the right. Determine the subsequent motion of the body, while acted on only by gravity.

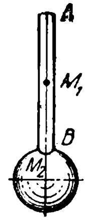


Fig. 412

Ans. The body rotates uniformly about its centre of gravity with angular velocity 6 sec^{-1}. The path of the centre of gravity is the parabola $y^2=117.5x$ (the origin of coordinates is at B, the axis y being directed along the horizontal to the right and the axis x downwards).

655. Two balls M_1 and M_2, weighing $p_1=2$ kgf and $p_2=1$ kgf, respectively, are connected by a rod of length $l=60$ cm. At $t=0$ the rod M_1M_2 is horizontal and the ball M_2 is in equilibrium while the velocity of the ball M_1 is $v_1=60\pi$ cm/sec directed vertically upwards. Neglecting the air resistance as well as the weight of the rod and the dimensions of the balls, determine: (1) the motion of both balls under the action of gravity; (2) the distances h_1 and h_2 between the balls and the horizontal M_1M_2, on which the balls were initially, after 2 sec have elapsed; (3) the tension T in the rod.

Ans. (1) The centre of gravity C of the balls moves along the vertical: $y_c=-\frac{2}{3}v_1t+\frac{1}{2}gt^2$, and the rod rotates about the centre of gravity with angular velocity π sec^{-1};

(2) $h_1=h_2=1711$ cm;

(3) $T=\frac{1}{g}\frac{p_1p_2}{p_1+p_2}l\omega^2=0.4$ kgf.

656. An axle of the driving wheel of a car moves along a horizontal surface in a straight line. It is acted upon by a horizontal motive force F The radius of gyration of the wheel about an axis, passing through the centre of gravity perpendicular to its plane, equals ρ. The coefficient of sliding friction between the wheel and

the pavement is f. The radius of the wheel is r and its weight is P. What conditions must be satisfied by the force F to make the wheel run without sliding? Neglect resistance to rolling.

Ans. $F \leqslant fP \frac{r^2+\rho^2}{\rho^2}$.

657. The coefficient of friction between a homogeneous heavy ball and an inclined plane is f. Determine the value of the angle α between the inclined plane and the horizontal, when the ball just rolls along the plane without sliding. Neglect resistance to rolling.

Ans. $\alpha \leqslant \arctan \frac{7}{2} f$.

658. A homogeneous cylinder, with horizontal axis, rolls sliding down an inclined plane by virtue of its weight. The coefficient of sliding friction is f. Determine the angle of inclination between the plane and the horizontal and the acceleration of the axis of the cylinder.

Ans. $\alpha > \arctan 3f$; $w = g(\sin\alpha - f\cos\alpha)$.

659. A homogeneous wheel of radius r rolls without sliding down a plane inclined at an angle α to the horizontal. At what value of the coefficient of rolling friction k will the centre of gravity of the wheel move uniformly while the wheel rotates uniformly about the axis, passing through its centre of gravity perpendicular to its plane?

Ans. $k = r\tan\alpha$.

660. A roller has the weight P and radius r It is pulled along a rough horizontal floor by a horizontal force T applied to the end of a string wound round the drum, as shown in Fig. 413. The force T is applied at an angle α to the horizontal. The radius of the drum is a and the radius of gyration of the roller is ρ. Find the equation of motion of the axis O of the roller.

Ans. $x = \frac{T}{P} \frac{rg(r\cos\alpha - a)}{2(\rho^2+r^2)} t^2$;

the axis x is directed from the left to the right.

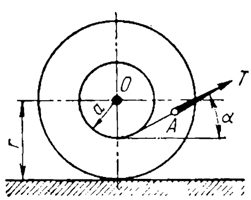


Fig. 413

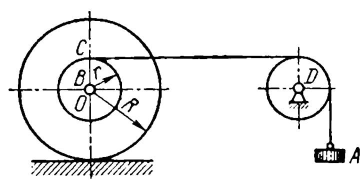


Fig. 414

661. Referring to Fig. 414, find the acceleration of the load A of weight P which descends on the weightless unstretched string. The other end of the string, which passes over the fixed weightless pulley D, is wound on a drum and so makes the wheel C roll without sliding along a horizontal rail. The drum B of radius r is rigidly connected with the wheel C of radius R, their total weight being Q and the radius of gyration about the horizontal axis O equals ρ.

Ans. $w = g\dfrac{P(R+r)^2}{Q(\rho^2+R^2)+P(R+r)^2}$.

662. A uniform rod AB of weight P is suspended horizontally from a ceiling by two vertical threads fastened to the ends of the rod.

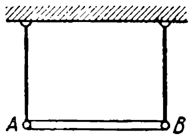


Fig. 415

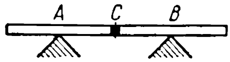


Fig. 416

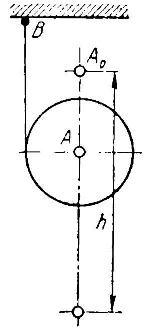


Fig. 417

Find the tension in one of the threads at the moment when the other thread breaks (Fig. 415).

Hint. To solve this problem set up the equation of motion of the rod just after the thread breaks. Neglect the change in the direction of the rod and the change in the distance between the centre of gravity of the rod and the other thread.

Ans. $T = \dfrac{P}{4}$.

663. A uniform thin rod of length $2l$ and weight P rests on two supports A and B (Fig. 416). The centre of gravity C of the rod is located at equal distances from the supports. $CA = CB = a$. The pressure on each support equals $\frac{1}{2}P$. If the support B is suddenly removed, what instantaneous change in the pressure on the support A occurs?

Ans. The difference in the pressures on the support A is

$$\frac{l^2-3a^2}{2(l^2+3a^2)}P.$$

664.* A heavy circular cylinder of mass m is suspended by a cord, one end of which is wound round the middle part of the cylinder while the other end is fixed at B (Fig. 417). The cylinder is allowed to fall from rest so that the cord unwinds. Determine the velocity v of the axis of the cylinder after it has fallen a distance h. Also find the tension T in the cord.

Ans. $v=\frac{2}{3}\sqrt{3gh}$; $T=\frac{1}{3}mg$.

665. Two cylindrical shafts of weight P_1 and P_2 are connected by an unstretched cord its ends being respectively wound round the shafts (Fig. 418). The shafts roll down two planes inclined at angles α and β to the horizontal. Determine the tension in the cord and its acceleration during the motion on the inclined planes. The shafts are assumed to be homogeneous circular cylinders. Neglect the weight of the cord.

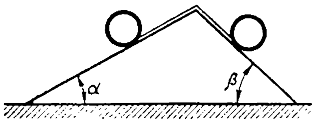


Fig. 418

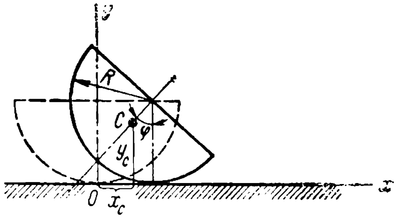


Fig. 419

Ans. $w=g\frac{P_1\sin\alpha-P_2\sin\beta}{P_1+P_2}$; $T=\frac{P_1P_2(\sin\alpha+\sin\beta)}{3(P_1+P_2)}$.

666. A homogeneous semicircular disk of radius R rolls without sliding on a rough horizontal surface, as shown in Fig. 419. Determine the period of small oscillations of the disk.

Ans. $T=\frac{\pi}{2g}\sqrt{2g(9\pi-16)R}$.

42. Forces Acting on Axis of a Rotating Body

667. A flywheel weighing 3000 kgf is mounted on a shaft. Its centre of gravity is at 1 mm distance from the horizontal axis of the shaft. The distances between the wheel and the bearings are equal. Find the pressure on the bearings, when the flywheel rotates with a speed of 1200 rpm. The flywheel has a plane of symmetry perpendicular to the axis of rotation.

Ans. The pressure on each bearing is the resultant of two forces: one equals 1500 kgf and is directed along the vertical, and the other equals 2400 kgf and is directed parallel to the straight line, which connects the geometrical centre of the wheel, which is on the axis of the shaft, with the centre of gravity of the wheel.

668. A rod AB of length $2l$, carrying two loads of equal weight P on its ends, rotates uniformly with angular velocity ω about a vertical axis Oz passing through the centre O of the rod (Fig. 420). The distance between the point O and the bearing C is a and OD is b. The angle between the rod AB and the axis Oz has a constant

value α. Neglecting the weight of the rod and the dimensions of the loads, determine the projections of the pressures on the bearing C and the bearing D at the instant, when the rod is in the plane Oyz.

Ans. $X_C = X_D = 0$; $Y_C = -Y_D = \dfrac{Pl^2\omega^2 \sin 2\alpha}{g(a+b)}$; $Z_D = -2P$.

669. Two identical cranks AC and BD of length l and weight Q each are mounted on the ends of the axle AB of a bicycle (Fig. 421). The cranks are fixed at an angle of 180° to each other. The axle AB of length $2a$ and weight P rotates with

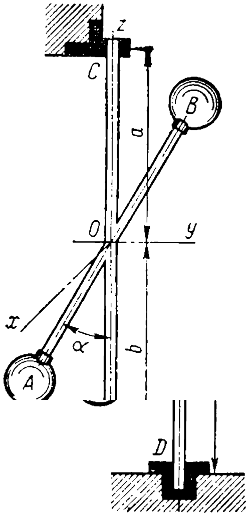


Fig. 420

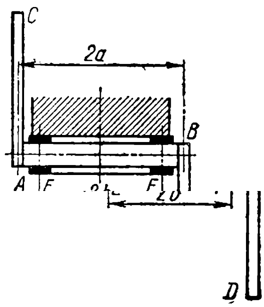


Fig. 421

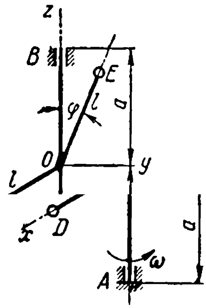


Fig. 422

constant angular velocity ω in the bearings E and F located symmetrically at distance $2b$. Determine the pressures N_E and N_F on the bearings at the instant, when the crank AC is directed vertically upwards. The mass of each crank is assumed to be distributed uniformly along its axis.

Ans. The pressure $N_E = \dfrac{1}{2} P + Q - \dfrac{al\omega^2}{2bg} Q$;

when $N_E > 0$, it is directed vertically downwards, and for $N_E < 0$ it is directed vertically upwards. The pressure $N_F = \dfrac{1}{2} P + Q + \dfrac{al\omega^2}{2bg} Q$ and is directed vertically downwards.

670. A vertical shaft AB rotates with constant angular velocity ω (Fig. 422). It carries two rods OE and OD. The rod OE makes an angle φ with the shaft while the rod OD is perpendicular to the plane of the shaft AB and the rod OE. $AB = 2a$. $OE = OD = l$. Two balls E and D each of mass m are attached to the free ends of the rods. Determine the dynamical pressures on the shaft in the

supports A and B. The balls D and E are considered as point masses and the masses of the rods are to be neglected.

Ans. $X_A = N_{Bx} = \dfrac{ml\omega^2}{2}$; $Y_A = \dfrac{ml\omega^2(a - l\cos\varphi)\sin\varphi}{2a}$;

$$Y_B = \frac{ml\omega^2(a + l\cos\varphi)\sin\varphi}{2a}.$$

671. A straight homogeneous circular cylinder of length $2l$, weight P and radius r rotates with constant angular velocity ω about the vertical Oz passing through the centre of gravity O of the cylinder (Fig. 423). The angle between

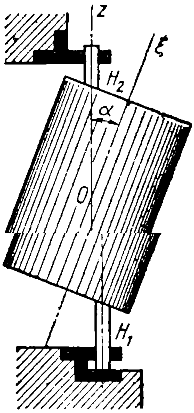


Fig. 423

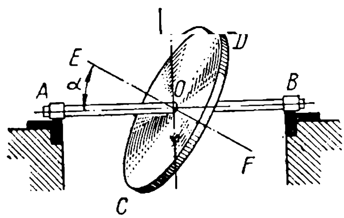


Fig. 424

the axis of the cylinder $O\xi$ and the axis Oz has a constant value α. The distance H_1H_2 between the bearings is h. Find the lateral pressures N_1 and N_2 on the bearings.

Ans. The pressures N_1 and N_2 have the same value:

$$P\frac{\omega^2 \sin 2\alpha}{2gh}\left(\frac{1}{3}l^2 - \frac{1}{4}r^2\right) \text{ and are opposite in sense.}$$

672. Fig. 424 represents a thin homogeneous circular disk CD of a steam turbine which rotates about the axle AB passing through the centre O of the disk. Due to improper boring of the bushing the axle AB makes an angle $AOE = \alpha = 0.02$ radian with the perpendicular to the plane of the disk. The other data are as follows: the weight of the disk is 3.27 kgf and its radius is 20 cm, the speed of rotation is 30,000 rpm, $AO = 50$ cm, $OB = 30$ cm, the axle AB is absolutely rigid, and the angle 2α is small ($\sin 2\alpha = 2\alpha$).

Determine the pressures on the bearings A and B.

Ans. The pressure exerted by the weight of the disk on the bearing A is 1.23 kgf and on the bearing B is 2.04 kgf. The pressures on the bearings caused by the rotation of the disk are equal in value, 822 kgf, and opposite in sense.

673. A uniform rectangular lamina of weight P rotates uniformly about its diagonal AB with angular velocity ω, as shown in Fig. 425. Determine the dynamical pressure of the lamina on the supports A and B, if the lengths of the sides are a and b, respectively.

Ans. $X_A=0;\quad Y_A=\dfrac{-Pab\omega^2(a^2-b^2)}{12g(a^2+b^2)^{3/2}};$

$X_B=0;\quad Y_B=\dfrac{Pab\omega^2(a^2-b^2)}{12g(a^2+b^2)^{3/2}}.$

674. A thin homogeneous disk is mounted in the middle of a horizontal shaft with eccentricity $OC=e$ at an angle of $90°-\alpha$ to

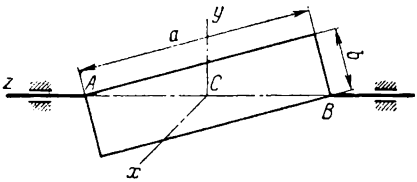


Fig. 425

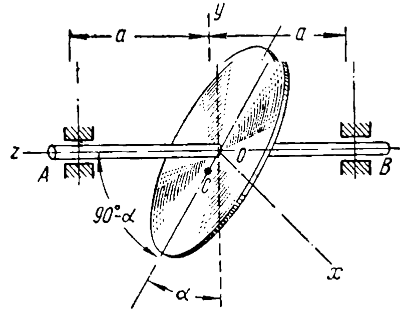


Fig. 426

the axis of the shaft (Fig. 426). The disk is of weight P and has radius r Determine the static and dynamic reactions of the supports, when the disk and the shaft rotate with angular velocity ω. The distance between the supports is $AB=2a$.

Ans. (1) The static reactions are directed along the vertical and equal

$$N'_A=P\frac{a+e\sin\alpha}{2a};\qquad N'_B=P\frac{a-e\sin\alpha}{2a}.$$

(2) The dynamic reactions are directed along the axis Oy and are equal to

$$N''_A=\frac{P}{2g}\left[e\cos\alpha+\frac{\sin 2\alpha}{2a}\left(e^2+\frac{r^2}{4}\right)\right]\omega^2;$$

$$N''_B=\frac{P}{2g}\left[e\cos\alpha-\frac{\sin 2\alpha}{2a}\left(e^2+\frac{r^2}{4}\right)\right]\omega^2.$$

43. Review Problems

675. A uniform heavy beam AB of length $2l$ and weight Q, its ends fixed, is held in a horizontal position. At a certain moment the end A is released and the beam starts to fall, rotating about a horizontal axis passing through the end B. At the instant when the beam is vertical the end B is also released. Determine the path

of the centre of gravity and the angular velocity ω in the subsequent motion of the beam (Fig. 427).

Ans. (1) The parabola $y^2=3lx-3l^2$;

(2) $\omega=\sqrt{\dfrac{3g}{2l}}$.

676. A heavy uniform rod of length l is hinged at its upper end to a horizontal axle O, as shown in Fig. 428. When the rod is vertical, it is given an angular velocity $\omega_0=3\sqrt{\frac{g}{l}}$. Because of this the rod performs half of a turn and leaves the axle O. Determine the path of the centre of gravity and the angular velocity ω of the rod in its subsequent motion.

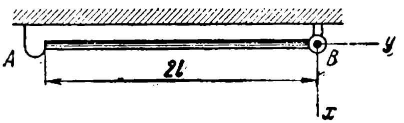


Fig. 427

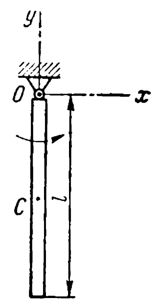


Fig. 428

Ans. (1) The parabola $y_C=\dfrac{l}{2}-\dfrac{2}{3l}x_C^2$; (2) $\omega=\sqrt{\dfrac{3g}{l}}$.

677.* Two heavy homogeneous circular cylinders A and B, of weight P_1 and P_2 and radius r_1 and r_2, respectively, have two threads wound around them. The threads are wound symmetrically with respect to the middle plane parallel to the base of the cylinders, whose generatrices are perpendicular to this plane (Fig. 429). The axle of the cylinder A is held stationary, and the cylinder B is let fall from rest under gravity. Assuming that the threads remain wound around both cylinders after time t, determine: (1) the angular velocities ω_1 and ω_2 of the cylinders; (2) the height s through which the centre of gravity of the cylinder B has fallen; (3) the tension T in the threads.

Ans. (1) $\omega_1=\dfrac{2gP_2}{r_1(3P_1+2P_2)}t$; $\omega_2=\dfrac{2gP_1}{r_2(3P_1+2P_2)}t$;

(2) $s=\dfrac{g(P_1+P_2)}{3P_1+2P_2}t^2$;

(3) $T=\dfrac{P_1P_2}{2(3P_1+2P_2)}$.

678.* A uniform rod AB of length a is placed in a vertical plane at an angle φ_0 to the horizontal in such a way that one

end A leans against a smooth vertical wall while the other end B rests on a smooth horizontal floor, as shown in Fig. 430. The rod starts to fall from rest. Determine: (1) the angular velocity and angular acceleration of the rod; (2) the angle φ_1 which the rod makes with the horizontal at the moment, when it leaves the wall.

Ans. (1) $\dot{\varphi}=\sqrt{\frac{3g}{a}(\sin\varphi_0-\sin\varphi)}$; $\quad \ddot{\varphi}=-\frac{3g}{2a}\cos\varphi$,

(2) $\sin\dot{\varphi}_1=\frac{2}{3}\sin\varphi_0$.

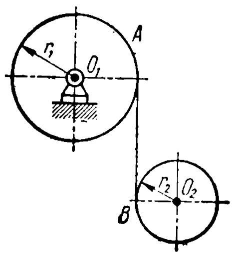


Fig. 429

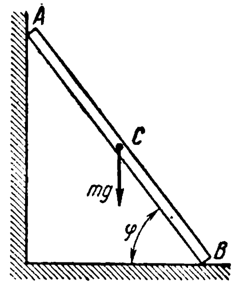


Fig. 430

679. Two disks rotate about a common axis with angular velocities ω_1 and ω_2, respectively. Their moments of inertia about this axis are J_1 and J_2, respectively. When both disks are connected suddenly by a rod, determine the loss of kinetic energy. Neglect the mass of the rod.

Ans. $\Delta T=\frac{1}{2}\frac{J_1J_2}{J_1+J_2}(\omega_1-\omega_2)^2$.

680. A slider-crank mechanism has a crank of length r which rotates with constant angular velocity ω_0. The crank drives a connecting rod of weight P and length l which is hinged to the crank and the slider. Determine the forces acting on the connecting rod (principal vector and principal moment about the centre of gravity of the connecting rod) at the vertical and horizontal positions of the crank. The connecting rod is assumed to be a thin uniform rod.

Ans. At the vertical positions of the crank the principal vector is applied to the centre of gravity of the connecting rod and is perpendicular to the straight line passing through the centre of rotation of the crank and the centre of gravity of the connecting rod. The magnitudes of the principal vector and principal moment are

$$V=\frac{Prl\omega_0^2}{2g\sqrt{l^2-r^2}};\quad M_C=\frac{Prl^2\omega_0^2}{12g\sqrt{l^2-r^2}}.$$

At the horizontal position of the crank the principal vector is applied at the centre of gravity of the connecting rod and is directed along the connecting rod to the axis of rotation of the crank.

$$V=\frac{Pr\omega_0^2}{g}\left(1+\frac{r}{2l}\right);\quad M_C=0.$$

681. A rod AB of mass m moves in a plane and at a given moment has angular acceleration ε (Fig. 431). The radius of gyration of the rod about the axis passing through the centre of gravity C perpendicular to the plane of rotation of the rod is ρ. The distances between the centre of gravity C and the ends A and B equal a and b, respectively. The mass of the rod is replaced by two point masses located on the ends of the rod A and B in such a way that the sum of these masses equals that of the rod. The centre of gyration of the given masses coincides with the centre of gravity of the rod. Determine whether the principal vector and the principal moment of the forces of inertia of the given masses equal the principal vector and the principal moment of the forces of inertia of the rod, respectively.

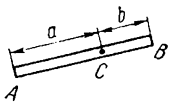


Fig. 431

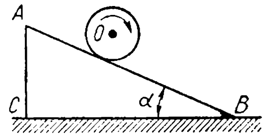


Fig. 432

Ans. The principal vectors of the forces of inertia of the given masses and the rod are geometrically equal while the principal moments differ in magnitude by $m\ (ab-\rho^2)\varepsilon$.

682.* Fig. 432 shows a triangular prism ABC of weight P which is free to slide without friction on a smooth horizontal plane. A homogeneous circular cylinder of weight Q rolls without sliding down the face AB of the prism. Determine the motion of the prism.

Ans. The prism moves to the left with the constant acceleration: $\dfrac{Q\sin 2\alpha}{3(P+Q)-2Q\cos^2\alpha}g$

683.* A circular cylinder is free to rotate without friction about its vertical axis. A smooth helical groove with an angle of pitch α is cut in the side of the cylinder. Initially the cylinder is in equilibrium position. A small heavy ball is dropped into the groove. It rolls down the groove from rest and thus sets the cylinder in motion. The cylinder has a mass M and its radius is R, the mass of the ball is m and the distance between the ball and the axis is R.

The moment of inertia of the cylinder equals $\frac{1}{2}MR^2$. Determine the angular velocity ω which the cylinder attains after the ball has fallen the height h.

Ans. $\omega=\frac{2m\cos\alpha}{R}\sqrt{\frac{2gh}{(M+2m)(M+2m\sin^2\alpha)}}$.

684. A rigid body of weight P oscillates about a horizontal axle O perpendicular to the plane of the sketch (Fig. 433). The distance between the axis of suspension and the centre of gravity C is a. The radius of gyration of the body about the axis passing through the centre of gravity perpendicular to the plane of the sketch is ρ. Initially the body was held at an angle φ_0 from the position of equilibrium and then released from rest. Determine the components of the force of reaction of the axle R and N acting along the line passing through the point of suspension and the centre of gravity of the body and perpendicular to it. Express R and N as functions of the angle of inclination φ between the body and the vertical.

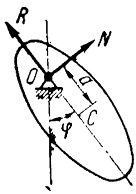


Fig. 433

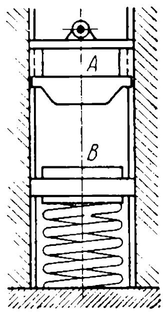


Fig. 434

Ans. $R=P\cos\varphi+\frac{2Pa^2}{\rho^2+a^2}(\cos\varphi-\cos\varphi_0)$; $N=P\frac{\rho^2}{\rho^2+a^2}\sin\varphi$.

44. Impact

685. A ram impact machine falls from a height of 4.905 m and strikes an anvil B fixed to a spring (Fig. 434). The ram weighs 10 kgf and the anvil 5 kgf. Determine the velocity of the anvil after impact if the ram and the anvil then move together.

Ans. 6.54 m/sec.

686. Two identical elastic balls A and B move towards one another. For what relation between the velocities of approach before the collision does the ball A come to rest after the collision? The coefficient of restitution at the impact is k.

Ans. $\frac{v_A}{v_B}=\frac{1+k}{1-k}$.

687. Determine the ratio of the masses m_1 and m_2 of two balls in the following two cases: (1) the first ball is at rest; a central impact occurs after which the second ball remains at rest; (2) the balls meet with equal and opposite velocities; after the impact the second ball remains at rest. The coefficient of restitution is k.

Ans. (1) $\frac{m_2}{m_1}=k$; (2) $\frac{m_2}{m_1}=1+2k$.

688. Three perfectly elastic balls of masses m_1, m_2, and m_3 rest in a smooth groove with certain distances between them. The first ball is given a certain initial velocity and strikes the second ball which is at rest. Then the second ball collides with the third one also at rest. For what mass m_2 of the second ball does the third ball receive the maximum velocity?

Ans. $m_2=\sqrt{m_1 m_3}$.

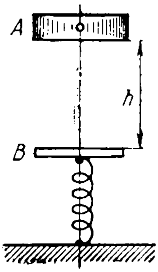


Fig. 435

689. A load A of weight P falls from rest from the height h on the plate B of weight p, which is fixed on a spring, as shown in Fig. 435. The coefficient of stiffness of the spring is c. Assuming that the coefficient of restitution is zero, find the value of the spring compression after the collision.

Ans. $s=\frac{P}{c}+\sqrt{\frac{P^2}{c^2}+2h\frac{P^2}{c\,(P+p)}}$.

690. A small elastic ball falls vertically from the height h onto a horizontal plate, rebounds from it and then falls again onto the plate. This motion is repeated several times. If the coefficient of restitution is k, find the path travelled by the ball before coming to rest.

Ans. $s=\frac{1+k^2}{1-k^2}h$.

691. A ram of a steam hammer, weighing 12,000 kgf, falls on an anvil with the velocity of 5 m/sec. The weight of the anvil together with a forging on it is 250,000 kgf. Determine the work A_1 done on the forging, and the work A_2 lost due to vibrations of the foundation. Compute efficiency η of the steam hammer. The impact is perfectly inelastic.

Ans. $A_1=14{,}600$ kgfm; $A_2=700$ kgfm;
$\eta=0.95$.

692. A ball of mass m_1 moving with a velocity v_1 collides with another ball of mass m_2 being at rest. The velocity of the first ball at impact makes an angle α with the line connecting the centres of the balls. Determine: (1) the velocity of the first ball after

the collision, assuming the impact to be perfectly inelastic; (2) the velocity of each ball after the impact, assuming that the impact is elastic with the coefficient of restitution k.

Ans. (1) $u_1=v_1\sqrt{\sin^2\alpha+\left(\frac{m_1}{m_1+m_2}\right)^2\cos^2\alpha}$;

(2) $u_1=v_1\sqrt{\sin^2\alpha+\left(\frac{m_1-km_2}{m_1+m_2}\right)^2\cos^2\alpha}$;

$u_2=v_1\frac{m_1(1+k)\cos\alpha}{m_1+m_2}$.

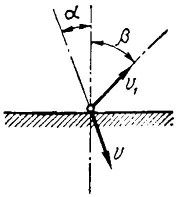


Fig. 436

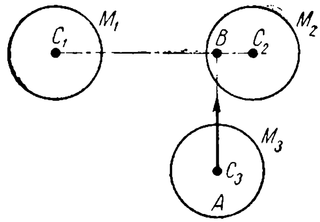


Fig. 437

693. A small ball with velocity v falls obliquely on a fixed horizontal plate and rebounds from it with the velocity $v_1=\frac{v\sqrt{2}}{2}$. Determine the angle of fall α and the angle of rebound β, if the coefficient of restitution is $k=\frac{\sqrt{3}}{3}$ (Fig. 436).

Ans. $\alpha=\frac{\pi}{6}$; $\beta=\frac{\pi}{4}$.

694. Fig. 437 shows three identical balls M_1, M_2, and M_3 each of radius R. $C_1C_2=a$. Find the line AB perpendicular to C_1C_2 on which the centre of the third ball must be located so that on acquiring a certain velocity in the direction of AB the third ball after the collision with the ball M_2 has a central impact with the ball M_1. The balls are assumed to be perfectly elastic.

Ans. The distance between the straight line AB and the centre C_2 is $BC_2=\frac{4R^2}{a}$.

695. To pack the earth under the building foundation piles each weighing $P=50$ kgf are driven in by a pile-driver. The ram of the pile-driver weighs $P_1=450$ kgf. It falls through a height $h=2$ m from rest. The coefficient of restitution is zero. During ten blows the pile penetrates $\delta=5$ cm into the ground. Find the average resistance of the ground.

Ans. $S=16{,}200$ kgf.

696. Two balls of masses m_1 and m_2 are suspended by parallel threads of length l_1 and l_2, respectively, in such a way that their centres are on the same level. The first ball is pulled back through the angle α_1 to the vertical and then released from rest. If the coefficient of restitution equals k, determine the maximum angle of deviation α_2 of the second ball.

Ans. $\sin\frac{\alpha_2}{2}=\frac{m_1(1+k)}{m_1+m_2}\sqrt{\frac{l_1}{l_2}}\sin\frac{\alpha_1}{2}$.

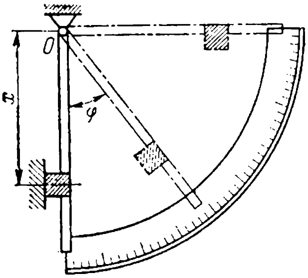


Fig. 438

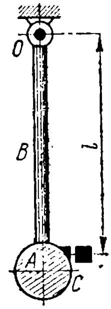


Fig. 439

697. An impact testing machine, shown in Fig. 438, is used to determine the coefficient of restitution on impact. It consists of a rod which is free to rotate in a vertical plane about a horizontal axle O. A piece of test material is placed on the rod at a certain distance from O. The rod starts to fall from rest in the horizontal position under the action of its weight, and when vertical it strikes the material to be tested. If, after impact, the rod rebounds at an angle φ, determine the coefficient of restitution k. At what distance x from the axle of rotation of the rod should the piece of a test material be placed to prevent additional forces in the bearing of the axle O when the impact occurs?

Ans. $k=\sqrt{2}\sin\frac{\varphi}{2}$; $x=\frac{2}{3}l$.

698. A pendulum of an impact testing machine consists of steel disk A 10 cm in radius and 5 cm in thickness and a round rod B 2 cm in diameter and 90 cm long (Fig. 439). At what distance l from the horizontal plane, containing the axle of rotation O, should a test piece C be placed so that no impulse acts on the axle O during impact? The impact is considered to be horizontal.

Ans. $l=97.5$ cm.

699. Two pulleys rotate in the same plane about their axles with angular velocities ω_{10} and ω_{20}. Determine the angular veloc-

ities of the pulleys ω_1 and ω_2 after a belt is put on them. It is assumed that both pulleys are round disks of equal density with radii R_1 and R_2. Neglect the sliding effects and the mass of the belt.

Ans. $\omega_1=\dfrac{R_1^3\omega_{10}+R_2^3\omega_{20}}{R_1(R_1^2+R_2^2)}$; $\quad\omega_2=\dfrac{R_1^3\omega_{10}+R_2^3\omega_{20}}{R_2(R_1^2+R_2^2)}$.

700. A ram is driven by a knuckle gear which carries a hand-wheel, as shown in Fig. 440. Assuming the impact to be inelastic, determine the angular velocity of the gear and the velocity of the ram after the impact. Also determine the mean value of the force Q caused by the impact. The angular velocity of the

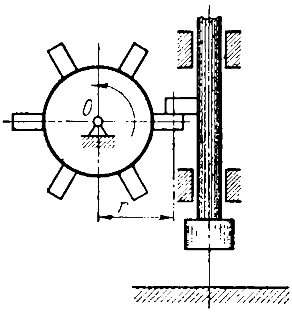


Fig. 440

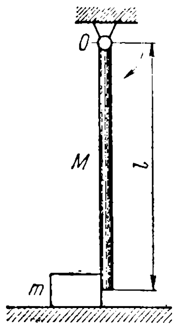


Fig. 441

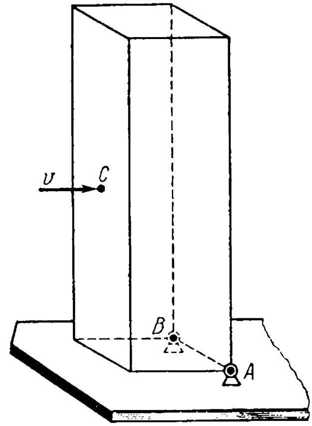


Fig. 442

gear before the impact is $\omega_{10}=2\pi$ sec^{-1}. The initial velocity of the ram is zero, the impact period is $\tau=0.05$ sec, the moment of inertia of the gear with the hand-wheel on it about the axle of rotation is $I_0=500$ kgfcmsec^2, the distance between the point of impact and the axle of the gear is $r=20$ cm, and the ram weighs $P=25$ kgf.

Ans. $\omega_1=6.15$ sec^{-1}; $v=1.23$ m/sec; $Q=62.8$ kgf.

701. A uniform rod of mass M and length l is hinged at its upper end O (Fig. 441). The rod starts to fall from rest in the horizontal position. On reaching the vertical it collides with the weight of mass m causing the latter to move along the rough horizontal surface. The coefficient of sliding friction is f. Determine the path of the weight assuming that the impact is inelastic.

Ans. $s=\dfrac{3l}{2f}\dfrac{M^2}{(M+3m)^2}$.

702. A homogeneous straight prism with a square base rests on a horizontal plane and is free to rotate about its edge AB (Fig. 442). The edge of the prism base is a, its height is $3a$ and its mass

is $3m$. A ball of mass m and horizontal velocity v strikes the middle C of the side face opposite to AB. Assuming that the impact is inelastic, that the mass of the ball is concentrated at its centre, and after the impact it remains at a point C, determine the minimum value of the velocity v for which the prism tips over.

Ans. $v=\frac{1}{3}\sqrt{53ga}$.

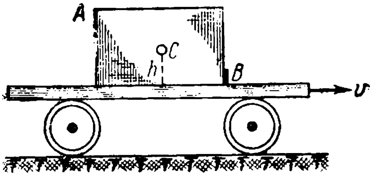


Fig. 443

703. A flat car with a load AB on it runs along the horizontal rails with the velocity v (Fig. 443). There is a jut on the flat car which butts against the rib B of the load thus preventing the latter from slipping forward on the flat car, but it does not prevent the load from rotating about the rib. The following data are given: h is the height of the centre of gravity of the load above the floor of the flat car, and ρ is the radius of gyration of the load about the rib B. Determine the angular velocity ω of the rotation of the load about the rib B at the moment when the flat car is instantaneously halted.

Ans. $\omega=\frac{hv}{\rho^2}$.

45. Dynamics of a System of Variable Mass*

704. Derive the differential equation of motion of a pendulum of variable mass in a medium where the resistance is proportional to the velocity. The mass of the pendulum varies in accordance with a given equation $m=m(t)$ by ejection of particles with relative velocity equal to zero. The length of the string of the pendulum is l. The latter is also acted upon by a force of resistance which is proportional to its angular velocity: $R=-\beta\dot{\varphi}$.

Ans. $\ddot{\varphi}+\frac{\beta}{m(t)l}\dot{\varphi}+\frac{g}{l}\sin\varphi=0$.

705. Find the differential equation of motion of a rocket. The relative escape velocity of the gases v_r from the rocket is assumed to be constant. The mass of the rocket changes with time in accordance with the law: $m=m_0f(t)$ (the law of combustion). The

* Some of the problems in this section can be solved by applying the differential equation of motion of a particle having variable mass known as Meshchersky's equation

$$M\frac{d\bar{v}}{dt}=\bar{F}^e+\bar{u}\frac{dM}{dt}.$$

This equation was first developed by him in 1897.

atmospheric resistance is a given function of the velocity and the position of the rocket $R=R(x, \dot{x})$.

Integrate also the equation of motion of the rocket, if $m=m_0\ (1-\alpha t)$, $R=0$, and the initial velocity of the rocket fired from the earth is zero. What altitude will the rocket attain after times of $t=10$; 30; 50 sec, when $v_r=2000$ m/sec and $\alpha=\frac{1}{100}$ sec^{-1}?

Ans. (1) $\ddot{x}=-g-\frac{\dot{f}(t)}{f(t)}v_r-\frac{R(x,\dot{x})}{m_0 f(t)}$.

(2) $x(t)=\frac{v_r}{\alpha}[(1-\alpha t)\ln(1-\alpha t)+\alpha t]-\frac{gt^2}{2}$;

$x(10)=0.54$ km; $x(30)=5.65$ km; $x(50)=18.4$ km.

706. Applying Meshchersky's equation $M\frac{d\bar{v}}{dt}=F^e+\bar{u}\frac{dM}{dt}$ and neglecting the action of the external forces ($F^e=0$), determine the velocity v of the rocket which it attains after the fuel is burned out. The mass of the rocket (payload) equals Mr and the mass of the fuel is Mf. The escape velocity of the gases issued from the nozzle of the rocket is denoted by u.

Note. The answer to this problem may be obtained by using Tsiolkovsky's* formula

$$v=v_0+u\ln\left(1+\frac{Mf}{Mr}\right).$$

The initial velocity of the rocket v_0 equals zero.

Ans. $v=u\ln\left(1+\frac{Mf}{Mr}\right)$.

707. The mass of the rocket, described in Problem 705, varies with time in accordance with the law $m=m_0e^{-\alpha t}$. Neglecting the resistance of the atmosphere, determine the motion of the rocket. Assuming that at the instant t_0 the fuel is completely burned out, determine the maximum altitude of the rocket. Initially the rocket was on the ground and its velocity was zero.

Ans. $H=\frac{\alpha v_r}{2g}[\alpha v_r-g]t_0^2$.

708. Referring to the preceding problem, determine the value of α corresponding to the maximum possible altitude H_{max} of the rocket and also compute H_{max} (the value $\mu=\alpha t_0=\ln\frac{m_0}{m_1}$ should be considered as constant; m_1 is the mass of the rocket at time t_0).

* Konstantin Eduardovich Tsiolkovsky (1857-1935), is a famous Russian scientist and inventor. The equation, mentioned in this problem, first appeared in his work published in 1903.

Ans. $\alpha=\infty$ (instantaneous combustion),

$$H_{\max}=\frac{\mu^2}{2g}v_r^2.$$

709. A balloon of weight Q takes off, dragging one end of a rope folded on the ground. The balloon is acted upon by a lifting force P, the force of gravity and the force of resistance, which is proportional to the square of the velocity $R=-\beta\dot{x}^2$. The weight per unit length of the rope is γ. Derive the equation of motion of the balloon. If initially the bolloon is in equilibrium at the height H_0, find its velocity of ascent.

Ans.

$$\ddot{x}=-g+\frac{Pg}{Q+\gamma x}-\frac{\beta g+\gamma}{Q+\gamma x}\dot{x}^2.$$

$$\dot{x}^2=\frac{Pg}{(\beta g+\gamma)}\left[1-\left(\frac{Q+\gamma H_0}{Q+\gamma x}\right)^{2\left(1+\beta\frac{g}{\gamma}\right)}\right]-$$

$$-\frac{2g}{2\beta g+3\gamma}\left[1-\left(\frac{Q+\gamma H_0}{Q+\gamma x}\right)^{3+2\beta\frac{g}{\gamma}}\right](Q+\gamma x).$$

710. A round water drop falls in air saturated with vapour. Due to condensation the mass of the drop increases proportionally to the area of its surface (the coefficient of proportionality is α). Given that the initial radius of the drop is r_0, its initial velocity v_0 and initial height h_0, determine the velocity of the drop and its altitude as a function of time.

Note. Prove that $dr=\alpha dt$ and take a new independent variable $r=r_0+\alpha t$.

Ans. $x=h_0+\frac{v_0r_0}{2\alpha}\left[1-\left(\frac{r_0}{r}\right)^2\right]-\frac{g}{8\alpha^2}\left[r^2-2r_0^2+\frac{r_0^4}{r^2}\right];$

$$v=v_0\frac{r_0^3}{r^3}-\frac{g}{4\alpha}\left[r-\frac{r_0^4}{r^3}\right].$$

711. A chain is folded up on the ground with one end attached to a trolley which rests on an inclined track making an angle α with the horizontal. The coefficient of friction between the chain and the ground is f. The weight per unit length of the chain is γ, and the trolley weighs P. The initial velocity of the trolley is v_0. Determine the velocity of the trolley at any given time. Under what conditions may the trolley come to rest?

Ans. $\frac{\dot{x}^2}{2}=\frac{P^2v_0^2}{2(P+\gamma x)^2}+\frac{Pg}{3\gamma}\sin\alpha\left[1-\frac{P^2}{(P+\gamma x)^2}\right]+$

$$+\frac{1}{3}gx\sin\alpha+\frac{fPg}{6\gamma}\left[1-\frac{P^2}{(P+\gamma x)^2}\right]\cos\alpha-\frac{1}{3}fgx\cos\alpha.$$

Stopping is only possible, when the inequality $f > \tan\alpha$ is satisfied.

712. A particle of mass m is attracted to a fixed centre in accordance with Newton's law of gravitation. The mass of the centre changes with time as $M = \frac{M_0}{1+\alpha t}$ Determine the motion of the particle.

N o t e. Use new coordinates $\xi = \frac{x}{1+\alpha t}$, $\eta = \frac{y}{1+\alpha t}$, and the scaled time $\tau = \frac{1}{\alpha(1+\alpha t)}$.

Ans. The equations of motion in ξ, η coordinates (f is a constant of gravitation) are

$$\frac{d^2\xi}{d\tau^2} + f\frac{M_0\xi}{\rho^3} = 0; \quad \frac{d^2\eta}{d\tau^2} + f\frac{M_0\eta}{\rho^3} = 0; \quad \rho = \sqrt{\xi^2+\eta^2},$$

and they correspond to the usual equations if the masses are constant. According to initial conditions on the variables ξ and η, elliptic, parabolic, and hyperbolic orbits may occur.

46. Analytical Statics

713. Six equal uniform rods each of weight p form a hinged hexagon linkage placed in a vertical plane. Its upper side AB is held stationary in the horizontal position while all the other sides are located symmetrically with respect to the vertical, passing through the mid-point of AB, as shown in Fig. 444. Determine the value of vertical force Q which must be applied at the centre of the horizontal side, opposite to AB, to make the system rest in neutral equilibrium.

Ans. $Q = 3p$.

714. A uniform rod AB of length $2a$ and weight Q is suspended by two threads each of length l (Fig. 445). A couple of

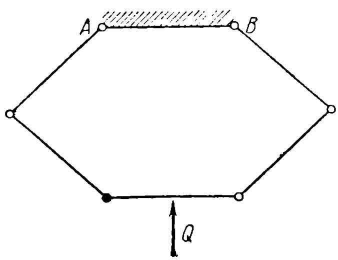


Fig. 444

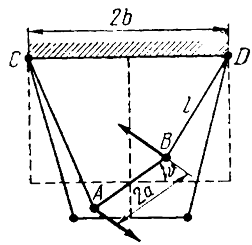


Fig. 445

forces with moment M is applied to the rod. Both points of suspension C and D are located on the same horizontal line, and the distance between them is $2b$. Find the value of an angle ϑ, when the rod is in equilibrium.

Ans. In equilibrium the angle ϑ is to be found from the equation

$$M\sqrt{l^2-(a-b)^2-4ab\sin^2\frac{\vartheta}{2}}=Qab\sin\vartheta.$$

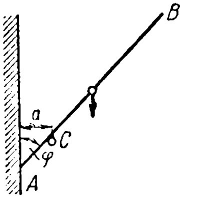


Fig. 446

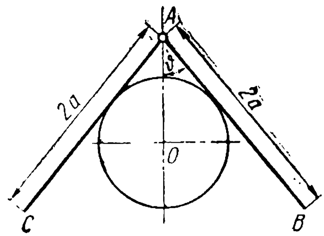


Fig. 447

715. A rectilinear uniform rod AB of length $2l$ leans with its bottom end A at an angle φ against a vertical wall. The rod is supported by a nail C parallel to the wall (Fig. 446). The distance between the nail and wall is a. Determine the value of an angle φ, when the rod rests in equilibrium.

Ans. $\sin\varphi=\sqrt[3]{\frac{a}{l}}$.

716. Two heavy uniform rods connected by a hinge A lie on a smooth cylinder of radius r, as shown in Fig. 447 The length of each rod is $2a$. Determine the angle of tapering 2ϑ of the rods when in equilibrium.

Ans. The angle ϑ is defined by the equation
$a\tan^3\vartheta-r\tan^2\vartheta-r=0.$

717. Both ends of a heavy uniform rod of length l are free to slide without friction along the curve, given by the equation $f(x, y)=0$. Determine the equilibrium positions of the rod. The axis y is directed vertically upwards and the axis x is directed horizontally to the right.

Ans. The coordinates of the ends of the rod, corresponding to the positions of equilibrium, will be the solutions of the following system of equations:

$$(x_2-x_1)^2+(y_2-y_1)^2-l^2=0;\quad f(x_1, y_1)=0,\ f(x_2, y_2)=0;$$

$$2(y_2-y_1)\frac{\partial f}{\partial x_1}\frac{\partial f}{\partial x_2}=(x_2-x_1)\left[\frac{\partial f}{\partial x_1}\frac{\partial f}{\partial y_2}+\frac{\partial f}{\partial y_1}\frac{\partial f}{\partial x_2}\right].$$

718. A heavy uniform rod of length l is free to slide without friction by its ends along the parabola $y=ax^2$. Determine possible positions of equilibrium. The y axis is directed vertically upwards and the x axis is directed horizontally to the right.

Ans. The first position of equilibrium

$$x_2=-x_1=\frac{l}{2};\qquad y_1=y_2=a\frac{l_2}{4}.$$

The second position of equilibrium is defined by the equation $\operatorname{ch}\xi=\sqrt{al}$

$$x_1=-\frac{1}{2a}e^{-\xi}.\quad y_1=\frac{1}{4a}e^{-2\xi}.\quad x_2=\frac{1}{2a}e^{\xi};\quad y_2=\frac{1}{4a}e^{2\xi}$$

719. Solve Problem 717, assuming that the curve is an ellipse $f(x, y)=\frac{x^2}{a^2}+\frac{y^2}{b^2}-1=0$, and the length of the rod satisfies the condition $l<2a$. Determine the possible positions of equilibrium of the rod. The y axis is horizontal.

Hint. Instead of Descartes' coordinates it is more convenient to introduce a coordinate φ (eccentric anomaly) by means of the equations $x=a\cos\varphi$, $y=b\sin\varphi$.

Ans. The positions of equilibrium correspond to values of the eccentric anomalies defined by the equations:

(a) $\varphi_1=2\pi-\varphi_2$, $\sin\varphi_2=\sqrt{\frac{l}{2b}}$ (this solution exists, if $l\leqslant 2b$).

(b) $\sin\frac{\varphi_2-\varphi_1}{2}=\sqrt{\frac{l}{2a}}$, $\cos\frac{\varphi_2+\varphi_1}{2}=\sqrt{\frac{1-\frac{l}{2a}}{1-\frac{b^2}{a^2}}}$ (this solution exists, if $a>b$ and $l<2a$).

720. A homogeneous square plate of weight P and side a is free to rotate in vertical plane about an axis passing through the angle O. One end of a thread is fastened to the corner A, and the other end that runs over a small pulley B carries a load of weight $Q=\frac{\sqrt{2}}{2}P$. The vertical distance $OB=a$ (Fig. 448). Find the positions of equilibrium of the system and examine their stability.

The length of the thread is l.

Ans. The positions of equilibrium are as follows:

$$\psi_1=\frac{\pi}{6},\qquad \psi_2=\frac{\pi}{2},\qquad \psi_3=\frac{3\pi}{2}.$$

The second and the third positions of equilibrium are stable.

721. A heavy uniform rod AB of length $2a$ rests on a curvilinear guide of semicircular shape with radius R (Fig. 449). Neglecting the effects of friction, find the position of equilibrium and examine its stability.

Ans. When in equilibrium the rod is inclined at an angle φ_0 to the horizontal, where φ_0 is defined by
$\cos\varphi_0=\frac{1}{8R}[a+\sqrt{a^2+32R^2}]$. (It is assumed that
$\sqrt{\frac{2}{3}}R<a<2R$.) This position of equilibrium is stable.

722. Fig. 450 represents a sketch of a draw-bridge OA which consists of a homogeneous plate of weight P and length $2a$. A cable of length l has one its end attached to the mid-point of the plate end while the other end C,

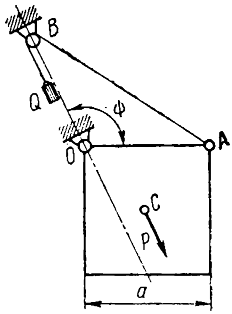


Fig. 448

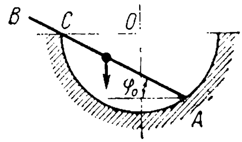


Fig. 449

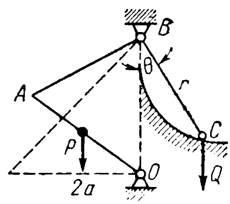


Fig. 450

after running over a small pulley, is connected to the counter-balance, which slides without friction on a curvilinear guide. The pulley is placed on the vertical at a distance $2a$ from the point O. Determine the shape of the guide and the weight of the counter-balance Q so that the system remains in neutral equilibrium. When the bridge is horizontal, the counter-balance C is on the line OB.

Ans. $Q=\frac{P}{\sqrt{2}}$, the equation of the guide in polar coordinates r, ϑ is $r^2=2(l-2\sqrt{2}a\cos\vartheta)r+4\sqrt{2}al-l^2-8a^2$.

723. Fig. 451 represents a pallograph which consists of a pendulum with a load M attached to the rod OM. The rod OM is free to pass through the oscillating cylinder O, and it is hinged at A to the rocker arm AO_1 which swings about the axle O_1. The rocker arm has length r, and the distance between the centre of gravity of the load and the hinge A is l. $OO_1=h$. Examine the stability of vertical equilibrium of the pendulum. Neglect the dimensions of the load and the weights of the rod and the rocker arm.

Ans. For $\sqrt{rl}>h-r$ the equilibrium is stable.
For $\sqrt{rl}<h-r$ the equilibrium is unstable.

724. A straight wire carrying a current i_1 attracts a parallel wire AB, carrying a current i_2 (Fig. 452). AB has a mass m. A spring of stiffness c is attached to AB. The length of each wire is l. When AB does not carry any current, the distance between the wires is a. Determine the positions of equilibrium of the system and examine their stability.

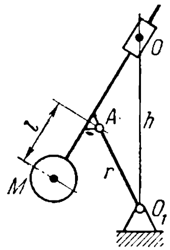


Fig. 451

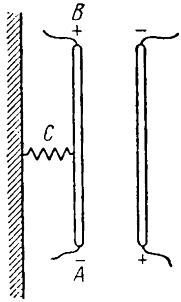


Fig. 452

Note. The force of attraction between two parallel wires, carrying currents i_1 and i_2, of length l and distance between them d, is defined by the formula

$$F=\frac{2i_1i_2}{d}\,l.$$

Ans. When $\alpha=\frac{2i_1i_2l}{c}<\frac{a^2}{4}$, there are two positions of equilibrium:

$x_1=\frac{a}{2}-\sqrt{\frac{a^2}{4}-\alpha}$, and $x_2=\frac{a}{2}+\sqrt{\frac{a^2}{4}-\alpha}$, where x_1 corresponds to stable equilibrium and x_2 to unstable. For $\alpha>\frac{a^2}{4}$ there is no position of stable equilibrium. For $\alpha=\frac{a^2}{4}$ there is one position of equilibrium which is unstable.

47. Lagrange's Equations

725. Two mutually perpendicular and intersecting shafts are driven by two bevel gears with z_1 and z_2 teeth, respectively. The moments of inertia of the shafts, which carry the gears on them, are J_1 and J_2, respectively. Determine the angular acceleration of the first shaft, if it is acted upon by a torque of moment M_1; while

the second shaft is under the action of a moment of force of resistance M_2. Neglect friction in the bearings.

Ans. $\varepsilon_1=\frac{M_1-kM_2}{J_1+k^2J_2}$, where $k=\frac{z_1}{z_2}$.

726. Fig. 453 shows a load P hanging on a homogeneous cable of weight P_1 and length l which is wound around a drum of radius a and weight P_2. The axis of rotation is horizontal. Neglect

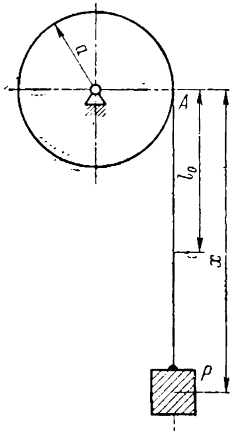


Fig. 453

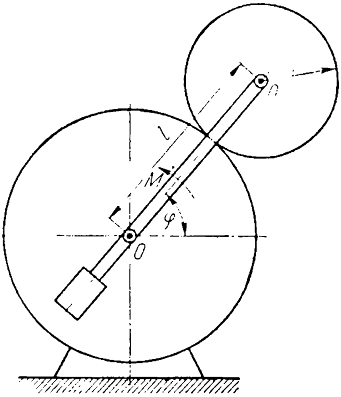


Fig. 454

any effects of friction. The mass of the drum is assumed to be distributed uniformly along its rim. At time $t=0$ the system is in equilibrium, and the length of the part of the cable hanging down is l_0. Determine the path of the load.

Note. Neglect the dimensions of the drum in comparison with the length of the part of the cable hanging down.

Ans. $x=-\frac{Pl}{P_1}+\left(l_0+\frac{Pl}{P_1}\right)\operatorname{ch}\sqrt{\frac{P_1g}{(P+P_1+P_2)l}}\,t.$

727. In an epicyclic train, shown in Fig. 454, the rolling gear of radius r_1 is mounted on the crank which carries a counter-balance on its other end. The crank rotates about an axle of the fixed gear under the action of a moment M. The distance between the axles of the gears is l, the moment of inertia of the crank with the counter-balance about the axis of rotation of the crank is J_0, the mass of the rolling gear is m_1, the moment of inertia of the gear about its axle is J_1, and the centre of gravity of the gear and the crank with the counter-balance lies on the axle of rotation of the crank. Determine the angular acceleration of the crank and

the circumferential stress S at the point of rolling contact between the gears. Neglect any effects of friction.

$$Ans.\ \varepsilon=\frac{M}{J_0+m_1l^2+J_1\frac{l^2}{r_1^2}};\quad S=\frac{J_1l}{r_1^2}\varepsilon.$$

728. Fig. 455 represents a planetary gear train with a gear fixed on the axle O_1. A torque of moment M is applied to the handle O_1O_3 of the system, which is placed in a horizontal plane. Determine the angular acceleration of the handle, assuming that the gears are

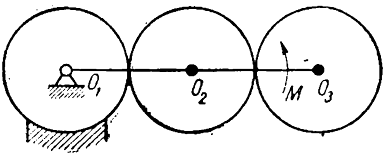


Fig. 455

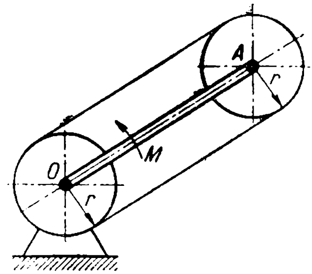


Fig. 456

homogeneous disks with identical mass m and radius r. Neglect the mass of the handle.

$$Ans.\ \varepsilon_1=\frac{M}{22mr^2}.$$

729. Determine the angular acceleration of the crank $OA=l$, which carries the movable pulley of radius r at its end A (Fig. 456). Under the action of a torque of moment M the crank rotates about the centre of the stationary pulley of radius r Both pulleys have an endless belt wrapped round them, which is stretched in such a way that no sliding over the rims of the pulleys takes place during the motion. The system lies in a horizontal plane. The weight of the crank (a uniform rod) is P and that of the pulley is Q.

$$Ans.\ \varepsilon=\frac{3gM}{(P+3Q)l^2}.$$

730. Determine the angular acceleration of the crank which moves the sliding bar of an ellipsograph. The latter is placed in horizontal plane. The axle of the crank is acted on by a torque of moment M_0. The crank and the sliding bar are homogeneous prismatic bars weighing p and $2p$, respectively. $OC=AC=BC=a$. The weights of the sliders A and B are equal: $q_1=q_2=q$. Neglect any effects of friction (Fig. 457).

$$Ans.\ \varepsilon=\frac{M_0g}{a^2(3p+4q)}.$$

731. Determine the angular acceleration of the driving and driven shafts *I* and *IV* connected by a speed-reducer gear (Fig. 458). The latter consists of a fixed gear *1* of radius r_1, two pairs of twin pinions *2* and *3* of radii r_2 and r_3, and a gear of radius r_4 mounted on the driven shaft. The moment of inertia of masses connected with the driving shaft about the axle of the shaft is J_1. The mass of each pair of twin pinions is m_2 and its moment of inertia about the axle is J_2. The moment of inertia of the masses connected to the driven shaft, about the axle

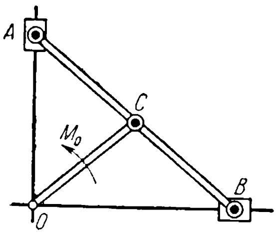


Fig. 457

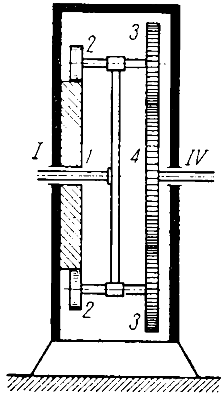


Fig. 458

of this shaft, is J_4. A torque of moment M_1 is applied to the driving shaft. The moment of force of resistance applied to the driven shaft equals M_4. Neglect any effects of friction.

$$Ans.\ \varepsilon_1 = \frac{M_1 - M_4\left(1-\dfrac{r_1 r_3}{r_2 r_4}\right)}{J_1 + 2m_2(r_1+r_2)^2 + 2J_2\left(1+\dfrac{r_1}{r_2}\right)^2 + J_4\left(1-\dfrac{r_1 r_3}{r_2 r_4}\right)^2};\quad \varepsilon_2 = \varepsilon_1\left(1-\frac{r_1 r_3}{r_2 r_4}\right).$$

732. A slider-crank mechanism, shown in Fig. 459, consists of a piston of mass m_1, a connecting rod AB of mass m_2, a crank OB, a shaft and a flywheel. The other data are as follows: the moment of inertia of the connecting rod about the point C is J_2, the moment of inertia of the crank OB, the shaft and the flywheel about the axle is J_3, the area of the piston is Ω, p is the pressure acting on the piston, l is the length of the connecting rod, S is a distance between a point A and the centre of gravity of the connecting rod, the length of the crank OB is r, and the moment of force of resistance acting on the shaft is M. Derive the equation of motion of the mechanism, assuming that the angle of rotation of the crank ψ is small, i. e., $\sin\psi=\psi$ and $\cos\psi=1$. The angle of the crank φ can be used as the generalized coordinate.

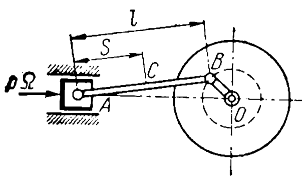


Fig. 459

$$Ans.\ \left[(m_1+m_2)\,r^2\,\sin^2\varphi+(J_2+mS^2)\left(\frac{r}{l}\right)^2\cos^2\varphi+J_3\right]\varphi+$$
$$+\left[(m_1+m_2)\,r^2-(J_2+mS^2)\left(\frac{r}{l}\right)^2\right]\cos\varphi\sin\varphi\dot{\varphi}^2=$$
$$=-M+p\Omega r\sin\varphi.$$

733. The bearings of the device for static balancing, shown in Fig. 460, are inclined at an angle α to the vertical. A rotor placed on the bearing has a moment of inertia J (about its own axis) and it carries an unbalanced mass m at distance r from the axis. Derive the differential equation of motion of the rotor and define the frequency of small rotations about the position of equilibrium.

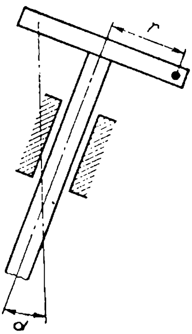


Fig. 460

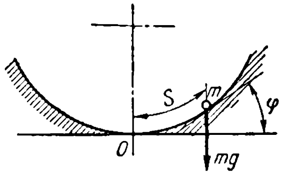


Fig. 461

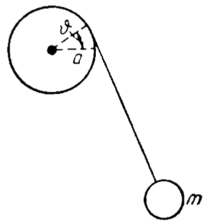


Fig. 462

Ans. $(mr^2+J)\,\varphi+mgr\sin\alpha\sin\varphi=0;\quad k=\sqrt{\dfrac{mgr\sin\alpha}{mr^2+J}},$
where φ is the angle of turn of the rotor.

734. A particle of mass m moves under gravity along the cycloidal guide, given by the equation: $s=4a\sin\varphi$, where s is an arc with the origin at the point O, and φ is the angle between the tangent to the cycloid and the horizontal axis. Determine the motion of the particle (Fig. 461).

Ans. $s=A\sin\left(\dfrac{1}{2}\sqrt{\dfrac{g}{a}}\,t+\varphi_0\right)$, where A and φ_0 are constants of integration.

735. Derive the equation of motion of a pendulum, consisting of a bob of mass m suspended by a thread wound around a fixed cylinder of radius r (Fig. 462). The length of the part of the thread which hangs down in equilibrium is l. Neglect the mass of the thread.

Ans. $(l+r\vartheta)\ddot{\vartheta}+r\dot{\vartheta}^2+g\sin\vartheta=0$, where ϑ is an angle between the pendulum and the vertical.

736. Derive the equation of motion of a pendulum, consisting of a bob with a mass m suspended by a thread, if the length of the thread is a function of time $l=l(t)$.

Ans. $\varphi+2\frac{\dot{l}}{l}\varphi+\frac{g}{l}\sin\varphi=0$, where φ is the angle between the thread and the vertical.

737. A pendulum consists of a bob of mass m which is suspended by an unstretched thread of length l. The point of suspension of the pendulum moves in accordance with the law $\xi=\xi_0(t)$ down the straight line making an angle α with the horizontal. Derive the equation of motion of the pendulum.

Ans. $\varphi+\frac{g}{l}\sin\varphi+\frac{\ddot{\xi}}{l}\cos(\varphi-\alpha)=0.$

738. Fig. 463 represents two shafts in the same plane, making an angle α with each other, and connected by a cardan joint. The moments of inertia of the shafts are J_1 and J_2, respectively. Derive the equation of motion of the first shaft, if it is acted on by a torque of moment M_1 while the second shaft is acted on by a moment of force of resistance M_2. Neglect the friction in the bearings.

Ans. Denoting the angle of rotation of the first shaft by φ, we have

$$\left[J_1+J_2\left(\frac{\cos\alpha}{1-\sin^2\alpha\cos^2\varphi}\right)^2\right]\varphi-\frac{J_2\sin^2\alpha\cos^2\alpha\sin 2\varphi}{(1-\sin^2\alpha\cos^2\varphi)^3}\dot{\varphi}^2=$$
$$=M_1-M_2\frac{\cos\alpha}{1-\sin^2\alpha\cos^2\varphi}.$$

739. Taking the same case as in the previous problem, determine the motion of the first shaft, if a small angle α is subtended between the two shafts. Calculations should be correct to order α^2.

Ans. $\varphi=\frac{1}{2}\frac{M_1-M_2}{J_1+J_2}t^2+C_1t+C_2$, where C_1 and C_2 are arbitrary constants.

740. A homogeneous circular cone rolls down a rough plane inclined at an angle α to the horizontal. The length of the generator of the cone is l and its semi-angle is β. Derive the equation of motion of the cone.

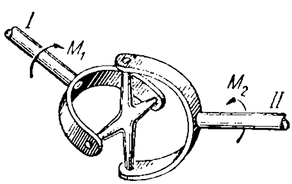


Fig. 463

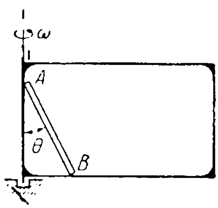


Fig. 464

Hint. The angle ϑ between the generator and the line of maximum slope of the plane is taken as the coordinate.

Ans. $\ddot{\vartheta}+\dfrac{g\sin\alpha}{l\left(\cos^2\beta+\dfrac{1}{5}\right)}\sin\vartheta=0.$

741. A uniform heavy rod AB of length $2a$ and mass M slides without friction on the horizontal and vertical rods of the frame, shown in Fig. 464. The frame rotates with constant angular velocity ω about its vertical side. Derive the equation of motion of the rod and find the position of relative equilibrium.

Ans. $\frac{4}{3}Ma^2\ddot{\vartheta}-\frac{4}{3}M\omega^2a^2\sin\vartheta\cos\vartheta-Mga\sin\vartheta=0$, where ϑ is the angle between the rod and the vertical. In equilibrium $\vartheta=0$ (unstable equilibrium).

742. A lever carrying two masses m_1 and m_2 at its ends is hinged to the circumference of a homogeneous disk of radius R, as shown in Fig. 465. The distances between the masses and the hinge are l_1 and l_2, respectively. The disk rotates about a vertical axis perpendicular to its plane with angular velocity ω. Derive the equation of motion of the lever and find its relative equilibrium position. Neglect the mass of the lever. The axis of rotation of the lever is parallel to that of the disk.

Ans. $(m_1l_1^2+m_2l_2^2)\ddot{\psi}-R\omega^2(m_1l_1-m_2l_2)\cos(\psi-\omega t)=0.$
For $m_1l_1=m_2l_2$ the lever is in neutral relative equilibrium, for $m_1l_1\neq m_2l_2$ there are two positions of relative equilibrium, when $\psi=\omega t\pm\frac{\pi}{2}$, e. the lever is directed along the radius.

743. A particle M moves under gravity along the straight line AB rotating with constant angular velocity ω about a fixed vertical

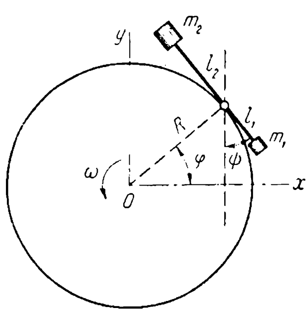


Fig. 465

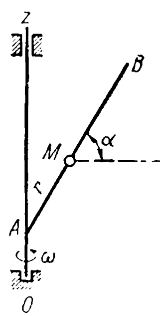


Fig. 466

axle (Fig. 466). AB makes an angle α with the horizontal. Find the law of motion of the particle.

Ans. The distance between the moving particle and the point of intersection of AB with the vertical axle is

$$r = C_1 e^{\omega t \cos\alpha} + C_2 e^{-\omega t \cos\alpha} + \frac{g}{\omega^2}\frac{\sin\alpha}{\cos^2\alpha},$$

where C_1 and C_2 are constants of integration.

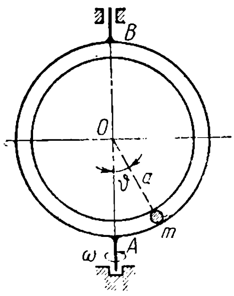


Fig. 467

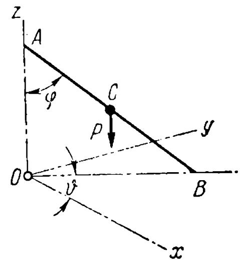


Fig. 468

744. A particle of mass m moves round the circumference of a circle of radius a, which rotates with constant angular velocity ω about the vertical diameter AB (Fig. 467). Derive the equation of motion of the particle and find the moment M required to keep the angular velocity constant.

Ans. $\ddot{\vartheta} + \left(\frac{g}{a} - \omega^2 \cos\vartheta\right)\sin\vartheta = 0, \quad M = 2ma^2 \sin\vartheta \cdot \cos\vartheta \cdot \omega\dot{\vartheta}.$

745. A particle of mass m moves inside a smooth pipe which is free to rotate about a vertical diameter. The axis of the pipe is a circumference of radius a, and its moment of inertia about the vertical diameter is J. Assuming that the pipe rotates under the action of a constant torque of moment M, derive the equation of motion of the system (see Fig. 467).

Ans. $ma^2\ddot{\vartheta} - ma^2 \sin\vartheta \cdot \cos\vartheta \cdot \dot{\varphi}^2 + mga \sin\vartheta = 0,$

$J\ddot{\varphi} + ma^2 \sin^2\vartheta \cdot \varphi + 2ma^2 \sin\vartheta \cos\vartheta \cdot \dot{\vartheta}\varphi = M$

(ϑ is the angle which defines the position of the particle in the pipe, and φ is an azimuth of the pipe).

746. A uniform thin rod AB of weight p and length $2l$ slides with its upper end A against a vertical straight wall while its other end B slides on a horizontal plane. Derive the equations of motion of the rod and find their first integrals (Fig. 468).

Ans. The equations of motion $\varphi - \dot{\vartheta}^2 \sin\varphi \cos\varphi = \frac{3}{4}\frac{g}{l}\sin\varphi;$

$\ddot{\vartheta} \sin^2\varphi + 2\dot{\vartheta}\dot{\varphi} \sin\varphi \cos\varphi = 0$

(φ is the angle between the rod and the vertical, ϑ is the angle of projection of the rod on the horizontal plane and the axis Ox).

The first integrals are $\dot{\vartheta}\sin^2\varphi = C_1$,

$\dot{\varphi}^2+\dot{\vartheta}^2\sin^2\varphi+\frac{3}{2}\frac{g}{l}\cos\varphi=C_2$.

(C_1 and C_2 are arbitrary constants.)

747. Derive the equation of motion of a mathematical pendulum of mass m which is suspended by an elastic string. In equilibrium the string has a length l and its stiffness is c.

Ans. If φ is the angle between the pendulum and the vertical and z is a relative elongation of the string, then the equations of motion are $(1+z)\ddot{\varphi}+2\dot{z}\dot{\varphi}+\frac{g}{l}\sin\varphi=0$;

$\ddot{z}-(1+z)\dot{\varphi}^2+\frac{c}{m}z+\frac{g}{l}(1-\cos\varphi)=0$.

748. Referring to the preceding problem, determine the motion of the pendulum, if it performs negligibly small oscillations.

Ans. $z=A\sin\left(\sqrt{\frac{c}{m}}\,t+\alpha\right)$, $\varphi=B\sin\left(\sqrt{\frac{g}{l}}\,t+\beta\right)$, where A, α, B, β are arbitrary constants.

749. Two masses m_1 and m_2 are mounted on a smooth horizontal rod (axis Ox), as shown in Fig. 469. Both masses are connected by a spring of stiffness c and perform translatory motions along the rod; l is a distance between the centres of gravity of the masses when the spring is not compressed. At time $t=0$ the initial position of the system is defined by the following values of velocities and coordinates of the centres of gravity of the masses: $x_1=0$, $\dot{x}_1=u_0$, $x_2=l$, $\dot{x}_2=0$. Determine the motion of the system.

Ans. $x_1=\frac{1}{m_1+m_2}\left\{m_1u_0t+\frac{m_2u_0}{k}\sin kt\right\}$;

$x_2-l=\frac{1}{m_1+m_2}\left\{m_1u_0t-\frac{m_1u_0}{k}\sin kt\right\}$;

$k=\sqrt{c\left(\frac{1}{m_1}+\frac{1}{m_2}\right)}$.

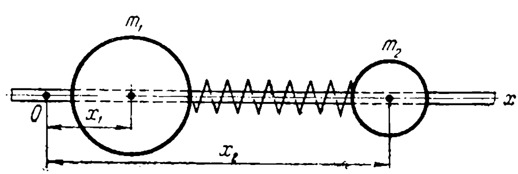


Fig. 469

750. A flywheel *1* rotates about a vertical axle O_1 under the action of a constant torque of moment M (Fig. 470). It carries a gear *2* having an axle of rotation O_2. The gear *2* is in mesh with a gear *3* which is free to rotate quite separately from the rotation of the flywheel. The motion of the gear *3* is hindered by a helical spring (the spring is not shown in the sketch), which has a moment of force of reaction $-c\psi$, proportional to the angle of turn ψ of the

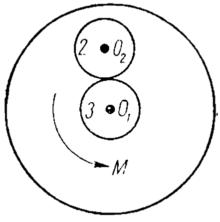


Fig. 470

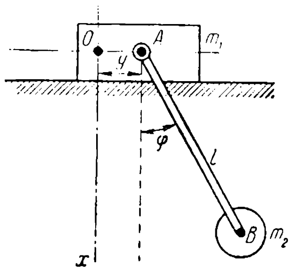


Fig. 471

gear *3*. Determine the motion of the system, assuming that the gears are all homogeneous disks each of radius a and mass m. The moment of inertia of the flywheel about the axle O_1 equals 20 ma^2. Initially the system is in equilibrium.

$$Ans.\ \psi = \frac{M}{26c}\left(1-\cos 1.02\sqrt{\frac{c}{ma^2}}\,t\right);$$

$$\varphi = \frac{Mt^2}{52ma^2}+\frac{M}{676c}\left(1-\cos 1.02\sqrt{\frac{c}{ma^2}}\,t\right),$$

where φ is the angle of turn of the flywheel.

751. An elliptic pendulum, shown in Fig. 471, consists of a slider of mass m_1 which moves without friction along a horizontal plane and a small ball of mass m_2 connected to the slider by a rod AB of length l. The rod is free to rotate about the axle A, connected with the slider and perpendicular to the plane of the sketch. Derive the equations of motion of the pendulum; neglect the mass of the rod.

$$Ans.\ \frac{d}{dt}\left[(m_1+m_2)\dot{y}+m_2 l\dot{\varphi}\cos\varphi\right]=0;$$

$$l\ddot{\varphi}+\cos\varphi\,\ddot{y}+g\sin\varphi=0.$$

752. Referring to the preceding problem, determine the period of small oscillations of the elliptic pendulum.

$$Ans.\ T=2\pi\sqrt{\frac{m_1}{m_1+m_2}\,\frac{l}{g}}.$$

753. Fig. 472 represents a rough cylinder of mass m and radius r which rolls without sliding on the inside circumference of another hollow cylinder of mass M and radius R. The second cylinder is free to rotate about its horizontal axis O. The moments of inertia of the cylinders about their axes are MR^2 and $\frac{1}{2}mr^2$, respectively. Derive the equations of motion of the system and find their first integrals.

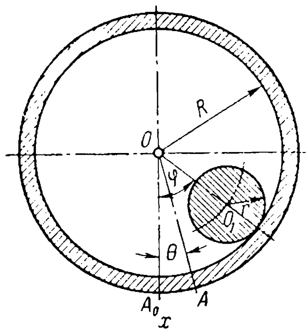


Fig. 472

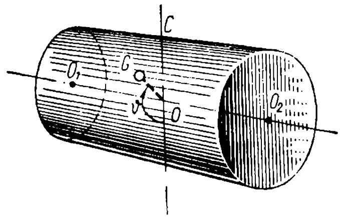


Fig. 473

Ans. $MR^2\dot{\vartheta}-\frac{1}{2}mR[(R-r)\dot{\varphi}-R\dot{\vartheta}]=C_1$, $\frac{1}{2}MR^2\dot{\vartheta}^2+$
$$+\frac{1}{4}m[(R-r)\dot{\varphi}-R\dot{\vartheta}]^2+\frac{m}{2}(R-r)^2\dot{\varphi}^2-mg(R-r)\cos\varphi=C_2,$$
where φ is the angle of rotation of the line, connecting the axes of the cylinders, and ϑ is the angle of turn of the second cylinder.

754. A body of weight P is free to rotate about a horizontal axis O_1O_2 which also rotates with constant angular velocity ω about a vertical axis OC (Fig. 473). The centre of gravity of the body G is located on the straight line perpendicular to O_1O_2 at a distance l from the point O. The axes O_1O_2 and OG are assumed to be the principal axes of inertia of the body at the point O. Derive the equation of motion of the system. The moments of inertia of the body about the principal axes are A, B and C.

Ans. $A\ddot{\vartheta}-\omega^2(C-B)\sin\vartheta\cos\vartheta=-Pl\sin\vartheta$, where ϑ is the angle of turn about O_1O_2.

755. Fig. 474 represents a system consisting of two identical wheels each of radius a. The wheels are connected by a spring of stiffness c which is acted on by torsional moment. The wheels rotate separately about the common axle O_1O_2 of length l which is perpendicular to the wheels. Each

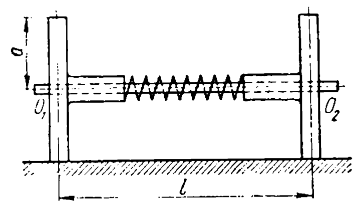


Fig. 474

wheel has mass M; C is the moment of inertia of the wheel about the axis of rotation. The moment of inertia of the wheel about the diameter is A. Derive the equations of motion of the system, and also find the motion corresponding to initial conditions $\varphi_1=0$, $\dot{\varphi}_1=0$, $\varphi_2=0$, $\dot{\varphi}_2=\omega$ (φ_1, φ_2 are the angles of turn of the wheels). Neglect the mass of the axle.

Ans. $\varphi_1=\frac{1}{2}\left(\omega t-\frac{\omega}{k}\sin kt\right)$; $\varphi_2=\frac{1}{2}\left(\omega t+\frac{\omega}{k}\sin kt\right)$;

$$k=\sqrt{\frac{2c}{Ma^2+C+4A\left(\frac{a}{l}\right)^2}}.$$

756. A circular cylinder of mass M_1 rolls without sliding on the platform of a trolley, which also runs without sliding down a plane parallel to the platform and inclined at an angle α to the horizontal. The generators of the cylinder are perpendicular to the line of maximum slope of the platform. The mass of the trolley without wheels is M, and the total mass of the wheels which are assumed to be homogeneous solid disks, is m. Determine the acceleration of the trolley.

Ans. $w=\frac{6M+6m+2M_1}{6M+9m+2M_1}g\sin\alpha.$

757. A smooth horizontal table has a hole O pierced in it, as shown in Fig. 475. The end B of the thread, passing through the hole, carries a point mass m_2 while the other end A is attached to a point mass m_1. The mass m_1 rests on the surface of the table while the mass m_2 moves along the vertical passing through the point O. At the initial moment $OA=r_0$, and the mass m_2 is at rest while the mass m_1 has velocity v_0 directed perpendicular to the initial position of thread section OA. Prove that under this condition the mass m_2 will perform oscillations and determine the amplitude and the period of the oscillations. Assume that the thread is weightless, inextensible and perfectly flexible.

Ans. $a=|r_0-r_1|$,

$$T=\sqrt{\frac{2(m_1+m_2)}{m_2g}}\left|\int_{r_0}^{r_1}\frac{r\,dr}{\sqrt{(r_0-r)(r-r_1)(r+r_2)}}\right|,$$

where

$$r_1=\sqrt{\frac{m_1v_0^2}{4m_2g}\left(2r_0+\frac{m_1v_0^2}{4m_2g}\right)}+\frac{m_1v_0^2}{4m_2g},$$

$$r_2=\sqrt{\frac{m_1v_0^2}{4m_2g}\left(2r_0+\frac{m_1v_0^2}{4m_2g}\right)}-\frac{m_1v_0^2}{4m_2g}.$$

758. A homogeneous disk of radius R and mass M is free to rotate about its horizontal axle O, as shown in Fig. 476. A particle of mass m is suspended to the disk by a thread AB of length l. Derive the equations of motion of the system.

Ans. $\left(m+\frac{M}{2}\right)R^2\ddot{\varphi}+mRl\cos(\varphi-\psi)\ddot{\psi}+mRl\sin(\varphi-\psi)\dot{\psi}^2+$ $+mgR\sin\varphi=0,\quad mRl\cos(\varphi-\psi)\ddot{\varphi}+ml^2\ddot{\psi}-mRl\sin(\varphi-$ $-\psi)\dot{\varphi}^2+mgl\sin\psi=0$, where φ is the angle of turn of the disk, and ψ is the angle between the thread and the vertical.

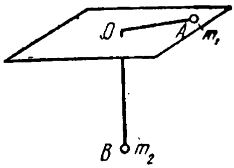


Fig. 475

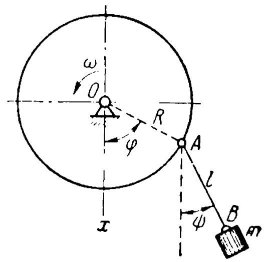


Fig. 476

759. The problem is the same as the preceding one, except that the disk rotates with constant angular velocity ω. Derive the equation of motion of the particle and, neglecting gravity, determine the equivalent length of the mathematical pendulum l_e.

Ans. $\ddot{\psi}-\omega^2\frac{R}{l}\sin(\omega t-\psi)+\frac{g}{l}\sin\psi=0,$

and $l_e=\frac{g}{\omega^2 R}l.$

760. Fig. 477 represents a circuit of an electrodynamical pick-up used for recording mechanical vibrations. The mass of the armature is M and the stiffness of the spring is c. The coefficient of self-induction of the coil changes due to alteration of the air-gap in the magnetic wire $L=L(x)$; x is a vertical displacement of the armature from the position when the springs are not compressed. An electric circuit with a battery of electromotive force E is attached to the coil. The resistance of the circuit equals R. Derive the equations of motion of the system and determine its "positions of equilibrium"

Hint. The displacement x of the armature and the charge q corresponding to the current i in the circuit $\left(i=\frac{dq}{dt}\right)$ are to be taken as absolute coordinates.

Ans. Equations of motion $L\ddot{q}+R\dot{q}+qx\dfrac{\partial L}{\partial x}=E$;

$$M\ddot{x}-\frac{1}{2}\frac{\partial L}{\partial x}\dot{q}^2+cx=Mg.$$

In the "position of equilibrium" $x=x_0$ and $i=\dot{q}=i_0$, where $i_0=\dfrac{E}{R}$; $cx_0=Mg+\dfrac{1}{2}\left(\dfrac{\partial L}{\partial x}\right)_0 i_0^2$.

761. Referring to the preceding problem, derive the equations of small oscillations about to the position of equilibrium of the electrodynamic pick-up.

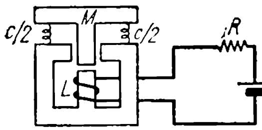


Fig. 477

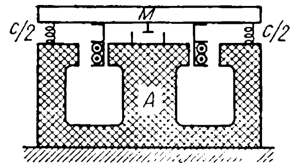


Fig. 478

Hint. The change of the charge e and the vertical displacement of the armature from the position of equilibrium ξ should be taken as absolute coordinates. The function $L(x)$ is expanded in a series $L=L(x_0+\xi)=L_0+L_1\xi+$ and we retain only first two terms.

Ans. $L_0\ddot{e}+R\dot{e}+L_1 i_0\dot{\xi}=0$;
$M\ddot{\xi}+c\xi-L_1 i_0\dot{e}=0$.

762. Fig. 478 shows an electromechanical system which consists of the permanent cylinder magnet with concentric poles A, generating a radial field, and the armature of mass M, resting on the spring of stiffness c. The armature is connected with a coil of n turns and with a mechanical damper whose resistance is proportional to the velocity of the armature. (The coefficient of resistance is β.) The average radius of the turns of the coil is r and its coefficient of self-induction is L, the resistance is R and the magnetic induction in the gap is B. The alternating voltage $V(t)$ is applied to the clamps of the coil. Derive the equation of motion of the system.

Hint. The absolute forces between the coil and the magnet equal $Q_q=-2\pi rnB\dot{x}$, and $Q_x=2\pi rnB\dot{q}$ (Q_q is the electromotive force in the electric circuit, and Q_x is the force between the magnet and the coil).

Ans. $L\ddot{q}+R\dot{q}+2\pi rnB\dot{x}=V(t)$;
$M\ddot{x}+\beta\dot{x}+cx-2\pi rnB\dot{q}=0$.

48. Small Oscillations of Systems with a Single Degree of Freedom

763. Find the period of free vibrations of the foundation of the machine placed on the solid ground and then displaced from the position of equilibrium (Fig. 479). The machine together with the foundation weighs $Q=147{,}000$ kgf, the area of the foundation is $S=50$ m^2, the specific stiffness of the ground is $\lambda=3$ kgf/cm^3 (the coefficient of the ground stiffness is $c=\lambda S$).

Ans. $T=0.0628$ sec.

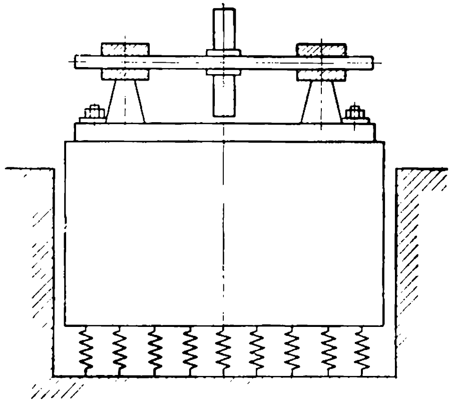


Fig. 479

764. One end O of a rigid rod OB of length l is free to swing in the ball joint, and the other end B carries a small ball of weight Q (Fig. 480). The rod is held in a horizontal position by an unstretched vertical rope of length h. $OA=a$. The ball is pulled sideways perpendicular to the plane of the sketch and then released. The system starts to oscillate. Neglecting the mass of the rod, determine the period of small oscillations of the system.

Ans. $T=2\pi\sqrt{\dfrac{hl}{ag}}$.

765. An astatic pendulum, shown in Fig. 481, is used in some seismographs to record the vibrations of the earth. It consists of a rigid rod of length l with a mass m at its end. The mass is gripped between two horizontal springs of stiffness c whose ends are fixed. Determine the period of small oscillations of the pendulum, assuming that the springs are not compressed while resting in equilibrium. Neglect the mass of the rod.

Ans. $T=\dfrac{2\pi}{\sqrt{2\dfrac{c}{m}-\dfrac{g}{l}}}$.

766. A pendulum consists of a rigid rod of length l, carrying a mass m at its end (Fig. 482). Two springs of stiffness c are attached to the rod at a distance a from its upper end. The other ends of both springs are rigidly fixed. Neglecting the mass of the rod, find the period of small oscillations of the pendulum.

Ans. $T=\dfrac{2\pi}{\sqrt{\dfrac{2ca^2}{ml^2}+\dfrac{g}{l}}}$.

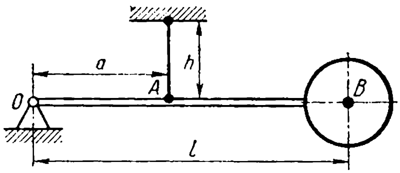


Fig. 480

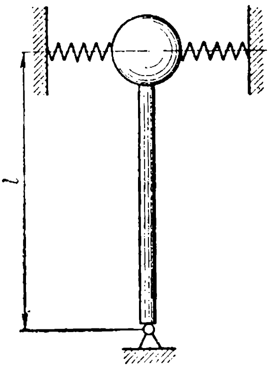


Fig. 481

767. Referring to the preceding problem, it is assumed that the pendulum is placed so that the mass is attached above the point of suspension, as shown in Fig. 483. What conditions must be satisfied for the vertical position to get stable equilibrium? Find the period of small oscillations of the pendulum.

Ans. $a^2 > \dfrac{mgl}{2c}$; $T=\dfrac{2\pi}{\sqrt{\dfrac{2ca^2}{ml^2}-\dfrac{g}{l}}}$.

768. A cylinder of diameter d and mass m is free to roll without sliding on a horizontal plane (Fig. 484). Two identical springs of stiffness c are attached to its mid-point at a distance a

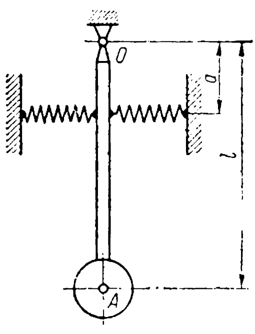


Fig. 482

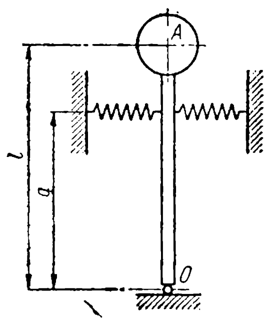


Fig. 483

from the axis of the cylinder. The other ends of both springs are rigidly fixed. Determine the period of small oscillations of the cylinder.

Ans. $T=\dfrac{\pi\sqrt{3}}{1+2\dfrac{a}{d}}\sqrt{\dfrac{m}{c}}$.

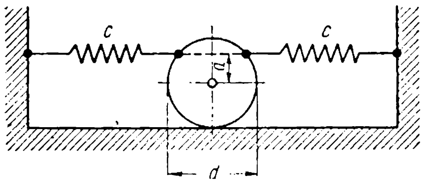


Fig. 484

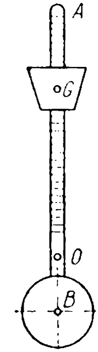


Fig. 485

769. A metronome consists of a pendulum and an additional sliding weight G of mass m (Fig. 485). The moment of inertia of the whole system about its horizontal axis of rotation changes by moving the weight G. The mass of the pendulum is M, and the distance between the centre of gravity of the pendulum and the axis of rotation O equals s_0. The distance $OG=s$. The moment of inertia of the pendulum about the axis of rotation is J_0. Determine the period of small oscillations of the metronome.

Ans. $T=2\pi\sqrt{\dfrac{J_0+ms^2}{(Ms_0-ms)g}}$.

770. A circular hoop is suspended from three fixed points by three identical unstretched springs each of length l in such a way that the plane of the hoop is horizontal. When the hoop is in equilibrium, the strings are vertical, and they divide the circumference of the hoop by three identical parts. Find the period of small oscillations of the hoop about the axis passing through the centre of the hoop.

Ans. $T=2\pi\sqrt{\dfrac{l}{g}}$.

771. A heavy square platform $ABCD$ of mass M is suspended from a fixed point O by four elastic ropes each of stiffness c (Fig. 486). When the system is in equilibrium, the distance between the point O and the centre E of the platform along the vertical is l. The diagonal of the platform has a length a. Determine the period of vertical vibrations of the system.

$$Ans.\ T=2\pi\sqrt{\frac{M}{c}\,\frac{(a^2+4l^2)}{16l^2}\,\frac{1}{1+\dfrac{Mga^2}{16cl^3}}}.$$

772. Fig. 487 represents an instrument for recording vertical vibrations of machine foundations. It consists of a load of weight Q which is connected to the vertical spring with the coefficient of stiffness c_1, and on the other side it is hinged to a pointer of a bell

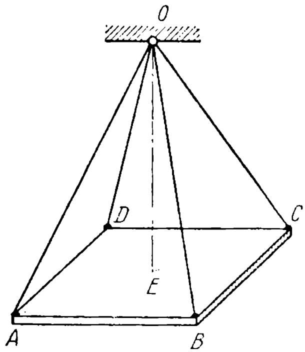


Fig. 486

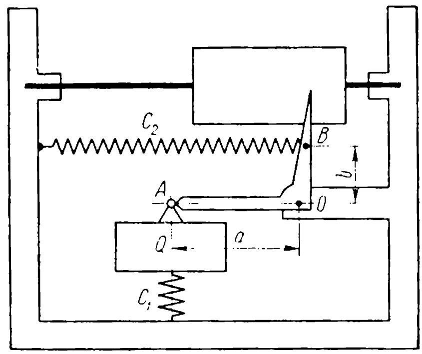


Fig. 487

crank shape, resting in static equilibrium. The bell crank pointer has a moment of inertia J about the axis of rotation O and is held by an unstretched horizontal spring with the coefficient of stiffness c_2. Determine the period of free oscillations of the pointer about its vertical equilibrium position. $OA=a$, $OB=b$. Neglect the dimensions of the load and the effect of initial stretching of the spring.

$$Ans.\ T=2\pi\sqrt{\frac{Jg+Qa^2}{g(c_1a^2+c_2b^2)}}.$$

773. A shock absorber device can be described schematically as consisting of a particle of mass m connected by n springs of stiffness c to the vertices of a rectilinear hexagon. The free length of each spring is a and the radius of the circumference traced through the vertices of the hexagon is b. Determine the frequency of horizontal free oscillations of the system located in the horizontal plane. (See Fig. 488.)

H i n t. If the potential energy is required to be exact up to terms of second order, then the extension of the spring must be determined to the same order of accuracy.

$$Ans.\ k=\sqrt{\frac{nc}{2m}\,\frac{2b-a}{b}}$$

774. Referring to the preceding problem, determine the frequency of oscillations perpendicular to the plane of the hexagon. Neglect the effect of gravity.

Ans. $k=\sqrt{\dfrac{nc(b-a)}{mb}}$

775. Determine the frequency of small vertical oscillations of the particle E in the system, shown in Fig. 489. The mass of the particle is m. $AB=BC$ and $DE=EF$ The springs have stiffness c_1,

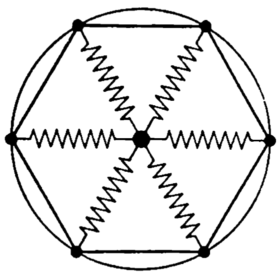


Fig. 488

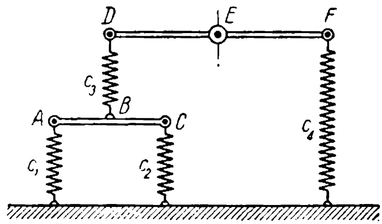


Fig. 489

c_2, c_3 and c_4. The rods AC and DF are considered as rigid and massless.

Ans. $k=\sqrt{\dfrac{4}{m\left(\dfrac{1}{4c_1}+\dfrac{1}{4c_2}+\dfrac{1}{c_3}+\dfrac{1}{c_4}\right)}}.$

776. An unstretched thread of length $4a$ carries three loads with masses m, M and m, respectively (Fig. 490). The thread is suspended symmetrically in such a way that it makes angles α and β with the vertical. The load M performs small vertical oscillations. Determine the frequency of free vertical oscillations of the load M.

Ans. $k=\sqrt{\dfrac{g(\cos^2\beta\sin\beta+\cos^2\alpha\sin\alpha)}{a\cos\beta\cos\alpha\sin(\beta-\alpha)\cos(\beta-\alpha)}}$,

where $2m=\dfrac{M\sin(\beta-\alpha)}{\sin\alpha\cos\beta}$.

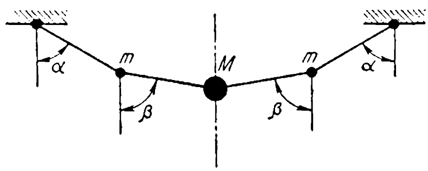


Fig. 490

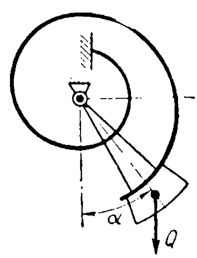


Fig. 491

777. Fig. 491 shows a vibrograph used for recording vibrations of foundations, machine parts, etc. It has a pendulum of weight Q which is held by a helical spring of stiffness c at an angle α with the vertical. The moment of inertia of the pendulum about an axis of rotation O is J The distance between the centre of gravity of the pendulum and the axis of rotation is s. Determine the period of free oscillations of the vibrograph.

Ans. $T = 2\pi\sqrt{\dfrac{J}{Qs\cos\alpha + c}}$.

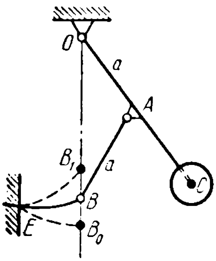


Fig. 492

778. A rod OA of a pendulum is connected to a small steel spring EB of stiffness c by means of a connecting rod AB (Fig. 492). The spring when unstretched is in position EB_1. To bring the spring to the position EB_0, corresponding to the equilibrium position of the pendulum, a force F_0 along OB should be applied. $OA = AB = a$. The masses of the rods may be neglected. A distance between the centre of gravity of the pendulum and the axis of rotation is $OC = l$. The weight of the pendulum is Q. To attain the maximum isochronous effect (i. e., the period of vibration does not depend on the angle of initial deflection) the system is adjusted so that the first term thrown away in the equation of motion of the pendulum $\ddot{\varphi} = f(\varphi) = -\beta\varphi +$ will be φ^5. Find the relation between the quantities Q, F_0, c, a, and l, and calculate the period of small oscillations of the pendulum.

Ans. $T = 2\pi\sqrt{\dfrac{l}{g}\dfrac{1}{\sqrt{1-\dfrac{2aF_0}{Ql}}}}$; $\quad Ql - 2aF_0 = 12a^2c$.

779. Referring to the previous problem, prove that increase in the period of oscillation does not exceed 0.4 per cent, when the pendulum is displaced by an angle $\varphi_0 = 45°$ from the position of equilibrium. How will the period of the simple pendulum be changed?

Ans. Preserving terms up to φ^5 in the equation of the motion of the pendulum, we have

$$T = 2\pi\sqrt{\frac{l}{g}\frac{1}{\sqrt{1-\dfrac{2aF_0}{Ql}}}}\left(1 + \frac{\varphi_0^4}{96}\right).$$

The period will be changed by 4 per cent.

780. The rod OM of a pallograph is free to pass through a rotating cylinder O and it is hinged at A to a rocker arm AO_1,

oscillating about a fixed axle O_1. (The dimensions are shown in Fig. 451.) A load M is attached to a free end of the rod. Under what conditions will the vertical position of the rod OM be in the position of stable equilibrium? Determine the period of small oscillations about this position. The dimensions of the load and the weight of the rods should be neglected.

$$Ans.\ T=2\pi(h-r+l)\sqrt{\frac{r}{[rl-(h-r)^2]g}};\quad h-r<\sqrt{rl}.$$

781. Neglecting the mass of the rods, find the period of small oscillations of the pendulum, shown in Fig. 493. The centre of gravity of the weight is located on the prolongation of the connecting rod of the linkage $OABO_1$ (straight-line guiding mechanism). In equilibrium the rods OA and BC are vertical while O_1B is horizontal. $OA=AB=a$. $AC=s$.

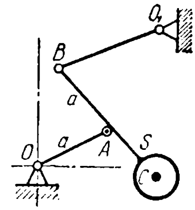


Fig. 493

$$Ans.\ T=2\pi\frac{s+a}{\sqrt{g(s-a)}}.$$

782. Determine a period of oscillations of a weight P which is suspended by a spring with its upper end rigidly fixed. The coefficient of stiffness of the spring is c and its weight is P_0.

Hint. To determine approximately the frequency of oscillations of the systems, mentioned in Problems 782-788, the energy method may be used.

$$Ans.\ T=2\pi\sqrt{\frac{P+\frac{1}{3}P_0}{cg}}$$

783. A round elastic vertical rod has its upper end rigidly fixed while its bottom end is attached to the centre of a horizontal disk, with moment of inertia J about a vertical axis passing through its centre. The moment of inertia of the rod about its axis is J_0. The coefficient of stiffness of the rod against twisting is c (i. e., a moment required to twist the bottom end of the rod per radian). Determine the period of oscillations of the system.

$$Ans.\ T=2\pi\sqrt{\frac{J+\frac{1}{3}J_0}{c}}.$$

784. A beam of length l rests freely on its ends. A weight Q is applied at the middle of the beam. A moment of inertia of a cross-section of the beam is J, and the modulus of elasticity of the

material is E. Neglecting the weight of the beam, determine the number of oscillations performed by the weight each minute.

Ans. $n=2080\sqrt{\frac{EJ}{Ql^3}}$ where one centimetre is taken as the unit of length.

785. A beam of length l and weight Q_1 is freely supported at both ends. It carries a weight Q at its mid-point. The moment of inertia of the cross-section of the beam is J, and its modulus of elasticity is E. Determine, approximately, the number of free oscillations performed by the weight each minute.

Ans. $n=2080\sqrt{\frac{EJ}{\left(Q+\frac{17}{35}Q_1\right)l^3}}$ where one centimetre is taken as the unit of length.

786. A beam supporting a crane is made of channel iron. Its linear weight is $q=49$ kgf/m, the moment of inertia of a cross-section is $J=8360$ cm^4, and its length is $l=10$ m. In the middle of the beam there is a wheel of weight $Q=700$ kgf. The beam is supported at both ends, and the modulus of elasticity of the material is $E=2\times10^6$ kgf/cm^2. Find the frequency of oscillations of the beam regarding and then disregarding its mass.

Ans. $k_1=4.56$ sec^{-1}; $k_2=5.34$ sec^{-1}

787. A beam with modulus of elasticity $E=2\times10^6$ kgf/cm^2 rests on two identical elastic springs of stiffness $c=150$ kgf/cm (Fig. 494). It has a moment of inertia of a cross-section $J=180$ cm^4 and a length $l=4$ m. A weight $Q=200$ kgf rests on the beam. Neglecting the weight of the beam, determine the period of free oscillations of the system.

Ans. $T=0.238$ sec.

788. A rod AB is built into a wall, as shown in Fig. 495. A load Q performing oscillations of period T is suspended at the free end of the rod. The moment of inertia of the rod cross-section about

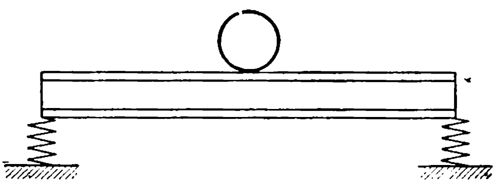


Fig. 494

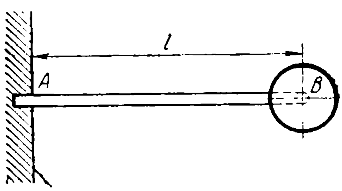


Fig. 495

a central axis perpendicular to the plane of oscillations is J. Determine a modulus of elasticity of the rod material.

Ans. $E = \dfrac{4\pi^2 Q l^3}{3JgT^2}$.

789. Fig. 496 represents a vibrograph which is used to record vertical vibrations. It consists of a rod OA swinging about a horizontal axis O and carrying the vibrations to the recording pen of the instrument. The weight Q suspended to the rod at A is held

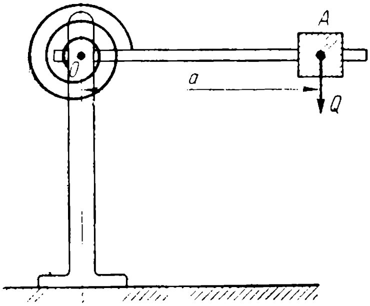


Fig. 496

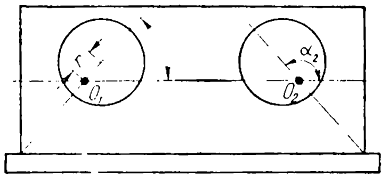


Fig. 497

horizontally by a helical spring. If the vibrograph is fastened to a foundation which performs vibrations according to the law $z=2\sin 25t$ mm, determine a relative motion of the rod. The coefficient of stiffness of the spring is $c=0.1$ kgf/cm, and the moment of inertia of the rod OA and weight Q about O equals $J=0.4$ kgfcmsec^2. $Qa=10$ kgfcm. Neglect the natural vibrations of the rod.

Ans. $\varphi=0.0051 \sin 25t$.

790. Fig. 497 represents a device used for generating vibrations. It consists of two identical disks mounted eccentrically on two parallel axles. Each disk is of weight Q and has equal eccentricity r. The total weight of the device is P Initially the disks make angles α_1 and α_2 with the horizontal. Both disks rotate in opposite directions with angular velocity ω. The device is fastened by bolts to a solid foundation of stiffness c. Determine the amplitude of forced vibrations in the vertical direction of the foundation, neglecting its weight.

Ans. $A = \dfrac{2Qr}{\dfrac{cg}{\omega^2} - (P+2Q)} \sin \dfrac{\alpha_1+\alpha_2}{2}$.

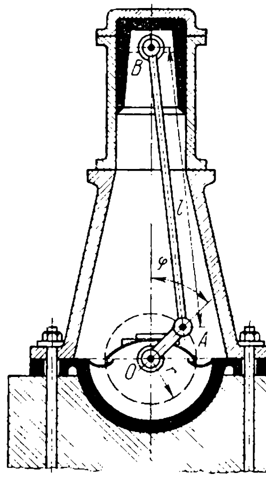


Fig. 498

791. A vertical engine of weight Q is fastened to a foundation of area S, as shown in Fig. 498. The other data are as follows: the specific stiffness of the ground is λ, the length of the crank is r, and the connecting rod is l, the angular velocity of the shaft is ω, the weight of the piston and unbalanced parts performing reciprocal motions is P, the foundation is of weight G, and the crank is assumed to be balanced by a counter-weight. The mass of the connecting rod should be neglected. Determine the amplitude of the forced vibrations of the foundation.

Hint. Neglect all terms higher than the first power in $\frac{r}{l}$.

Ans. $\xi = \frac{Pr\omega^2}{(Q+G)(k^2-\omega^2)} \cos \omega t + \frac{r}{l} \frac{Pr\omega^2}{(Q+G)(k^2-4\omega^2)} \cos 2\omega t$,
where $k = \sqrt{\frac{\lambda S g}{Q+G}}$.

792. An electric motor of weight $Q=1200$ kgf is installed on the free ends of two horizontal parallel beams whose other ends are imbedded in a wall, as shown in Fig. 499. The distance between the axle of the motor and the wall is $l=1.5$ m. The armature of the motor makes $n=1500$ rpm, it weighs $p=200$ kgf, and its centre of gravity is located at the distance $r=0.05$ mm from the axis of the shaft. The beams are made of a soft steel with the modulus of elasticity $E=2\times10^6$ kgf/cm². Determine the moment of inertia of a cross-section for such a position, so that the amplitude of forced vibrations does not exceed 0.5 mm. Neglect the weight of the beam.

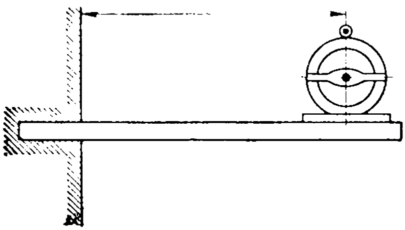


Fig. 499

Ans. $J=8740$ cm⁴, or $J=8480$ cm⁴.

793. A cam valve control mechanism can be described as a mass m, which is connected with one end to a fixed point by means of a spring of stiffness c (Fig. 500). The other end of the mass is connected to a spring of stiffness c_1, whose length varies as the cam moves. The contour of the cam face is such that the vertical stroke of the cam is defined by the following formulae:

$$x_1 = a[1-\cos \omega t] \text{ for } 0 \leq t < \frac{2\pi}{\omega}, \quad x_1 = 0 \text{ for } t > \frac{2\pi}{\omega}.$$

Determine the motion of the mass m.

Ans. For $0 < t < \frac{2\pi}{\omega}$

$$x=\frac{c_1 a}{m(k^2-\omega^2)}\,[\cos kt-\cos\omega t]+\frac{c_1 a}{mk^2}\,[1-\cos kt],$$

where $k=\sqrt{\dfrac{c+c_1}{m}}$.

For $t>\dfrac{2\pi}{\omega}$ $\quad x=\left[\dfrac{c_1 a}{m(k^2-\omega^2)}-\dfrac{c_1 a}{mk^2}\right]\left[\cos kt-\cos k\left(t-\dfrac{2\pi}{\omega}\right)\right],$

and the mass oscillates quite freely.

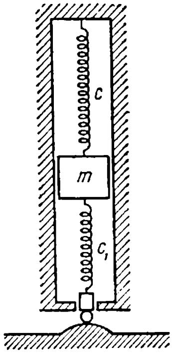


Fig. 500

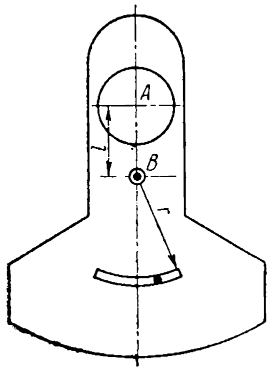


Fig. 501

794. To damp vibrations in the shaft of an aircraft motor a special slot in the shape of an arc of radius r is made in its counter-balance to contain a supplementary counter-weight, which is to be considered as a point particle (Fig. 501). The latter is free to slide along the slot whose centre is displaced a distance $AB=l$ from the axis of rotation. The angular velocity of the shaft is ω. Neglecting gravity effect, determine the frequency of small oscillations of the supplementary counter-weight.

Ans. $k=\omega\sqrt{\dfrac{l}{r}}$

795. A weight P suspended by a spring is initially acted on by a constant force F After time τ the action of the force is stopped. The spring has stiffness c. Determine the motion of the weight.

Ans. For $0<t<\tau$ $\quad x=\dfrac{F}{c}\left[1-\cos\sqrt{\dfrac{cg}{P}}\,t\right].$

For $\tau<t$ $\quad x=\dfrac{F}{c}\left[\cos\sqrt{\dfrac{cg}{P}}\,(t-\tau)-\cos\sqrt{\dfrac{cg}{P}}\,t\right].$

796. Referring to the preceding problem, determine the maximum deviation from the position of equilibrium of the system, if it is acted on by forces of different durations:

(1) $\tau=0,\ \lim\limits_{\tau\to 0} F\tau=S$ (impact); (2) $\tau=\frac{T}{4}$

(3) $\tau=\frac{T}{2}$, where T is the period of free oscillations of the system.

Ans. (1) $x_{\max}=\sqrt{\frac{g}{cP}}S$; (2) $x_{\max}=\sqrt{2}\,\frac{F}{c}=\sqrt{2}\,x_{st}$;

(3) $x_{\max}=2\frac{F}{c}\,2x_{st}$.

797. A weight P suspended from a spring is acted on by a disturbing force defined by the conditions: $F=0$ for $t<0$; $F=\frac{t}{\tau}F_0$ for $0<t<\tau$; $F=F_0$ for $\tau<t$. The spring has a stiffness c. Determine the motion of the weight and find the amplitude of oscillations for $t>\tau$.

Ans. $x=\frac{F_0}{c}\left[1-\frac{2}{k\tau}\cos k\left(t-\frac{\tau}{2}\right)\sin\frac{k\tau}{2}\right]$; $k=\sqrt{\frac{cg}{P}}$;

$A=\frac{2F_0}{kc\tau}\sin\frac{k\tau}{2}$.

798. A weight P suspended from a spring is acted on by a disturbing force which changes in accordance with the law $Q(t)=F|\sin\omega t|$. The stiffness of the spring is c. Determine the oscillations of the system which have the same frequency as the disturbing force.

Ans. For $0<t<\frac{\pi}{\omega}$

$$x=\frac{F_\omega}{mk(\omega^2-k^2)}\left[\sin kt+\cot\frac{k\pi}{2\omega}\cos kt\right]-\frac{F}{m(\omega^2-k^2)}\sin\omega t;$$

$$k=\sqrt{\frac{cg}{P}}$$

799. A light shaft of length l carries a disk of weight P at its centre and its bending stiffness equals EJ Determine the critical angular velocity (with respect to transverse vibrations) in two cases: (1) both ends of the shaft rest on long bearings (the ends are built in); (2) one end of the shaft rests on a long bearing (the end is built in) while the other rests on a short bearing (the end is simply supported)

Ans. (1) $\omega_{cr}=\sqrt{\frac{192EJg}{Pl^3}}$ (2) $\omega_{cr}=\sqrt{\frac{768EJg}{7\,Pl^3}}$

800. Determine a critical velocity of a heavy shaft which rests with its one end in a short bearing while with the other end it rests in a long bearing. The length of the shaft is l, its bending stiffness is EJ, and its weight per unit length is q.

Ans. $\omega_{cr}=15.4\sqrt{\frac{EJg}{ql^4}}$.

49. Small Oscillations of Systems with Several Degrees of Freedom

801. Fig. 502 represents a device which is used to adjust turbine blades. It consists of a turbine with a rotor and a flywheel connected by an elastic shaft C. The moment of inertia of the rotor about the axle of rotation is $J_1=5$ kgfcmsec^2 and the moment of inertia of the flywheel is $J_2=150$ kgfcmsec^2. The shaft has a length $l=1552$ mm, a diameter $d=25.4$ mm and a modulus of

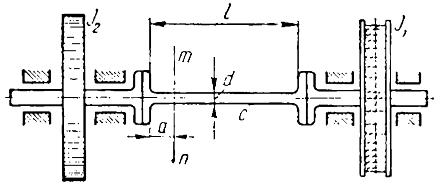


Fig. 502

transverse elasticity $G=880{,}000$ kgf/cm^2. Neglecting the mass of the shaft and the torsion of its thick ends, determine the cross-section mn of the shaft which remains in equilibrium (nodal cross-section) during the oscillations of the system. Also compute the period T of free oscillations of the system.

Ans. $a=50$ mm; $T=0.09$ sec.

802. Determine the frequency of free torsional oscillations of the steel shaft of a ship's propeller. The shaft has a length $l=50$ m and a diameter $d=35$ cm. The moment of inertia of the oscillating masses mounted on one end of the shaft is $J_1=390{,}000$ kgfcmsec^2, and the moment of inertia of a screw propeller mounted on the other end of shaft is $J_2=69{,}000$ kgfcmsec^2. Neglect the effects of the mass of the shaft on the frequency of free oscillations. The modulus of transverse elasticity of the steel is $G=880{,}000$ kgf/cm^2.

Ans. $k=21.4$ sec^{-1}.

803. Determine the frequencies of free torsional oscillations of a system, which consists of a shaft with one end fixed and carrying one disk at its middle and one disk on its free end. Both disks are considered to be homogeneous. The moment of inertia of each disk about the axle of the shaft is J. The torsion stiffness of the various parts of the shaft is $c_1=c_2=c$. Neglect the mass of the shaft.

Ans. $k_1=0.62\sqrt{\frac{c}{J}}$; $k_2=1.62\sqrt{\frac{c}{J}}$.

804. Two identical pendulums each of length l and mass m are connected at the level h by an elastic spring of stiffness c, as

shown in Fig. 503. The ends of the spring are fixed to the rods of the pendulums. Determine the small oscillations of the system in the plane of the equilibrium position of the pendulums after one of the pendulums has been displaced an angle α from equilibrium. The initial velocities of both pendulums equal zero. Neglect the masses of the rods of the pendulums and of the spring.

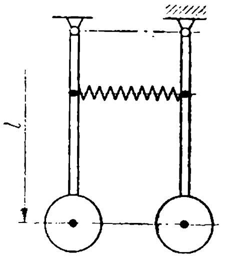


Fig. 503

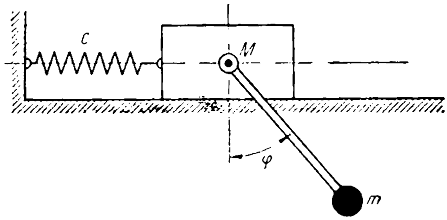


Fig. 504

Ans. $\varphi_1 = \alpha \cos \frac{k_1+k_2}{2} t \cos \frac{k_2-k_1}{2} t;$

$\varphi_2 = \alpha \sin \frac{k_1+k_2}{2} t \sin \frac{k_2-k_1}{2} t,$

where φ_1 and φ_2 are the angles between the pendulums and verticals, and $k_1 = \sqrt{\frac{g}{l}}$ $k_2 = \sqrt{\frac{g}{l} + \frac{2ch^2}{ml^2}}$

805. A pendulum consists of a slider of mass M and a ball of mass m connected by a rod of length l, as shown in Fig. 504. The slider slides without friction along the horizontal plane. The rod is free to oscillate about an axle which is connected to the slider. A spring of stiffness c is tied with its one end to the slider while its other end is rigidly fixed. Determine the frequencies of small oscillations of the system.

Ans. These frequencies are the roots of the equation

$$k^4 - \left[\frac{c}{M} + \frac{g}{l}\frac{M+m}{M}\right]k^2 + \frac{c}{M}\frac{g}{l} = 0.$$

806. Two identical physical pendulums are suspended from horizontal axles on the same level, and they are connected by an elastic spring whose free length is equal to the distance between the axes of oscillation of the pendulums. Each pendulum weighs $P = 0.45$ kgf, its moment of inertia about the axle of suspension is $J = 0.664$ kgfcmsec^2, and the distance between its centre of gravity and the axle of suspension is $a = 34.2$ cm. The distances between the ends of the spring and the axles of suspension are $h_1 = h_2 = 34.2$ cm. The spring has a stiffness of $c = 0.004$ kgf/cm.

Determine the frequencies of the normal modes of oscillations of the system and the corresponding ratio of the amplitudes. Neglect the mass of the spring.

Ans. $k_1=4.8\ \text{sec}^{-1}$; $k_2=6.1\ \text{sec}^{-1}$;

$$\frac{A_1^{(1)}}{A_2^{(1)}}=1\,;\ \frac{A_1^{(2)}}{A_2^{(2)}}=-1\,.$$

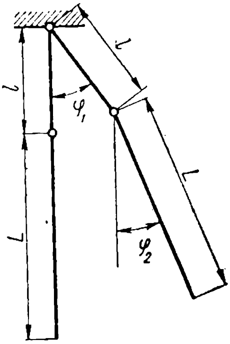


Fig. 505

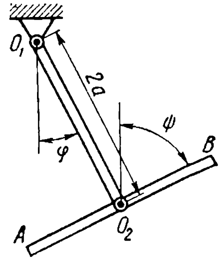


Fig. 506

807. A uniform rod of length L is suspended to a fixed point by a string of length $l=0.5L$. Neglecting the mass of the string, determine the frequencies of the normal modes of oscillations of the system and find the ratio of the deviations from the vertical of the rod and the string, for the first and the second normal modes of oscillations. (See Fig. 505.)

Ans. $k_1=0.677\sqrt{\frac{g}{l}}$; $k_2=2.558\sqrt{\frac{g}{l}}$;

for the first normal mode $\varphi_1=0.847\varphi_2$, for the second $\varphi_1=-1.180\varphi_2$, where φ_1 and φ_2 are the angles of the string and the rod with the vertical.

808. The problem is the same as the preceding one, except that the length of the string is rather long in comparison with that of the rod. Neglecting terms containing the square of the ratio $\frac{L}{l}$, determine the relation between the smallest frequency of free oscillations of the system and the frequency of oscillations of a simple pendulum of length l.

Ans. $1-\frac{1}{4}\,\frac{L}{l}$.

809. The compound physical pendulum, shown in Fig. 506, consists of a uniform straight bar O_1O_2 of length $2a$ and weight

P_1 oscillating about a fixed horizontal axle O_1, and another uniform straight bar AB of weight P_2 hinged at its centre to the end O_2 of the first bar. Originally the bar O_1O_2 was at an angle φ_0 to the vertical while the bar AB was vertical and had the initial angular velocity ω_0. Determine the motion of the system.

Ans. $\varphi=\varphi_0 \cos \sqrt{\frac{3}{4} \frac{P_1+2P_2}{P_1+3P_2} \frac{g}{a}}\, t$;

$\psi=\omega_0 t$, where ψ is an angle between AB and the vertical.

810. Describe the vibrations of a railway car in its central vertical plane, if the weight of the car on the springs is Q, the dis-

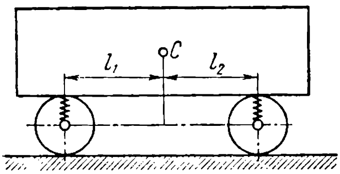


Fig. 507

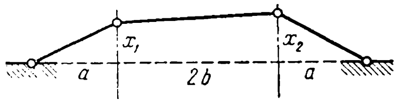


Fig. 508

tances between the centre of gravity and the vertical planes passing through the axles are $l_1=l_2=l$, the radius of gyration about the central axis parallel to the axles of the car is ρ and the stiffness of the springs is identical for both axles: $c_1=c_2=c$ (Fig. 507).

Ans. $x=A \sin (k_1 t+\alpha)$; $\psi=B \sin (k_2 t+\beta)$, where x is a vertical displacement of the centre of gravity of the car, and ψ is the angle made between the car floor and the horizontal; A, B, α, and β are integration constants;

$$k_1=\sqrt{\frac{2\,cg}{Q}} \qquad k_2=\sqrt{\frac{2cgl^2}{Q\rho^2}}.$$

811. Two identical material particles of weight Q are located symmetrically at equal distances from the ends of a stretched string of length $2\,(a+b)$. The tension in the string is p. Determine the frequencies of the normal modes of oscillations and find the principal coordinates (Fig. 508).

Ans. $k_1=\sqrt{\frac{pg}{Qa}}$; $k_2=\sqrt{\frac{pg}{Q}\left[\frac{1}{a}+\frac{1}{b}\right]}$.

The principal coordinates are: $\theta_1=\frac{1}{2}(x_1+x_2)$;

$\theta_2=\frac{1}{2}(x_2-x_1)$.

812. A heavy particle oscillates about its position of equilibrium on a smooth surface with a concave side upwards. The principal

radii of the surface curvature at the points of equilibrium are ρ_1 and ρ_2. Determine the frequencies of small oscillations.

Ans. $k_1=\sqrt{\frac{g}{\rho_1}}$; $k_2=\sqrt{\frac{g}{\rho_2}}$.

813. Determine the frequencies of small oscillations of a particle about its position of equilibrium, which coincides with the lowest point of a surface rotating with constant angular velocity ω about a vertical axis passing through this point. The principal radii of curvature at the lowest point of the surface are ρ_1 and ρ_2.

Ans. The frequencies of small oscillations are the roots of the equation:

$$k^4-\left[2\omega^2+\frac{g}{\rho_1}+\frac{g}{\rho_2}\right]k^2+\left(\omega^2-\frac{g}{\rho_1}\right)\left(\omega^2-\frac{g}{\rho_2}\right)=0.$$

814. A homogeneous circular disk of radius r and mass M is hinged to a rod OA of length l which is free to rotate about a fixed horizontal axle (Fig. 509). A particle of mass m is fixed on a circumference of the disk. Determine the frequencies of free oscillations of the system. The mass of the rod should be neglected. The disk is free to rotate in the plane of rotation of the rod OA.

Ans. The frequencies of free oscillations are the roots of the equation:

$$k^4-\frac{M+m}{M+3m}\left[1+2\frac{m}{M}\frac{r+l}{r}\right]\frac{g}{l}k^2+\frac{2m(M+m)}{M(M+3m)}\frac{g^2}{lr}=0.$$

815. A mechanical system A consists of a mass m_1 which is rigidly connected at B to the piston of a dashpot (Fig. 510). The mechanical system is suspended by a spring of stiffness c_1 to a platform whose motion is defined by the law $\xi=\xi(t)$. The frame of the dashpot of mass m_2 is placed on a spring of stiffness c_2 with

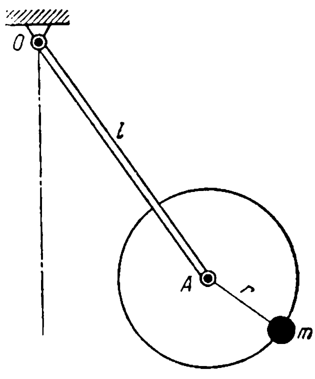


Fig. 509

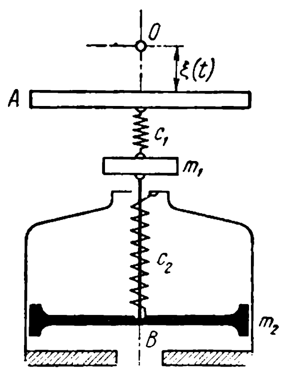


Fig. 510

its free end resting on the piston. The friction due to viscosity in the dashpot is proportional to the relative velocity of the piston and the frame; β is the coefficient of resistance. Derive the equation of motion of the system.

Ans. $m_1\ddot{x}_1+\beta\dot{x}_1-\beta\dot{x}_2+(c_1+c_2)x_1-c_2x_2=c_1\xi(t)$;
$m_2\ddot{x}_2-\beta\dot{x}_1+\beta\dot{x}_2-c_2x_1+c_2x_2=0$.

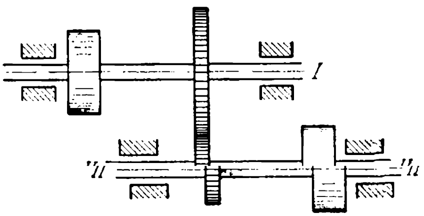


Fig. 511

816. Determine the frequencies of free torsional oscillations of the system, which consists of two shafts connected by a gear drive, as shown in Fig. 511. The moments of inertia of the masses mounted on the shafts and the moments of inertia of the gears about the axes of the shafts equal $J_1=87{,}500$ kgfcmsec², $J_2=56{,}000$ kgfcmsec², $i_1=302$ kgfcmsec², $i_2=10.5$ kgfcmsec², respectively. The gear ratio is $k=\frac{z_1}{z_2}=5$, and the coefficients of stiffness of the shafts under torsion are $c_1=316\times10^6$ kgfcm, $c_2=115\times\times10^6$ kgfcm. Neglect the masses of the shafts.

Ans. $k_1=54.8\ \text{sec}^{-1}$; $k_2=2.38\times10^3\ \text{sec}^{-1}$.

817. Referring to the preceding problem, determine the frequency of free torsional oscillations of the system, neglecting the masses of the gears.

Ans. $k=58.7\ \text{sec}^{-1}$.

818. A beam of length l rests on two supports and it carries two identical loads each of weight Q applied at points $x=\frac{1}{3}l$ and $x=\frac{2}{3}l$, respectively. The moment of inertia of a cross-sectional area of the beam is J, and the modulus of elasticity is E. Find the frequencies and contours of the normal transverse oscillations of the beam. Neglect the mass of the beam.

Ans. $k_1=5.69\sqrt{\frac{EJg}{Ql^3}}$;
$k_2=22.04\sqrt{\frac{EJg}{Ql^3}}$;
$\frac{A_1^{(1)}}{A_2^{(1)}}=1$; $\frac{A_1^{(2)}}{A_2^{(2)}}=-1$.
The contours of the normal oscillations are shown in Fig. 512.

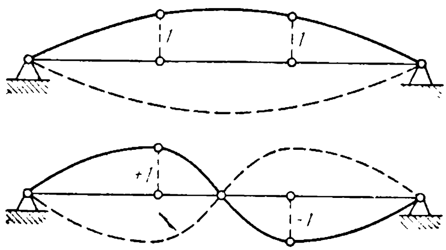


Fig. 512

819. A beam of length l resting on supports at its ends carries two loads $Q_1=Q$ and $Q_2=\frac{Q}{2}$ at distances $\frac{1}{3}l$ from the supports, respectively. Find the frequencies and contours of the normal transverse oscillations of the beam. Neglect the mass of the beam.

Ans. $k_1=6.55\sqrt{\frac{EJg}{Ql^3}}$ $\quad k_2=27.2\sqrt{\frac{EJg}{Ql^3}}$

$\frac{A_2^{(1)}}{A_1^{(1)}}=0.95$; $\quad \frac{A_2^{(2)}}{A_1^{(2)}}=-2.09.$

The contours of the normal transverse oscillations are shown in Fig. 513.

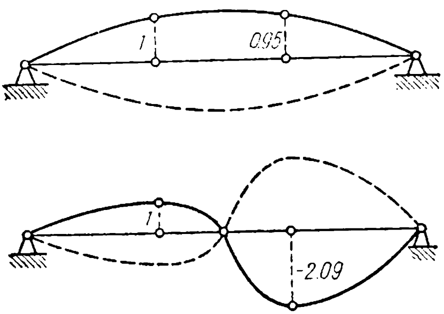


Fig. 513

820. A horizontal cantilever beam carries at its ends two identical loads each of weight Q applied at equal distances l from the beam supports (Fig. 514). The beam has a length $3l$ and it rests on two supports with distance l between them. The moment of inertia of the cross-sectional area of the beam is J, and Young's modulus of the beam material is E. Neglect the mass of the beam. Find the frequencies and contours of the normal transverse oscillations of the weights.

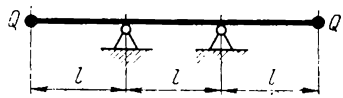


Fig. 514

Ans. $k_1=\sqrt{\frac{6}{5}\frac{EJ}{ml^3}}$ $\quad k_2=\sqrt{2\frac{EJ}{ml^3}}$.

821. A beam of length l has one end fixed while the other end A is attached to a homogeneous rectangular plate of mass m, as shown in Fig. 515. The system is in a horizontal plane and performs free oscillations in this plane about the position of equilibrium. Determine the frequencies and contours of these oscillations. The dimensions of the plate are following: $a=0.2l$, $b=0.1l$. Neglect the mass of the beam.

Hint. The force Q and the moment M, which must

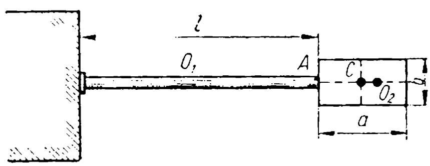


Fig. 515

be applied to the end A of the beam so as to produce a deflection f and a rotation φ of the tangential line to the curved axis of the beam, are defined by the formulae:

$$f=pQ+sM,\quad \varphi=sQ+qM,$$

and in this case:

$$p=\frac{l^3}{3EJ},\quad q=\frac{l}{EJ},\quad s=\frac{l^2}{2EJ}.$$

Ans. The frequencies equal $0.804\sqrt{\frac{3EJ}{ml^3}}$; $20.7\sqrt{\frac{3EJ}{ml^3}}$, respectively. The first normal oscillation can be considered as a rotation about the point O_1 located to the left of the point A on the axis of the beam at the distance $O_1A=0.612l$. The second normal oscillation is performed about the point O_2 located on the elongation of the axis of the beam at the distance $O_2A=0.106l$ to the right of the point A.

822. Two initially static disks, connected by an elastic shaft of stiffness c, are suddenly acted on by a constant torque of moment M_0. The moment of inertia of each disk is J. Neglecting the mass of the shaft, determine the subsequent motion of the system.

Ans. $\varphi_1=\frac{M}{4J}t^2+\frac{M}{4c}\left(1-\cos\sqrt{2\frac{c}{J}}t\right);$

$\varphi_2=\frac{M}{4J}t^2-\frac{M}{4c}\left(1-\cos\sqrt{2\frac{c}{J}}t\right).$

823. To absorb the torsional vibrations of a rotating mass system, a special pendulum is attached, as shown in Fig. 516. The sketch shows the outline of the system which consists of two masses I and II rotating with constant angular velocity ω. The second mass carries a pendulum of mass m. The moments of inertia of the masses about the axis of rotation are J_1 and J_2, respectively; the moment of inertia of the pendulum about the axis parallel to the axis of rotation of the system and passing through the centre of gravity

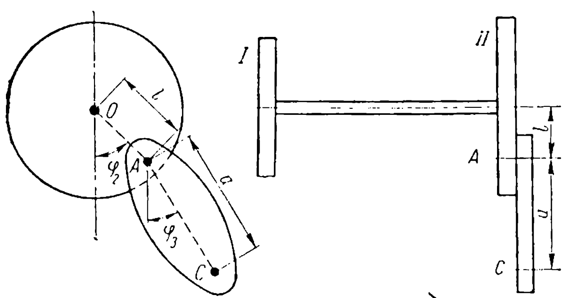


Fig. 516

of the pendulum is J_3. The distance between the axis of rotation of the system and the axis of suspension of the pendulum is $OA=l$, and the distance between the axis of suspension and the parallel one passing through the centre of gravity of the pendulum is $AC=a$. The coefficient of stiffness (rigidity against torsion) of the shaft section between masses is c_1. The second mass is acted on by an outside moment $M=M_0 \sin \omega t$. Derive the differential equations of motion for both the system and the pendulum.

Hint. When computing the potential energy of the system, neglect the potential energy of the pendulum due to gravity.

Ans. $J_1\ddot{\varphi}_1+c_1(\varphi_1-\varphi_2)=0;$

$(J_2+ml^2)\ddot{\varphi}_2+mal\ddot{\varphi}_3\cos(\varphi_2-\varphi_3)+mal\dot{\varphi}_3^2\sin(\varphi_2-\varphi_3)+$
$+c_1(\varphi_2-\varphi_1)=M_0\sin\omega t;$

$(J_3+ma^2)\ddot{\varphi}_3+mal\ddot{\varphi}_2\cos(\varphi_2-\varphi_3)-mal\dot{\varphi}_2^2\sin(\varphi_2-\varphi_3)=0.$

824. A reservoir of weight P in the shape of a cube rests with its bottom corners on four identical springs, as shown in Fig. 517 The side of the cube has length $2a$. The stiffness of each spring in the direction of the axes parallel to the sides of the cube is c_x, c_y and c_z. The moment of inertia of the cube about the principal central axes is J. Derive the equations of small oscillations and determine their frequencies in the case, when $c_x=c_y$.

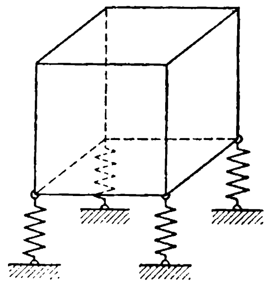


Fig. 517

Ans. $mx+c_x x-c_x a\varphi_2=0;$

$m\ddot{y}+c_y y+c_y a\varphi_1=0;$

$m\ddot{z}+c_z z=0;$

$J\ddot{\varphi}_1+c_y ay+c_y a^2\varphi_1+c_z a^2\varphi_1=0;$

$J\ddot{\varphi}_2+c_x a^2\varphi_2-c_x ax+c_z a^2\varphi_2=0;$

$J\ddot{\varphi}_3+c_x a^2\varphi_3+c_y a^2\varphi_3=0;$

where x, y and z are the coordinates of the centre of the cube, and φ_1, φ_2, φ_3 are the angles through which the cube turns with respect to the coordinate axes. If $c_x=c_y$, then

$$k_z=\sqrt{\frac{c_z g}{P}};\quad k_{\varphi 3}=\sqrt{\frac{2c_x a^2}{J}};$$

$$k^4-\frac{m(c_x+c_z)a^2+c_z J}{mJ}k^2+$$
$$+c_x c_z\frac{a^2}{mJ}=0.$$

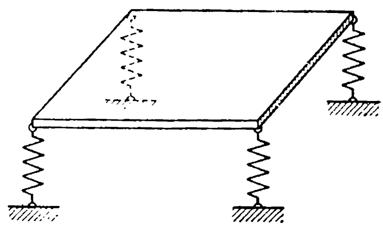


Fig. 518

825. A homogeneous horizontal rectangular lamina with sides

a and b rests with its corners on four identical springs of stiffness c. The mass of the lamina is M. Determine the frequencies of free oscillations. (See Fig. 518.)

Ans. $k_1=\sqrt{4\frac{c}{M}}$; $k_2=k_3=\sqrt{12\frac{c}{M}}$.

826. Three railway cars are coupled together in the way, shown in Fig. 519. The stiffness of each couple is c_1 and c_2. The cars weigh

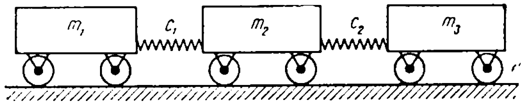


Fig. 519

Q_1, Q_2, and Q_3. At the initial moment two cars are in equilibrium while the third one, extreme right, is moved a distance x_0 from the position of equilibrium. Find the frequencies of normal modes of oscillations of the system.

Ans. $k_1=0$, while k_2 and k_3 are the roots of the equation:

$$k^4-g\left[\frac{c_1}{Q_1}+\frac{c_1+c_2}{Q}+\frac{c_2}{Q_3}\right]k^2+g^2\left[\frac{c_1c_2}{Q_1Q_2}+\frac{c_2c_1}{Q_2Q_3}+\frac{c_1c_2}{Q_3Q_1}\right]=0.$$

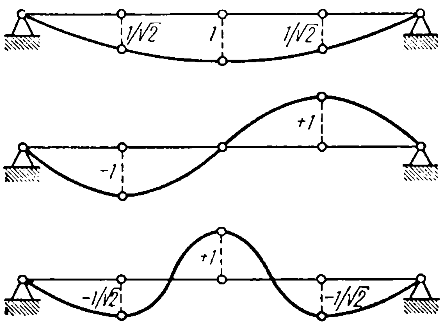


Fig. 520

827 Find the frequencies and contours of the normal modes of oscillations of the system consisting of three identical masses m fixed on a beam equally distant from themselves and from the supports (Fig. 520). The beam is assumed to rest freely on the supports, its length is l, the moment of inertia of the cross-sectional area is J, and Young's modulus for the beam material is E.

Ans. $k_1=4.93\sqrt{\frac{EJ}{ml^3}}$, $k_2=19.6\sqrt{\frac{EJ}{ml^3}}$, $k_3=41.8\sqrt{\frac{EJ}{ml^3}}$

The contours of the normal modes of oscillations are shown in the sketch.

50. Stability of Motion

828. A compound pendulum, consisting of two weightless bars of length l and two particles of mass m, is suspended from a

horizontal axis which rotates with constant angular velocity ω about a vertical axle z. Describe the stability of the vertical equilibrium of the pendulum.

Ans. At $\frac{g}{l\omega^2} > 1 + \sqrt{\frac{1}{2}}$ the vertical equilibrium of the pendulum is stable.

829. A small heavy ball rests inside the surface of a smooth pipe bent in the shape of an ellipse $\frac{x^2}{a^2} + \frac{z^2}{c^2} = 1$, which rotates about a vertical axis Oz with constant angular velocity ω (the axis Oz is directed downwards). Determine the positions of relative equilibrium of the ball and describe the stability of these positions.

Ans. For $\omega^2 \leqslant \frac{gc}{a^2}$ there are two positions of equilibrium:

(a) $x=0$, $z=c$ (stable), (b) $x=0$, $z=-c$ (unstable).

For $\omega^2 > \frac{gc}{a^2}$ there are three positions of equilibrium:

(a) $x=0$, $z=+c$ (unstable); (b) $x=0$, $z=-c$ (unstable);

(c) $z = \frac{gc^2}{\omega^2 a^2}$ (stable).

830. A particle is free to move along a smooth plane curve, rotating about a vertical axis with angular velocity ω. Its potential energy $V(s)$ is already given, and it depends only on the position of the particle defined by an arc s, calculated along the curve; $r(s)$ is the distance between the particle and the axis of rotation. Determine the frequency of small oscillations of the particle about its position of relative equilibrium.

Ans. $k^2 = \frac{1}{m}\left(\frac{d^2V}{ds^2} - \frac{d}{ds}\left[mr\frac{dr}{ds}\right]\omega^2\right)_{s=s_0}$ where s_0 is defined by the equation:

$$\left(\frac{dV}{ds}\right)_{s=s_0} = \omega^2 \left(mr\frac{dr}{ds}\right)_{s=s_0}$$

831. A particle of mass m describes a circumference of a circle of radius r_0 under the action of a central force of attraction proportional to the n power of the distance: $F=ar^n$ The particle is slightly perturbed. For what values of n is the motion stable?

Ans. At $n<-3$ the motion is unstable, while at $n>-3$ the motion is stable.

832. A thin homogeneous circular disk of radius r and weight Q has a vertical axle of symmetry which is free to oscillate about a point A (Fig. 521). At B the axle is held by two springs whose axes are horizontal and mutually perpendicular. The stiffnesses

of the springs are c_1 and c_2, respectively, and $c_2 > c_1$. The springs are fixed to the axle of the disk at a distance L from the bottom support while the distance between the disk and the bottom support is l. Determine the angular velocity ω of the disk, when the motion is stable.

Ans. For $Ql < c_1L^2$ the system is stable for any angular velocity; for $Ql > c_2L^2$ the system is stable, if $\omega > \omega^*$, where

$$\omega^* = \frac{\sqrt{gl(r^2+4l^2)}}{r^2}\left\{\sqrt{1-\frac{c_1L^2}{Ql}}+\sqrt{1-\frac{c_2L^2}{Ql}}\right\}.$$

For $c_1L^2 < Ql < c_2L^2$ the system is unstable for any angular velocity.

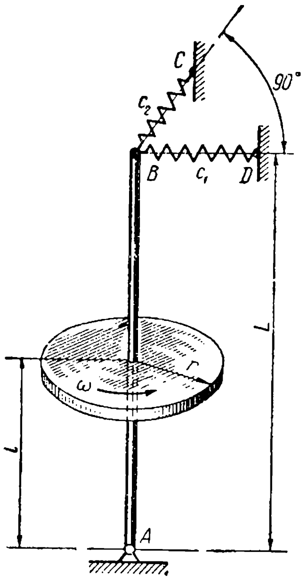


Fig. 521

833. A material particle is constrained to move along a smooth surface of the torus defined by the following parametrical equations: $x = \rho\cos\psi$; $y = \rho\sin\psi$; $z = b\sin\vartheta$; $\rho = a + b\cos\vartheta$. (Axis z is directed vertically upwards. Fig. 522.) Find all possible motions of the particle having the angle ϑ constant and describe their stability.

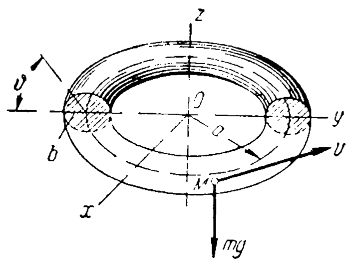


Fig. 522

Ans. The values $\vartheta = \vartheta_i = \text{const}$ are to be found from the equation $(1 + \alpha\cos\vartheta_i) = -\beta\cot\vartheta_i$, where $\alpha = \frac{b}{a}$; $\beta = \frac{g}{a\omega^2}$; $\dot{\psi} = \omega = \text{const}$. This equation provides two different solutions:

$$-\frac{\pi}{2} < \vartheta_1 < 0, \ \frac{\pi}{2} < \vartheta_2 < \pi.$$

In the first case the motion is stable, in the second case it is unstable.

834. Describe the stability of motion of a hoop which rolls uniformly with angular velocity ω along a horizontal plane. The plane of the hoop is vertical and its radius is a.

Ans. The motion is stable, if $\omega^2 > \frac{g}{4a}$.

835. A particle of mass m, disturbed from its position of equilibrium, is acted on by a force F_r which is proportional to the deviation $OM = r = \sqrt{x^2+y^2}$ from its original position and is direc-

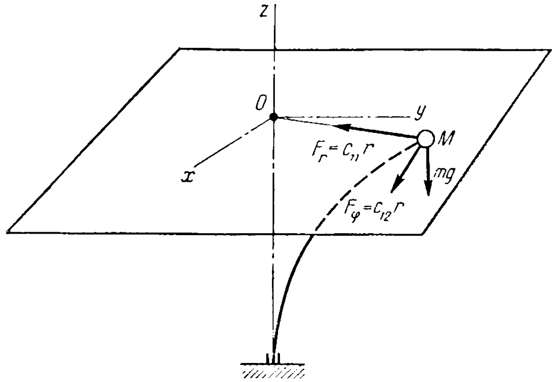


Fig. 523

ted to it, and a force F_φ which is perpendicular to the first (side force) and is proportional to the deviation r: $|F_r| = c_{11} r$, $F_\varphi = c_{12} r$.

Describe the stability of equilibrium of the particle using the method of small oscillations. (See Fig. 523.)

Hint. Consider particle as one end of a rod, whose other end is firmly fixed. The rod stands upright in the equilibrium position but here is under stress and twisting forces. The coefficients c_{11} and c_{12} depend on the force of stress, the moment of the twisting forces, the length of the rod and the stiffness against stress and twisting.

Ans. The equilibrium is unstable.

836. The motion of the sleeve of a centrifugal governor of a steam machine is defined by the equation:

$m\ddot{x} + \beta\dot{x} + cx = A(\omega - \omega_0)$, where x is the displacement of the sleeve of the governor, m is the inertial coefficient of the system, β is the coefficient of resistance, c is the stiffness of the governor springs, ω is the instantaneous and ω_0 is the average angular velocity of the machine, A is a constant. Motions of the machine are defined by the equation $J\,\frac{d\omega}{dt} = -Bx$ (B is a constant, J is the

reduced moment of inertia of rotating parts of the machine). Determine the conditions of stability of the system which consists of the engine and the governor.

Ans. The system is stable at $0<AB<J\frac{c\beta}{m}$ (c, β and J are assumed to be positive).

837. A symmetrical top, with its peak located in a fixed seat, is spinning about its vertical axle. It carries another symmetrical top also spinning about a vertical axle. The peak of the second top rests in a seat on the axle of the first top. M and M' are the masses of the upper and bottom tops, C and C' are their moments of inertia about the axis of symmetry, A and A' are the moments of inertia about the horizontal axes passing through the peaks, c and c' are the distances between the centres of gravity of the tops and their respective peaks, h is a distance between the peaks, and Ω and Ω' are the angular velocities of the tops. Determine the position of equilibrium of the system.

Ans. The system is in equilibrium when all roots of the equations in the fourth power are different and real:

$$[AA'+Mh^2(A-Mc^2)]\lambda^4+[A'C'\Omega'+C\Omega(A'+Mh^2)]\lambda^3+$$
$$+[A(M'c'+Mh)g+(A'+Mh^2)Mcg+CC'\Omega\Omega']\lambda^2+$$
$$+[C\Omega(M'c'+Mh)g+C'\Omega'Mcg]\lambda+Mc(M'c'+Mh)g^2=0.$$

SOLUTIONS OF SOME TYPICAL PROBLEMS

63(3)*. First determine the reaction of the beam at points of contact of the crane wheels D_1 and E_1 (Fig. 524). The resultant moments of all the forces applied to the crane about the points D_1 and E_1 are $\Sigma m(D_1)$ and $\Sigma m(E_1)$, which are zero if the crane is in equilibrium.

$$\Sigma m(D_1)=R_2 D_1 E_1 - P(D_1 C + KL) - QDC = 0. \tag{1}$$

$$\Sigma m(E_1) = -R_1 D_1 E_1 + QCE_1 - P(KL - CE_1) = 0. \tag{2}$$

From (1) and (2) we obtain $R_1=1000$ kgf, $R_2=5000$ kgf. Then we may consider the equilibrium of both parts AC and CB of the beam separately by replacing the reactions of the supports and the hinge at points A, B and C by the reaction force (Fig. 524, *a* and *b*).

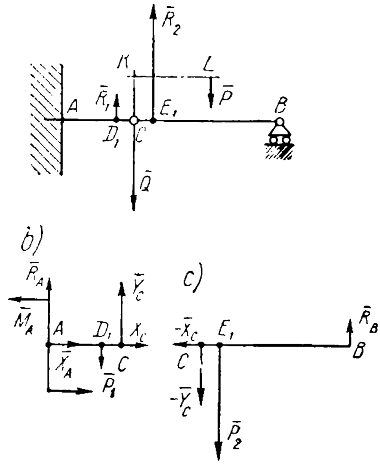


Fig. 524

Forces $\overline{P}_1=-\overline{R}_1$ and $\overline{P}_2=-\overline{R}_2$ are applied to points D_1 and E_1, respectively. As the end A of the beam is built into the wall, A is acted upon by the components of the reaction force R_A and X_A and by a couple M_A. A point C is under the action of the components of the reaction force X_C and Y_C. Applying the principle of equality of action and reaction, we see that the moduli of the components of reaction of the hinge C applied to the part AC are equal and lie in the opposite direction to the respective components of reaction of the hinge C applied to the part CB.

As all the external forces applied to the beam are perpendicular to its axis, then $X_A=X_C=0$.

We find from the condition of equilibrium of the part CB that the resultant moment of the applied forces and the reactions of the supports $\Sigma m(C)$ about the point C equal zero.

$$\Sigma m(C) = P_2 CE_1 - R_B CB = 0,$$

or $R_B=575$ kgf.

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* The first number denotes the number of a Problem, and the second the number of a Paragraph.

From the condition of equilibrium we also have

$$Y_C+P_2-R_B=0.$$

Substituting the values of P_2 and R_B in (2), we obtain

$$Y_C=-4375 \text{ kgf}.$$

Considering the equilibrium of the part AC, we find

$$\Sigma m(C)=-P_1CD_1+M_A-R_ACA=0; \tag{3}$$

$$R_A+Y_C-P_1=0. \tag{4}$$

Taking (3) and (4), we obtain the required magnitudes R_A and M_A. Finally, it should be noted that this problem can be solved easier by applying the method of virtual displacements.

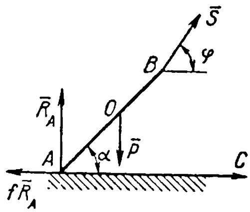


Fig. 525

100(4). The weight $\bar{P}$ is applied at the centre O of the beam. Let us replace the actions of the rope and the floor by the reaction forces S and R_A (Fig. 525). From the equilibrium of the beam we see that the resultant projection of all the forces applied to the beam on the axis AC equals zero:

$$-fR_A+S\cos\varphi=0. \tag{1}$$

Now derive the equation of the resultant moment of the forces about the point O when the beam is at equilibrium

$$\Sigma m(O)=S\sin(\varphi-\alpha)\,\frac{l}{2}-R_A(1+f)\,\frac{l}{2\sqrt{2}}=0, \tag{2}$$

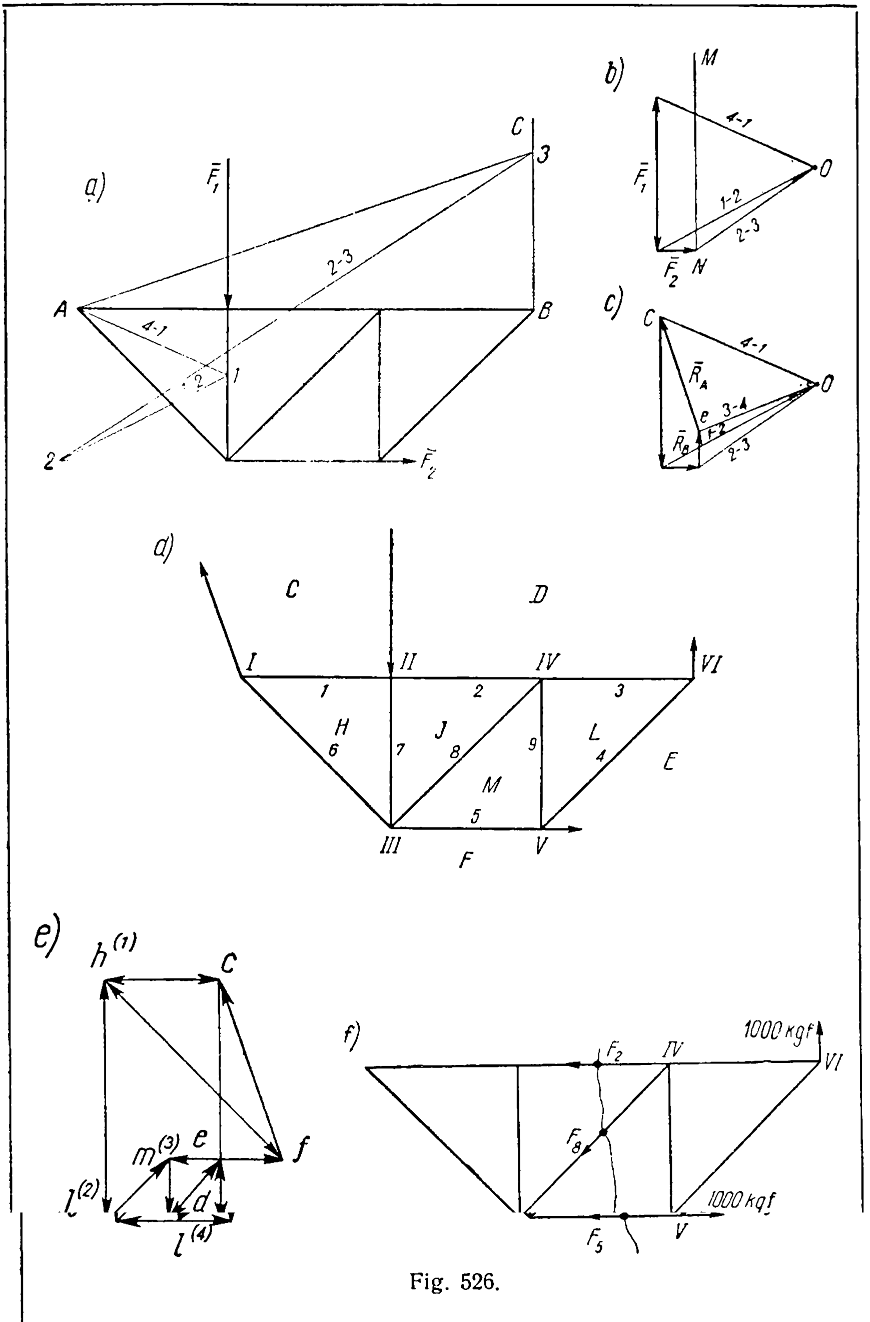
where $l=AB$ is the length of the beam.

Eliminating R_A and S from equations (1) and (2), we finally obtain, after some simple substitutions, the following expression:

$$\tan\varphi=2+\frac{1}{f}.$$

109(5). To determine the reactions of the supports, substitute the actions of the supports by reactions in their members. (See Fig. 526, *a-f*.) At B the reaction $\bar{R}_B$ of the smooth plane which supports the rollers is directed perpendicular to this plane. The modulus and the direction of the reaction $\bar{R}_A$ at A are unknown. From the point C (Fig. 526,*b*) draw a vector with magnitude equal to that of the force $\bar{F}_1$, from its end draw a vector $\bar{F}_2$, and from the end of this vector draw a straight line parallel to the vector $\bar{R}_B$.

Then take an arbitrary pole O and draw the following rays: *1-2* to the origin of the vector $\bar{F}_2$, *2-3* to the origin of the vector $\bar{R}_B$, and *4-1* to the origin of the vector $\bar{F}_1$. To determine the reactions



a)
C
3
$\bar{F}_1$
2-3
A
B
4-1
1
2
2
$\bar{F}_2$
b)
M
4-1
O
$\bar{F}_1$
1-2
2-3
$\bar{F}_2$
N
c)
C
4-1
$\bar{R}_A$
O
e
3-4
F-2
$\bar{R}_B$
2-3
d)
C
D
I
II
IV
VI
1
2
3
H
J
L
6
7
8
9
4
E
M
5
III
V
F
e)
$h^{(1)}$
c
$m^{(3)}$
e
f
$l^{(2)}$
d
$l^{(4)}$
f)
1000 kgf
F_2
IV
VI
F_8
1000 kgf
F_5
V

Fig. 526.

of the supports, draw a straight line parallel to the ray *4-1* through the point A to intersect the line of action of the force $\vec{F}_1$, and from there draw a line parallel to the ray *1-2* to intersect the line of action of the force $\vec{F}_2$ and, finally, draw a line from there parallel to the ray *2-3* to intersect the line of action of the force $\vec{R}_B$ (point *3*). The straight line between the point *3* and the point A of the polygon gives the direction of the ray *3-4*. We can now draw a ray *3-4* from O to intersect the line MN to determine the reactions $\vec{R}_A$ and $\vec{R}_B$ (Fig. 526, *c*) using the fact that the truss is in equilibrium, and so the end of the force $\vec{R}_A$ should coincide with the beginning of the force $\vec{F}_1$.

We may determine the tension in the truss members by the construction of the Maxwell-Cremona polygon of forces. Draw in Fig. 526, *d* all the given external forces acting on the truss by placing all the vectors outside the periphery of the truss. Denote the regions on either side of the forces and the periphery of the truss by the letters C, D, E, and F The regions between the members of the truss will be denoted by H, I, L, and M. The truss joints will be denoted by *I*, *II*,. ., *VI*. The members will be denoted by *1*, *2*, . . . , *9*. The forces should be denoted by lower-case letters corresponding to the labels of the adjacent regions taken in the clockwise direction. We thus obtain the polygon of external forces *fcde*, where the forces are drawn in the order in which they occur passing clockwise round the truss. (Thus, the force $\vec{R}_A$ is denoted by *fc*, $\vec{F}_1$ by *cd*, etc.)

To construct the force polygons for all the joints of the truss we start from a two-member joint *I*. The reaction of the support $\vec{R}_A$ (vector *fc*) and the reactions of the members *1* and *6* (*ch* and *hf*), directed along the members, are applied to the joint *I*. From the point *c* draw a line parallel to the member *1*, and from the point *f* draw a line parallel to the member *6*. Denote the intersection of these lines by the letter *h*. The force *cd* ($\vec{F}_1$), a known reaction of the member *hc*, and two unknown reactions of the members *dj* and *jh* are applied to the joint *II*. To construct the force polygon for this joint from points *d* and *h* draw lines parallel to the members *2* and 7 and their point of intersection denote by *j*. Let us now consider the joint *III*. Two unknown forces *jm* and *mf* are applied to it. From point *j* draw a line parallel to the member *8* and from point *f* a line parallel to the member *5*, thus determining the position of the point *m*, etc. (The force polygons for other joints are constructed similarly.)

Fig. 526, *e* represents a force diagram for the whole truss. In order to determine the tension in the members we shift the reactions on the truss members from the diagram. For example, let us shift a reaction *hc* on the member *1*. It is directed to the joint *I* and

so the member *1* is under compression. If the reaction is directed from the joint, that means that the member is stretched.

Determine the tensions in the members analytically.

Let us "cut out" the truss by members *2*, *8*, and *5* (Fig. 526, *f*). We find the left part of the truss in equilibrium, if the action of the members is replaced by forces $\overline{F}_2$, $\overline{F}_5$, $\overline{F}_8$. In order to find the moduli of these forces, derive the equation of moments about the points *IV* and *VI*.

$$\Sigma m(IV) = a - aF_5 + a = 0;$$

$$\Sigma m(VI) = F_8 \frac{a\sqrt{2}}{2} - aF_5 + a = 0. \qquad (1)$$

Hence, from the system of equations (1) we can determine the forces $\overline{F}_5$ and $\overline{F}_8$, and then, by projecting the forces on the direction of the force $\overline{F}_5$, we obtain

$$F_2 + F_8 \frac{\sqrt{2}}{2} + F_5 - 1 = 0. \qquad (2)$$

From (2) we know the magnitude of the force F_2.

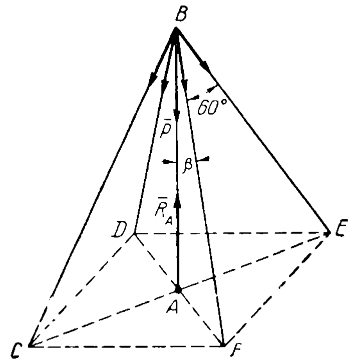


Fig. 527

120(6). The forces applied to the mast are shown in Fig. 527, where $P = 200$ kgf is the weight of the mast and $S = 100$ kgf is the tension in each guy. A modulus of the reaction $\overline{R}_A$ is the same as the pressure of the mast on the earth. Projecting the reaction of the support $\overline{R}_A$ and the force $\overline{S}$ on the direction of the axis of the mast AB, we obtain

$$-4S\cos\beta + R_A = 0,$$

$$\text{or } R_A = 4S\cos\beta; \qquad (1)$$

$$\cos\beta = \frac{AB}{BF}\cdot \qquad (2)$$

The triangle BFE is equilateral. $FE = BF$ As the angle $EAF = 90°$, then $AF = \frac{\sqrt{2}}{2} FE = \frac{\sqrt{2}}{2} BF = AB$.

Substituting the value AB in the equation (2), we obtain $\cos\beta = \frac{\sqrt{2}}{2}$, and from (1) we finally obtain the required value.

125(7). The point O is taken as the centre of reduction of the system of forces. Find the projections of the principal vector $\overline{V}$ on the coordinate axes, as shown in Fig. 110.

$$X = 0; \quad Y = P_2 = 12 \text{ kgf}; \quad Z = P_1 = 8 \text{ kgf}.$$

The modulus of the principal vector equals

$$V=\sqrt{Y^2+Z^2}=14.4 \text{ kgf}.$$

Now determine the principal moments m_x, m_y, m_z of the system of forces with respect to the axes x, y, z.

$$m_x=0; \quad m_y=0; \quad m_z=1.3P_2=15.6 \text{ kgfm}.$$

Compute the scalar quantity

$$\overline{V}\overline{m}_0=Xm_x+Ym_y+Zm_z=125.$$

$\overline{V}\overline{m}_0\neq 0$ and so the given system of forces is reduced to a wrench.

The magnitude of the principal moment M with respect to a point on the central axis of the wrench will be

$$M=\frac{\overline{V}\overline{m}_0}{V}=\frac{125}{144}=8.65 \text{ kgfm}.$$

Write, next, the equation of the axis of the wrench

$$\frac{m_x-yZ-zY}{X}=\frac{m_y-zX+xZ}{Y}=\frac{m_z-xY+yX}{Z} \tag{1}$$

Substituting the values in (1), we obtain

$$\frac{-8y+12z}{O}=\frac{8x}{12}=\frac{15.6-12x}{8}, \text{ or } x=0.9 \text{ m}; \tag{2}$$

$$-2y+3z=0. \tag{3}$$

It follows from (2), that the central axis of the wrench is located in the plane perpendicular to the axis x at an angle $\alpha=90°$ Hence, from (3) we obtain

$$\tan\beta=\frac{z}{y}=\frac{2}{3};$$

$$\tan\gamma=\cot\beta=\frac{3}{2}$$

147(8). The forces applied to the rod are shown in Fig. 528, *a*. Replace the action of the threads by the force denoted by $\overline{T}$ The couple M is also applied to the rod. The system is in equilibrium, so projecting the forces $\overline{T}$ and $\overline{P}$ on the direction OO_1 and considering the resultant moment of the forces with respect to the axis OO_1 we obtain

$$-P+2T\cos\beta=0; \tag{1}$$

$$-2Tr\sin\beta\cos\frac{\alpha}{2}+M=0. \tag{2}$$

Thus, $T\sin\beta\cos\frac{\alpha}{2}$ is the projection of the force $\overline{T}$ on the direction perpendicular to the axis of the rod AA_1 in the plane perpendicular

to the axis OO_1 (Fig. 528, *b*). Next, it follows from the triangle *ABC* that

$$\cos\beta=\frac{\sqrt{AB^2-BC^2}}{l}=\sqrt{1-\left(\frac{2r}{l}\sin\frac{\alpha}{2}\right)^2} \tag{3}$$

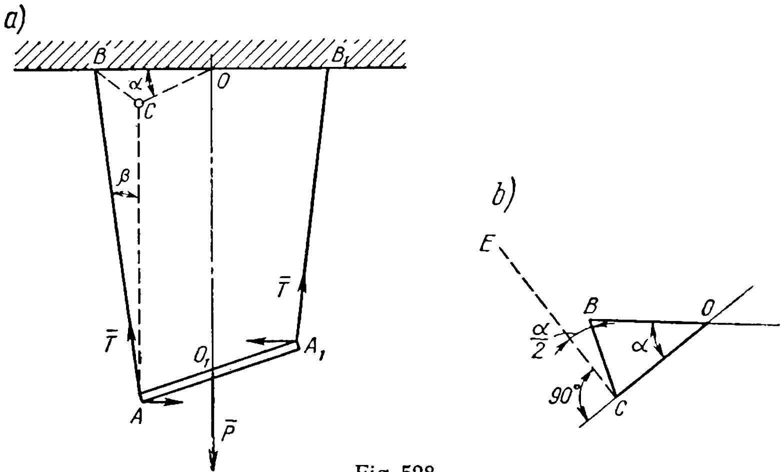


Fig. 528

From the equation (1) we find

$$T=\frac{P}{2\cos\beta}=\frac{lP}{2\sqrt{l^2-4r^2\sin^2\frac{\alpha}{2}}} \tag{4}$$

$\sin\beta=\frac{2r}{l}\sin\frac{\alpha}{2}$, hence, from (2) we obtain

$$M=\frac{2r^2}{l}T\sin\alpha. \tag{5}$$

And, finally, substituting (4) in (5), we get the required result.

157(9). Consider the equilibrium of the right half of the cylinder. Draw coordinate axes x and y in the directions, shown in Fig. 529. A force $\frac{\bar{Q}}{2}$ is applied to the centre of gravity of this half. This centre of gravity is located on the axis of symmetry Ox and thus its coordinate $y_c=0$. Then find the coordinate x_c of the centre of gravity of the cylinder. We know that if a plane area revolves about an axis in its plane not intersecting its

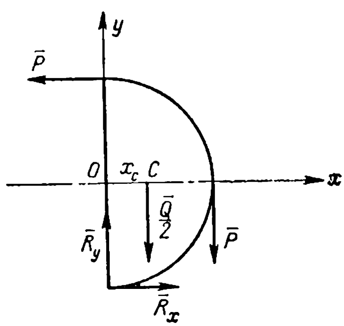


Fig. 529

outline the volume generated is equal to the area S multiplied by the length of the path $2\pi x_c$ of its centre of gravity (Guldin's theorem). Thus, the volume of the body of rotation generated by the cross-section of a half of the cylinder is equal to the volume of a ball of radius R.

Consequently, we obtain

$$\frac{4}{3}\pi R^3 = \frac{\pi R^2}{2} \times 2\pi x_c,$$

whence $x_c = \frac{4}{3}\frac{R}{\pi}$.

Substituting the threads between the two halves of the cylinder by the corresponding forces, we obtain the following equations for equilibrium of the semi-cylinder

$$\Sigma X = R_x - P = 0;$$

$$\Sigma Y = R_y - \frac{Q}{2} - P = 0;$$

$$\Sigma m(O) = -x_c \frac{Q}{2} + RR_x + RP - RP = 0. \qquad (1)$$

In this case R_x, R_y are the components of the reaction of the plane on which the cylinder rests. ΣX, ΣY are the resultants of the projections of the forces on the axes x and y; $\Sigma m(O)$ is the resultant moment of the forces with respect to the point O. Hence, solving the system of equations (1), we finally obtain

$$P = \frac{x_c Q}{2R} = \frac{2Q}{3\pi} \text{ kgf.}$$

163(10). First, derive the equation of motion of the particle M. Denote the angle $AOB = \varphi = \omega t$.

From the equilateral triangle OAB it follows

$$y = \frac{OA}{2} \sin\varphi; \quad x = (OA + AM) \cos\varphi. \qquad (1)$$

Substituting the values of the lengths of the segments OA and AM in the equations (1), we obtain $x = 120 \cos\omega t$, $y = 40 \sin\omega t$, or

$$\frac{x}{120} = \cos\omega t, \quad \frac{y}{40} = \sin\omega t. \qquad (2)$$

Squaring each equation and adding their left and right parts, we obtain the required equation of the path

$$\frac{x^2}{(120)^2} + \frac{y^2}{(40)^2} = 1.$$

Thus, the relation between the coordinate of the slider AB and the time is

$$x = OA \cos\varphi + AB \cos\varphi = 160 \cos\omega t.$$

172(11). First, derive the equation of motion of a point M on the rim of the wheel. Draw the coordinate axes in the way, shown in Fig. 530. Initially $t=0$, $x=-R$, and $y=0$, and the angular velocity of the wheel is $\omega=\frac{v_0}{R}$ In this case we have

$$x=v_0t-R\cos\omega t; \tag{1}$$

$$y=R\sin\omega t. \tag{2}$$

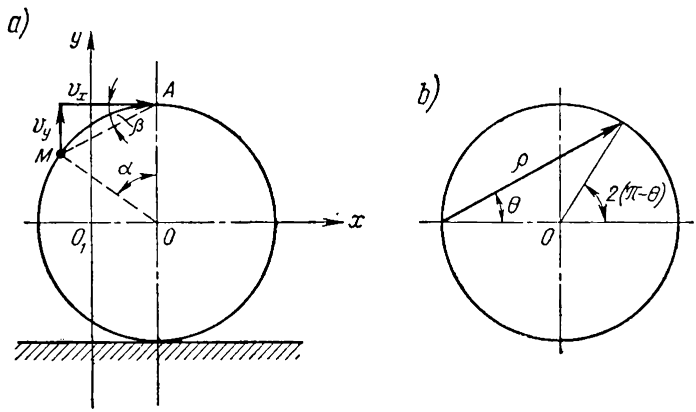


Fig. 530

The projections of the velocity of the point M on the axes x and y will be equal to:

$$v_x=\dot{x}=R\omega(1+\sin\omega t); \tag{3}$$

$$v_y=\dot{y}=R\omega\cos\omega t. \tag{4}$$

Thus, the velocity of the point M is given by the relation

$$v=\sqrt{\dot{x}^2+\dot{y}^2}=2R\omega\cos\frac{\alpha}{2}; \tag{5}$$

$$\alpha=\frac{\pi}{2}-\omega t.$$

Then, the angle β is found from the equation

$$\sin\beta=\frac{\dot{y}}{v}=\sin\frac{\alpha}{2};\ \beta=\frac{\alpha}{2} \tag{6}$$

Taking the triangle OMA and considering the equation (6), we conclude that the velocity $\overline{v}$ is directed along a straight line MA.

It should be noted that there is an easier way of finding the direction and the modulus of the velocity of the point by using the theorem on composition of velocities.

From the equation (5) we can find the equation of the hodograph. Thus, the modulus of the velocity v_1 of the point, tracing

the hodograph, equals $r\omega_1$, where ω_1 is the angular velocity of the point on the hodograph and $r=v_0$ is the radius of the hodograph (Fig. 530, b).

$$\omega_1=\frac{d}{dt}\,(\pi-2\theta)=\omega, \text{ where } \theta=\frac{\alpha}{2}$$

$$v_1=r\omega_1=v_0\omega=\frac{v_0^2}{R}$$

192(12). The coordinate axes are chosen, as shown in Fig. 531. Derive the equation of motion of the projectile

$$x=v_0\cos\alpha_0 t. \qquad (1)$$

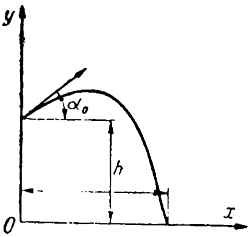


Fig. 531

Then, the vertical component of the velocity of the projectile as a function of time is $\frac{dy}{dt}=v_0\sin\alpha_0-gt$, where g is the acceleration of gravity.

$$y=h+\int_0^t \frac{dy}{dt}\,dt=v_0\sin\alpha_0 t-\frac{gt^2}{2}+h. \qquad (2)$$

The projectile will hit the target, when $y=0$. Equating (2) to zero and substituting the values $\sin\alpha_0=\cos\alpha_0=\frac{\sqrt{2}}{2}$, we obtain a quadratic equation in the time

$$t^2-\frac{v_0\sqrt{2}}{g}\,t-\frac{2h}{g}=0. \qquad (3)$$

It is evident that the time taken by the projectile to reach the target is defined by the positive root t_0 of the equation (3).

$$t_0=\frac{v_0\sqrt{2}}{2g}+\sqrt{\frac{v_0^2}{2g^2}+\frac{2h}{g}}. \qquad (4)$$

Hence, substituting the value of t_0 in (1), we obtain the required result.

205(13). The acceleration of the point on the rim of the wheel consists of a normal and tangential accelerations. The normal acceleration $\overline{w}_n$ is directed from the point on the rim of the wheel to its axis of rotation. Hence, the modulus of this acceleration is $w_n=\frac{v^2}{R}$, where v is the magnitude of the velocity of the point while R is the radius of the wheel.

The tangential acceleration $\overline{w}_\tau$ is directed along the tangent to the path of the point of the rim. Its modulus equals

$$w_\tau=w_0=\text{const}. \qquad (1)$$

Thus, the modulus of the velocity of the point equals $v=w_0t$, whence

$$w_n=\frac{w_0^2t^2}{R}. \tag{2}$$

Then, express w_n as a function of h

$$h=\frac{w_0t^2}{2}. \tag{3}$$

Multiplying (3) by $2w_0$, we obtain

$$w_0^2t^2=2w_0h,$$

whence

$$w_n=\frac{2w_0h}{R}.$$

And, finally, the modulus of the total acceleration is

$$w=\sqrt{w_n^2+w_\tau^2}=\frac{w_0}{R}\sqrt{R^2+4h^2}.$$

213(14). The velocities of the points of contact of the wheels A and B are equal. Denote the modulus of these velocities by the letter v. Then $v=\omega_I r=\omega_{II}d$, where ω_I is the angular velocity of the shaft I, and ω_{II} is the angular velocity of the shaft II. Hence,

$$\omega_{II}=\frac{\omega_I r}{d}; \tag{1}$$

$$\varepsilon=\frac{d\omega_{II}}{dt}=-\frac{\omega_I r}{d^2}\dot{d}=\frac{0.5\omega_I r}{d^2}=\frac{50\pi}{d^2}. \tag{2}$$

If $d=r$, then the magnitude of the angular velocity of the wheel B equals $\omega_{II}=\omega_I=20\pi$. Therefore, the normal acceleration of the point on the rim of the wheel will be given by $\omega^2{}_I R=\omega^2{}_{II}R$, and the tangential by $w_\tau=R\varepsilon$.

Thus, the modulus of the total acceleration is given by the equation

$$w=\sqrt{w_\tau^2+w_n^2} \quad \text{where } w_n=\omega_I{}^2R.$$

221(15). First, draw the axis ξ of the movable system of coordinates in the direction, shown in Fig. 532. The point O_1 is taken as the origin. Then the dependence of $\xi=O_1A$ on time will define the equation of relative motion of the block in the slot of the rocker. From the triangle O_1OA we obtain

$$\xi=\sqrt{a^2+r^2+2ar\cos\omega t}.$$

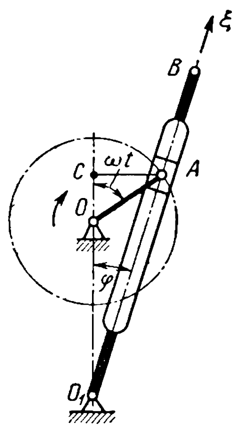


Fig. 532

The equation of rotation of the rocker is defined by the time dependence of the angle φ. From the triangle ACO_1 we have

$$\tan\varphi=\frac{AC}{O_1C}=\frac{r\sin\omega t}{r\cos\omega t+a}$$

232(16). We know that the angular velocity ω_E of the gear E equals $\omega_E=\omega\frac{R}{R_1}=2\ \text{sec}^{-1}$.

Draw the axis of the movable coordinate system along the line AB. Then we have

$$\bar{v}_A=\bar{v}_e+\bar{v}_r=\bar{\omega}_e\times\bar{r}+\bar{v}_r, \tag{1}$$

where $\bar{v}_A$, $\bar{v}_e$ and $\bar{v}_r$ are the absolute velocity, velocity of the coordinate system, and relative velocity, respectively, of the block A while ω_e is the angular velocity of the rocker. In the positions II and IV we have $\bar{v}_A=\bar{v}_r$ and, therefore, $\omega_e=0$.

In the positions I and III $v_r=0$, and thus, the value r equals O_1B+O_1A and O_1B-O_1A, respectively. So, from (1) we obtain

$$\omega_I=\frac{v_A}{O_1B+O_1A}=\frac{\omega_E O_1A}{O_1B+O_1A}=0.6\ \text{sec}^{-1};$$

$$\omega_{III}=\frac{v_A}{O_1B-O_1A}=1.5\ \text{sec}^{-1}.$$

241(17). Let us take the horizontal oscillations of the motor as the motion of the system of axes (transport motion). Draw the axis x parallel to the foundation of the motor. In case of translatory motion of the system of axes the absolute acceleration $\bar{w}_A$ is defined by the equation

$$\bar{w}_A=\bar{w}_e+\bar{w}_r, \tag{1}$$

where $\bar{w}_A$ is the absolute acceleration, $\bar{w}_e$ is the acceleration of the coordinate system, and $\bar{w}_r$ is the relative acceleration of the point.

Whence, the moduli of the components of the acceleration of the point A are

$$w_e=\ddot{x}=-a\omega^2\sin\omega t; \tag{2}$$

$$w_r=\omega^2 l. \tag{3}$$

As at time $t=\frac{\pi}{2\omega}$ the vector $\bar{w}_e$ is perpendicular to $\bar{w}_r$ and $w_e=-a\omega^2$, then

$$w_A=\sqrt{w_e^2+w_r^2}=\omega^2\sqrt{a^2+l^2}.$$

253(18). The connection between the accelerations of a point having absolute acceleration $\bar{w}_{a}$, relative acceleration $\bar{w}_r$, and acceleration of the coordinate system $\bar{w}_e$ is determined from the Coriolis theorem:

$$\overline{w}_a=\overline{w}_e+\overline{w}_r+\overline{w}_c; \quad (1)$$

$$\overline{w}_c=2\overline{\omega}_e\times\overline{v}_r, \quad (2)$$

where $\overline{\omega}_e$ is the angular velocity of the motion of the coordinate system of the point, and $\overline{v}_r$ is its velocity in relative motion.

Now, let us connect the movable system of coordinates to the disk. In this case the coordinate ξ of the point M in the relative motion will be equal to OM, i. e., $\xi=OM=4t^2$. Hence, the relative velocity and acceleration of the point M becomes

$$v_r=\frac{d\xi}{dt}=8t, \ w_r=\frac{d^2\xi}{dt^2}=8;$$

$\overline{v}_r$ and $\overline{w}_r$ are directed along OM. Thus, the angular velocity of the coordinate system will be equal to the angular velocity of the disk with respect to the axis O_1O_2.

$$w_c=2\omega v_r\sin(\overline{\omega}, \ \overline{v}_r)=16\sqrt{3}t^2.$$

Whence, the vector $\overline{w}_c$ is perpendicular to the plane of the disk.

Let us determine the components $w_{e\tau}$ and w_{en} of the acceleration of the coordinate system w_e.

$$w_{e\tau}=\xi ME=4\sqrt{3}t^2, \ \ w_{en}=\omega^2 ME=8\sqrt{3}t^4.$$

We note that the component $w_{e\tau}$ is perpendicular to the plane of the disk being directed to the side of rotation while w_{en} is directed along ME.

And, consequently, we can find the modulus w of the absolute acceleration of the point M (Fig. 533).

$$w^2=(w_{e\tau}+w_c)^2+(w_{en}\sin 60^\circ-w_r)^2+w^2_{en}\cos^2 60^\circ$$

When $t=1$ sec, we have $w=35.56$ cm/sec^2.

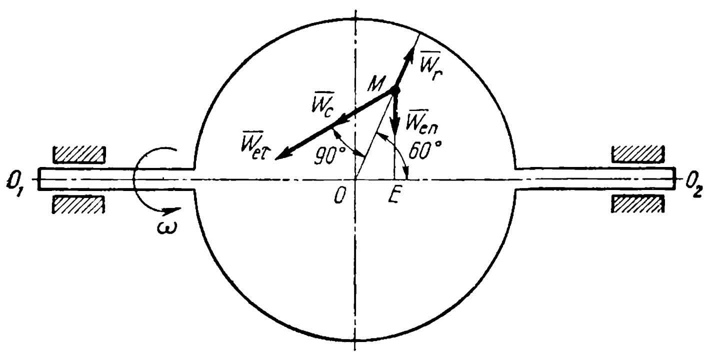


Fig. 533

262(19). The equations of motion of the point M are:

$$x = x_{0_2} + x_1 \cos\psi - y_1 \sin\psi; \qquad (1)$$
$$y = y_{0_2} + x_1 \sin\psi + y_1 \cos\psi,$$

where x_{0_2} and y_{0_2} are the coordinates of the pole O_2; x_1, y_1 are the coordinates of the point in the system of axes which is fixed to the body, ψ is the angle of turn of the movable system of coordinates.

The axes of the fixed system of coordinates are shown in Fig. 189.

Let us consider the point O_2 as pole. Take the axis y_1 along the straight line O_2M and the axis x_1 perpendicular to the axis y_1. In this particular case the coordinates of the point M in the movable system will be $x_1=0$; $y_1=r_2$.

Since $O_1O_2=r_1-r_2$, the coordinates of the pole are defined by the equations

$$x_{0_2} = (r_1-r_2)\sin\varphi; \; y_{0_2} = (r_1-r_2)\cos\varphi. \qquad (2)$$

And now find the angle of turn of the movable system of coordinates ψ as a function of angle φ.

$$\varphi = 9\pi t; \; \psi = \varphi\frac{r_2}{r_1} - \varphi = 6\pi t. \qquad (3)$$

Substituting the values of x_{0_2}, y_{0_2}, x_1, y_1 and ψ from (2) and (3) into (1), we finally obtain the equations of motion of the point M.

287(20). The magnitude of the velocity of the point A is

$$v_A = \omega_0 OA = \omega_0(r_1+r_2). \qquad (1)$$

And now, let us determine the velocity of the point B (Fig. 534). The instantaneous centre of velocity of the gear of radius r_2

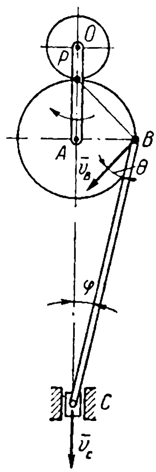


Fig. 534

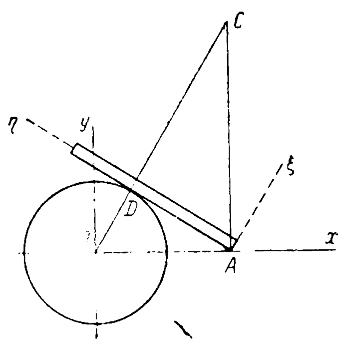


Fig. 535

is the point P of contact of the gears. The modulus of its angular velocity is

$$\omega = \frac{v_A}{AP} = \omega_0 \frac{r_1 + r_2}{r_2}, \tag{2}$$

and, hence, the modulus of the velocity v_B is given by

$$v_B = \omega PB = \omega r_2 \sqrt{2} = \omega_0 \sqrt{2} (r_1 + r_2). \tag{3}$$

The projections of the velocities of both points on a line, parallel to the one connecting these points, are equal. Taking into consideration that the velocity $\bar{v}_C$ is directed along AC, we have

$$v_B \cos \theta = v_C \cos \varphi; \quad v_C = v_B \frac{\cos \theta}{\cos \varphi}.$$

Hence, it follows from the triangles ABC and ABP, that $\theta = 45° - \varphi$, therefore,

$$v_C = \frac{v_B \sqrt{2}}{2} (1 + \tan \varphi);$$

$$\tan \varphi = \frac{AB}{AC} = 0.2; \quad v_C = 18 \text{ cm/sec}.$$

Thus, the angular velocity of the connecting rod is given by the equation

$$\omega_{\text{con.rod}} = \frac{v_B \sin \theta + v_C \sin \varphi}{BC} = 0.15 \text{ sec}^{-1}.$$

297(21). The coordinate systems of the space and body centrodes are shown in Fig. 535. The velocity $\bar{v}_C$ of the point D is directed along the line AD, and the velocity $\bar{v}_A$ of the point A is directed along the axis x.

The instantaneous centre of velocities of the rod lies at the point of intersection of the perpendiculars OD and AC to the velocities $\bar{v}_C$ and $\bar{v}_A$. Taking the triangle OCA, we have

$$x_C^2 + y_C^2 = (a + CD)^2. \tag{1}$$

Then, from the triangles ACO and ACD we obtain

$$AD^2 = x_C^2 - a^2; \quad CD^2 = -AD^2 + AC^2 = y_C^2 - x_C^2 + a^2. \tag{2}$$

Thus, eliminating CD from (1) and (2), we find the equation of the space centrode $x_C^2 \ (x_C^2 - a^2) = a^2 y_C^2$.

Now, we may find the equation for the body centrode. Using the equality of the triangles ADO and CDA the expression becomes

$$\frac{\xi_C}{\eta_C} = \frac{\eta_C}{a}, \text{ or } \eta_C^2 = a\xi_C.$$

310(22). The velocities of the points A and B of the link AB are parallel to the line OB (Fig. 536). Therefore, at this moment the link performs translatory motion. Hence, the magnitude of the velocity of any point on the link AB equals $v=\omega r=200$ cm/sec, whence, the tangent w_{At} and normal w_{An}, being the components of acceleration of the point A, equal

$$w_{At}=\varepsilon r, \quad w_{An}=\omega_0^2 r. \tag{1}$$

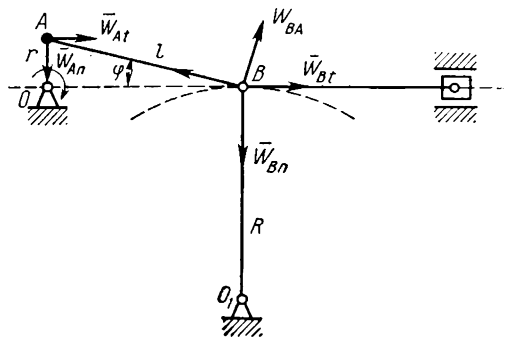


Fig. 536

Taking the point A as a pole, we find the acceleration of the point B from the equation

$$\bar{w}_B=\bar{w}_A+\bar{w}_{BA}^c+\bar{w}_{BA}^r, \tag{2}$$

where w_A is the acceleration of the pole A, $\bar{w}_{BA}^c$ is a centripetal acceleration with respect to the pole, and $\bar{w}_{BA}^r$ is the rotational acceleration about the pole. Because the link has an instantaneous translatory motion,

$$\bar{w}_{BA}^c=0, \quad w_{Bn}=\frac{v_B^2}{R}, \tag{3}$$

where $R=O_1B$. Then, by projecting (2) on the direction O_1B we obtain

$$w_{An}-w_{BA}^r\cos\varphi=w_{Bn},$$

whence

$$w_{BA}^r=\frac{w_{An}-w_{Bn}}{\cos\varphi}. \tag{4}$$

Substituting the values w_{An} and w_{Bn} from (1) and (3) in (4), we may define the value w_{BA}^r, and finally the tangential acceleration of the point B is defined by the equation:

$$w_{Bt}=w_{At}+w_{AB}^r\sin\varphi=\varepsilon r+\frac{w_{An}-w_{Bn}}{\cos\varphi}\sin\varphi.$$

318(23). First, determine the angular velocities of gear *I*, gear *II*, and gear *III*. For the pair of gears *I* and *II* we have the equation

$$\frac{\omega_{1z}-\Omega_z}{\omega_{2z}-\Omega_z}=-\frac{r_2}{r_1}, \tag{1}$$

and for the pair of gears *II* and *III*

$$\frac{\omega_{2z}-\Omega_z}{\omega_{3z}-\Omega_z}=-\frac{r_3}{r_2}, \tag{2}$$

where $\Omega_z=-\omega_0$ is the projection of the angular velocity of the crank on the axis perpendicular to the plane of the gears, and $\omega_{1z}=\omega_0$, ω_{2z} and ω_{3z} are the projections of the angular velocities of the first, second and third gears on the same axis. The radii of the gears are $r_1=R$, $r_2=\frac{R}{2}$, and $r_3=R$, respectively.

Multiplying (1) by (2) we obtain

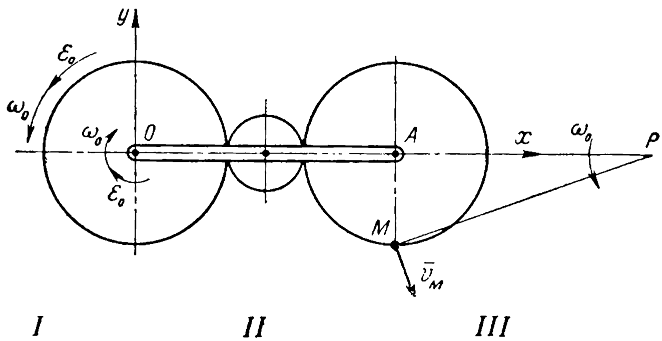
$$\frac{\omega_{1z}-\Omega_z}{\omega_{3z}-\Omega_z}=\frac{r_3}{r_1}, \tag{3}$$

whence from (3) we have $\omega_{3z}=\omega_0$.

Now, let us determine the velocity of the point M on the third gear. For this we find the velocity of the centre A of the third gear. We see that this point lies also on the crank, therefore, $v_A=\omega_0 3R$. On the other hand, $v_A=\omega_{3z}PA$, where PA is the segment which connects the instantaneous centre of velocities P with the centre of the gear. As $\omega_{3z}=\omega_0$, then $PA=3R$. Remembering the direction of the velocity v_A, we may determine the position of the instantaneous centre of velocities P on the straight line OA.

$$v_M=PM\omega_0=\omega_0 R\sqrt{10}.$$

Draw the axis x in the direction of OA and the axis y perpendicular to it, as shown in Fig. 537. Take the point A as a pole. Then the projections of the acceleration of the point A on the axes x



I *II* *III*

Fig. 537

and y will be

$$w_{Ax} = -3\omega_0^2 R, \quad w_{Ay} = -3\omega_0^2 R.$$

Taking the equation of composition of accelerations, we may define the acceleration of the point M.

$$\overline{w}_M = \overline{w}_A + \overline{w}_{MA}^r + \overline{w}_{MA}^c .$$

where $\overline{w}_{MA}^r$ is the rotational acceleration of the point M with respect to the point A, and $\overline{w}_{MA}^c$ is its centripetal acceleration with respect to this point. Thus, the moduli of the angular velocity and acceleration of the gear *III* with respect to its centre A are easily computed and equal ω_0 and ε_0, respectively. In this case $w_{MA}^c = \omega_0^2 R$, and the vector $\overline{w}_{MA}^c$ is directed along the axis y; $w_{MA}^r = \varepsilon_0 R$. The vector $\overline{w}_{MA}^r$ is directed along the axis x, and so the components of the total acceleration of the point M are

$$w_{M_x} = -3\omega_0^2 R + \varepsilon R, \quad w_{M_y} = -3\varepsilon R + \omega_0^2 R.$$

325(24). The projections of the angular velocity on the fixed coordinate axes ω_x, ω_y, and ω_z are coonnected with Euler's angles by the equations

$$\omega_x = \dot\varphi \sin\psi \sin\theta + \dot\theta \cos\psi;$$

$$\omega_y = -\dot\varphi \cos\psi \sin\theta + \dot\theta \sin\psi;$$

$$\omega_z = \varphi \cos\theta + \dot\psi. \tag{1}$$

Substituting the derivatives of angles $\varphi = n$, $\dot\psi = an$, $\dot\theta = 0$ in (1), we obtain

$$\omega_x = \frac{\sqrt{3}}{2} n \cos ant;$$

$$\omega_y = \frac{\sqrt{3}}{2} n \sin ant;$$

$$\omega_z = n\left(\frac{1}{2} + a\right).$$

Then, the equation of the instantaneous axis in the fixed coordinate system is

$$\frac{x}{\omega_x} = \frac{y}{\omega_y} = \frac{z}{\omega_z}; \quad \frac{x}{\frac{\sqrt{3}}{2} n \cos ant} = \frac{y}{\frac{\sqrt{3}}{2} n \sin ant} = \frac{z}{n\left(\frac{1}{2} + a\right)}. \tag{2}$$

Let us eliminate the time from (2). Thus, from the first equation we have $\tan ant = \frac{y}{x}$ Therefore, $\frac{1}{\cos^2 ant} = \frac{x^2 + y^2}{x^2}$; $\frac{1}{\sin^2 ant} = \frac{x^2 + y^2}{y^2}$.

Squaring the equation (2), we finally obtain the equation of the fixed axoid

$$\frac{4}{3}(x^2+y^2)=\frac{z^2}{n^2\left(\frac{1}{2}+a\right)^2}. \tag{3}$$

When $a=-\frac{1}{2}$, the equation (3) is $z=0$, i. e. the fixed axoid will be the plane Oxy.

Thus, the projections of the angular acceleration on the fixed axes are easily determined by applying such formulae as

$$\varepsilon_x=\dot{\omega}_x,\quad \varepsilon_y=\dot{\omega}_y,\quad \varepsilon_z=\dot{\omega}_z.$$

336(25). Let us take three systems of rectangular coordinates, the first, x'', y'', z'', is fixed with respect to the body of the car, the second, x', y', z', is fixed with respect to the gear II, and the third, x, y, z is firmly fixed to the satellite. Draw the axes z'', z' and z along the axis of gears I and II, choosing the origin of coordinates at the point of intersection O of the axes of rotation of the satellite and the gears. Draw the axis x along the axis of rotation of the satellite.

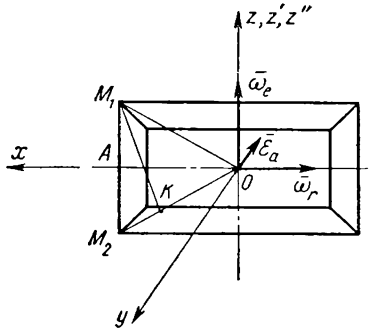


Fig. 538

The directions of other axes are shown in Fig. 538. Now we take the coordinate system x', y', z' as fixed and motion in it is considered to be absolute. Let us determine the acceleration of the point M_2 in this system. The angular velocities of the gear I ω_0 and the satellite ω_A are equal to:

$$\omega_0=\omega_1-\omega_2=2\ \text{sec}^{-1},\ \text{and}\ \omega_A=\frac{v_{M_1}}{MK}=\omega_0\frac{\sqrt{5}}{2},$$

where v_{M_1} is the magnitude of the velocity of the point M_1; M_1K is the distance between the point M_1 and the instantaneous axis of velocities of the satellite OM_2 in the coordinate system x', y', z' Then, the velocity of the point A is $v_A=\omega_A\frac{R}{\sqrt{5}}=\frac{\omega_0 R}{2}$.

The angular velocity of the coordinate axes of the satellite is

$$\overline{\omega}_e=-\overline{i}\,\frac{v_A}{R}=-\overline{i}\,\frac{\omega_0}{2},$$

whereas its relative angular velocity is

$$\overline{\omega}_r=2\overline{k}\,\frac{v_{M_2A}}{R}=\overline{k}\omega_0,$$

where v_{M_2A} is the velocity of the point M_2 with respect to the centre A. Here $\bar{i}$, $\bar{k}$ are unit vectors directed along the axes x and z, respectively. Therefore, if the angular acceleration of the coordinate system is zero, then the angular acceleration of the satellite, which is performing absolute motion, is defined by

$$\bar{\varepsilon}_a=\bar{\omega}_e\times\bar{\omega}_r. \tag{1}$$

Substituting the values of $\bar{\omega}_e$ and $\bar{\omega}_r$ in (1), we obtain

$$\bar{\varepsilon}_a=-\bar{j}\,\frac{\omega_0^2}{2}=-2\bar{j}\ \mathrm{sec}^{-2},$$

where $\bar{j}$ is a unit vector, directed along the axis y. Thus, we may determine the acceleration of rotation of the point M_2:

$$\bar{w}^r_{M_2}=\bar{\varepsilon}_a\times\bar{r}=\bar{i}R+\bar{k}2R,$$

where $\bar{r}=\bar{i}R-\bar{k}r$ is the radius vector of the point M_2.

Since the point M_2 is located on the instantaneous axis in the system x', y', z', its velocity is zero, and, hence, its acceleration $\bar{w}_2^a$ directed to the instantaneous axis of rotation is also zero. The total acceleration of the point M_2 in this coordinate system is $\bar{w}'_2=\bar{w}_2'^r$ The rotation of the system x', y', z' with respect to the car with angular velocity $\bar{\omega}_2=4\bar{k}$ sec^{-1} causes the acceleration $\bar{w}_2^a$ of the point M_2 in the coordinate system x'', y'', z''

$$\bar{w}_2^a=-\bar{i}\omega_2^2R=-16R\bar{i}.$$

Hence, the modulus of the total acceleration of the point M_2 equals

$$w_2=R\sqrt{15^2+2^2}=90.8\ \mathrm{cm/sec}^2.$$

And now, let us determine the acceleration of the point M_1 of the satellite w_1' in the coordinate system x', y', z' The acceleration towards the instantaneous centre of rotation equals

$$w_1'^{\gamma}=\omega_a^2MK=2\sqrt{5}R,$$

whereas the rotational acceleration is

$$w_1'^r=\bar{\varepsilon}_a\times\bar{r}_1=R\bar{i}-2R\bar{j},$$

where $\bar{r}_1$ is the radius vector of the point M_1.

$$\bar{w}_1'=\bar{w}_1'^r+\bar{w}_1'^a=3R\bar{i}-2R\bar{j}.$$

In the system x'', y'', z'' the Coriolis acceleration of the point is defined by the equation

$$\bar{w}_C''=2\bar{\omega}_e\times\bar{v}_r=2\bar{\omega}_2\times\bar{v}_1',$$

where $\overline{v}'_1$ is the velocity of the point M_1 in the coordinate system x', y', z'

$$\overline{v}'_1=\overline{j}\omega_0 R;\ \overline{w}''_{C_1}=16\,\overline{i}R.$$

The accelerations in the direction of the instantaneous centre of rotation of the points M_1 and M_2 equal $\overline{w}''^a=16\overline{i}R$.

$$\overline{w}_1=\overline{w}''^a+\overline{w}''_{C_1}+\overline{w}'_1=35\overline{i}R-2\overline{j}R.$$

The modulus of the acceleration is $w_1=R\sqrt{1229}=210.4$ cm/sec^2.

Let us now determine the acceleration of the point M_3 in the coordinate system x', y', z': $\overline{w}'_3=\overline{w}'_{e_3}+\overline{w}'_{r_3}+\overline{w}'_{C_3}$, where $\overline{w}'_{e_3}$ is the acceleration of the coordinate system, w'_{r_3} is a relative acceleration, and $\overline{w}'_{C_3}$ is the Coriolis acceleration of the point M_3 in the given coordinate system.

$$\overline{w}'_{e_3}=-\overline{i}\left(\frac{\omega_0}{2}\right)^2 R-\overline{j}\left(\frac{\omega_0}{2}\right)^2\frac{R}{2}=-\overline{i}R-\overline{j}\frac{R}{2}.$$

The relative acceleration is $\overline{w}_{r_3}=-\overline{j}\omega_0^2 r$.

It is evident that, as the vector $\overline{v}_{r_3}$ of the relative velocity of the point M_3 is parallel to ω_e, then $\overline{w}_{C_3}=0$. The radius vector $\overline{r}_3$ of the point M_3 is $\overline{r}_3=\overline{i}R+\overline{j}r$ The angular velocity of the satellite is $\overline{\omega}_s=\overline{i}\omega_0+\overline{k}\frac{\omega_0}{2}$ In this case the velocity of the point M_3 in the system x', y', z' equals

$$\overline{v}_{r_3}=\overline{\omega}_s\times\overline{r}_3=-\overline{i}r\frac{\omega_0}{2}+\overline{j}R\frac{\omega_0}{2}+\overline{k}r\omega_0.$$

The Coriolis acceleration of the point M_3 in the fixed coordinate system x'', y'', z'' equals $\overline{w}''_C=2\overline{\omega}_e\times\overline{v}_r=-8\overline{i}R-4\overline{j}R$, and the centripetal acceleration is $\overline{w}''^c=-16\overline{i}R-8\overline{j}R$. Therefore, the total acceleration of the point M_3 is

$$\overline{w}_3=\overline{w}''_s+\overline{w}''^c+\overline{w}'_3=-25\overline{i}R-14.5\overline{j}R.$$

And, consequently, we can easily determine its modulus. Finally, the acceleration of the point M_4 can be determined in the same way.

361(26). The acceleration of the particle is directed along the axis y. Hence, we have

$$F_x=m\ddot{x}=0; \tag{1}$$

$$F_y=m\ddot{y}. \tag{2}$$

Integrating (1), we obtain $x=C_1t+C_2$.

Knowing the initial values $t=0$, $x=0$, $y=b$, $v_0=\dot{x}(0)$, we can determine the constants $C_2=0$ and $C_1=0$, whence

$$x=v_0 t. \tag{3}$$

Substituting the equation (3) into the equation of path of the particle we find the coordinate y as a function of time, and then calculating we find the second derivative of the coordinate y. Substituting the value of $\ddot{y}$ into equation (2), we find the required answer.

388(27). Place the origin of the coordinates at the initial position of the body. Take the axis x along the horizontal, and the axis y perpendicular to the axis x. The body is acted on by gravity

$$\overline{P}=-\overline{j}mg, \tag{1}$$

and the resistance of the atmosphere

$$\overline{R}=-\overline{v}kP=-\overline{i}\dot{x}kP-\overline{j}\dot{y}kP, \tag{2}$$

where $\overline{v}$ is the velocity of the particle, $\overline{i}$ and $\overline{j}$ are unit vectors directed along the axes x and y. The equation of motion of the particle is

$$m\ddot{\overline{r}}=\overline{P}+\overline{R}, \tag{3}$$

or in the projections on the coordinate axes are

$$m\ddot{x}=-kmg\dot{x}; \tag{4}$$

$$m\ddot{y}=-mg-kmg\dot{y}, \tag{5}$$

Integrating the differential equations (4) and (5), we obtain

$$x=A+Be^{-kgt}; \tag{6}$$

$$y=A_1+B_1e^{-kgt}-\frac{t}{k}. \tag{7}$$

And now, let us determine the constants A, B, A_1, B_1 from the initial values, i. e., when $t=0$, $x=0$, $\dot{x}=v_0\cos\alpha$, $y=0$, $\dot{y}=v_0\sin\alpha$.

Substituting these values into the equations (6) and (7), and also into the equations $\dot{x}=-kBge^{-kgt}$ and $\dot{y}=-kB_1ge^{-kgt}-\frac{1}{k}$, which we obtain by differentiating (6) and (7), we, finally, find the system of equations for the constants

$$A+B=0, \quad A_1+B_1=0;$$

$$-kBg=v_0\cos\alpha, \quad -kgB_1-\frac{1}{k}=v_0\sin\alpha,$$

whence

$$A = \frac{v_0 \cos\alpha}{kg}, \quad B = -\frac{v_0 \cos\alpha}{kg};$$

$$A_1 = \frac{kv_0 \sin\alpha + 1}{k^2 g}, \quad B_1 = \frac{kv_0 \sin\alpha + 1}{k^2 g}. \tag{8}$$

And now from the equations (6), (7) and (8) we find the required dependence of the coordinates of the particle on the time. When the particle is at top position, the time t_0 can be found from the equation

$$\dot{y} = v_y = 0.$$

Thus, the magnitude S can be determined from the equation:

$$S = x(t_0) = \frac{v_0 \cos\alpha}{kg}(1 - e^{-kgt_0}).$$

421(29). The pulley pulls the spring with the force $F = F_{BC} - F_{DE} = 3$ kgf, where F_{BC} and F_{DE} are the tensions in the slack and tight sides.

The moment of force applied to the pulley equals $M = F\frac{d}{2}$.

The power of the motor P is defined by the expression.

$$P = M\omega = F\frac{d\omega}{2},$$

where ω is the angular velocity of the motor.

We note that $\frac{d}{2} = 0.318$ m, $\omega = 4\pi$ sec^{-1}.

$$P = 12 \text{ kgfm sec}^{-1} = 0.16 \text{ hp} = 117.8 \text{ w}.$$

$$1\text{w} = 0.102 \text{ kgfm sec}^{-1};$$
$$1\text{hp} = 75 \text{ kgfm sec}^{-1}.$$

439(30). First, let us compute the work required to place the projectile at the point where the gravitational forces of the moon and the earth are equal.

If we assume that the projectile moves along the straight line, connecting the centres of the earth and the moon, then at the distance r from the centre of the earth it will be acted on by the forces

$$F_{\text{earth}} = f\frac{m_1 M}{r^2}; \tag{1}$$

and

$$F_{\text{moon}} = -f\frac{m_1 m}{(d-r)^2} \tag{2}$$

The work required for passing a straight distance dr between the centres of the moon and the earth is $dA \cong (F_{\text{earth}} + F_{\text{moon}})\,dr$, where F_{earth} and F_{moon} are the forces acting on the

projectile, and m_1 is its mass. Thus, the coordinate of the point r_0, at which the gravitational forces of the moon and the earth are equal, is defined by the equation

$$F_{\text{earth}}+F_{\text{moon}}=0. \tag{3}$$

Substituting in (3) the vaules of F_{earth} and F_{moon} from (1) and (2), we obtain

$$r_0=\frac{d\sqrt{M}}{\sqrt{M}+\sqrt{m}}.$$

Whence, the work is defined by the integral

$$A=fm_1\int_R^{r_0}\left[\frac{M}{r^2}-\frac{m}{(d-r)^2}\right]dr=fm_1m\,\frac{\frac{M}{m}(d-R)^2-2R(d-R)\sqrt{\frac{M}{m}}+R^2}{Rd(d-R)}, \tag{4}$$

$$f=\frac{gR^2(d-R)^2}{M(d-R)^2-mR^2}. \tag{5}$$

Besides, substituting the value of f from (5) into (4), we have

$$A=m_1g\,\frac{R(d-R)}{d}\,\frac{\sqrt{\frac{M}{m}}(d-R)-R}{\sqrt{\frac{M}{m}}(d-R)+R}. \tag{6}$$

Hence, the applied work should be equal to the change of the kinetic energy of the projectile on its way from the surface of the earth to the point of equilibrium, e., $A=\frac{mv_0^2}{2}$ And, finally, using the equation (6), and the formula $A=\frac{mv_0^2}{2}$ we can determine the velocity v_0 of the projectile.

448(*31*). When the stone reaches the position defined by the angle φ, its kinetic energy of motion will be

$$T=\frac{mv^2}{2}=\frac{mv_0^2}{2}+R(1-\cos\varphi)mg, \tag{1}$$

where v is the modulus of the velocity of the stone.

Multiplying (1) by $\frac{2}{Rm}$, we obtain the force acting on the stone

$$\frac{v^2}{R}=\frac{v_0^2}{R}+2g(1-\cos\varphi). \tag{2}$$

Thus, the stone moves on the top of a hemispherical dome till the component of the gravitational force, directed along the radius of

the dome P_R and equal to the centripetal force, satisfies the condition

$$P_R = mg\cos\varphi \geqslant \frac{mv_0^2}{R} + 2mg(1-\cos\varphi). \qquad (3)$$

When the equality is satisfied, the stone leaves the dome and so we get the answer for the angle φ.

$$\cos\varphi = \frac{2}{3} + \frac{v_0^2}{3Rg}.$$

468(32). First, take the axis x vertically down and place the origin of the coordinate system in the position of static equilibrium of the system. When the resistance to motion is proportional to the velocity of the body, the equation of motion is

$$\ddot{x} + \frac{k}{m}\dot{x} + \frac{c}{m}x = 0, \qquad (1)$$

where c is the coefficient of elasticity of the spring.
Hence, in this case the auxiliary equation will be

$$p^2 + \frac{k}{m}p + \frac{c}{m} = 0.$$

The roots of this equation are defined by the expression

$$p_{1,2} = -\frac{k}{2m} \pm \sqrt{\left(\frac{k}{2m}\right)^2 - \frac{c}{m}}. \qquad (2)$$

Therefore, the solution of equation (1) is

$$x = Ae^{-\frac{k}{2m}t}\sin\left[\left(\sqrt{\frac{c}{\mathrm{m}} - \left(\frac{k}{2m}\right)^2}\right)t + \alpha\right]. \qquad (3)$$

We can determine the quantity $\frac{c}{m}$ from the period of oscillation of the body $T = 0.4\pi$ sec, when $k = 0$, and the quantity $\frac{k}{m}$ from the equation

$$\sqrt{\frac{c}{m} - \left(\frac{k}{2m}\right)^2} = \frac{2\pi}{T_1},$$

and finally the result is

$$\frac{c}{m} = 25, \text{ and } \frac{k}{m} = 6.$$

When the initial conditions $x = 4$ cm and $\dot{x} = 0$ at $t = 0$ are substituted into the equation (3), we finally obtain the system of equations for determining the constants A and α.

$$A\sin\alpha = 4.$$

$$-\frac{k}{2m}\sin\alpha + \frac{2\pi}{T_1}\cos\alpha = 0. \qquad (4)$$

Hence, from (4) $A = 5$; $\tan\alpha = \frac{4}{3}$.

508 (34). A force of gravity $\overline{P}$ is applied at the centre of mass of the rod AB. A part of the rod $\frac{dz}{\cos\varphi}$ is acted on by a force of inertia directed, as shown in Fig. 539.

The sum of the moments of a force $\overline{P}$ and all the forces of inertia about the point O equals zero.

$$\sum_{k=1}^{n} m_x(\overline{F}_k) = \frac{P(a-b)}{2}\sin\varphi - \int_0^{a\cos\varphi} z dI - \int_0^{b\cos\varphi} z dI \qquad (1)$$

where

$$dI = \frac{zdzP\omega^2}{(a+b)g}\,\frac{\sin\varphi}{\cos^2\varphi}. \qquad (2)$$

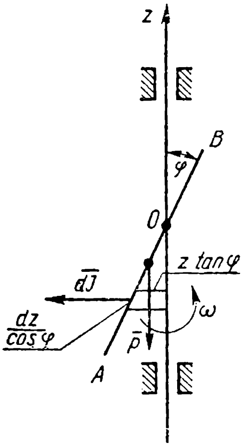


Fig. 539

And, finally, substituting the value for dI into (1) and integrating, we obtain the required result.

525(35). First, let us determine the force of the supporting reaction $\overline{R}_A$. The support A will be considered as "removed", and the

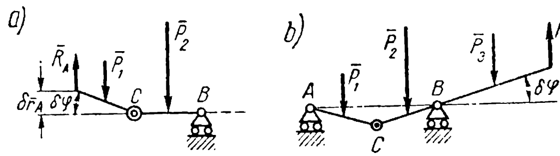


Fig. 540

reaction of the removed unit is denoted by $\overline{R}_A$. The point A will be displaced through distance $\delta\overline{r}_A$ vertically upwards. Then the beam assumes the position, shown in Fig. 540, *a*. Using the principle of virtual displacements, we find that the total work, done by the applied force, and the reaction force $\overline{R}_A$ when moved through distances $\delta\overline{r}_A$ and $\delta\overline{r}_1$, equals zero.

$$R_A\delta r_A - P_1\delta r_1 = 0. \qquad (1)$$

Express δr_A and δr_1 in terms of the angular displacement of the unit AC.

$$\delta r_A = 2a\delta\varphi; \quad \delta r_1 = a\delta\varphi.$$

Hence, from (1) we have

$$2aR_A\delta\varphi - P_1 a\delta\varphi = 0, \text{ or } R_A = \frac{P_1}{2} = 1000 \text{ kgf}.$$

Let us remove the support D and replace it by the reaction $\overline{R}_D$. The virtual displacement $\delta\overline{r}_D$ of the point D will be directed verti-

cally upwards, as shown in Fig. 540, *b*. Then we shall connect all virtual displacements $\delta\bar{r}_1$, $\delta\bar{r}_2$, $\delta\bar{r}_3$ of the forces $\bar{P}_1$, $\bar{P}_2$, $\bar{P}_3$ and displacement $\delta\bar{r}_D$ with the angular displacement $\delta\varphi$. We note from the Fig. 540, *b*, that $\delta r_1=\delta r_2=a\delta\varphi$, $\delta r_3=2a\delta\varphi$, $\delta r_D=4a\delta\varphi$. Applying the principle of virtual displacements, we obtain

$$P_1\delta r_1+P_2\delta r_2-P_3\delta r_3+R_D\delta r_D=0;$$

$$2a+4aR_D=0, \quad R_D=-500 \text{ kgf}.$$

Reasoning in the same way, we may find the magnitude R_B. But, if we know R_A and R_D it is much easier to compute R_B by equating to zero the sum of the projections of all the forces on the vertical axis.

534(36). As all the constraints in this problem are ideal, then the total virtual work done by all the active forces and all the inertia forces in any virtual displacement is zero at any instant. Thus,

$$\Sigma(\bar{F}_k-m_k\bar{w}_k)\,\delta\bar{r}_k=0, \tag{1}$$

where $\bar{F}_k$ denotes the active forces, and $-m_k\bar{w}_k$ denotes the forces of inertia.

Using (1) we obtain

$$P\sin\alpha\,dl-\frac{P}{g}\ddot{x}dl-I_0\varepsilon d\varphi=0, \tag{2}$$

where $\ddot{x}$ is the acceleration of the body A, I_0 is the moment of inertia of the drum, dl is the displacement of the body A on the inclined plane, $d\varphi$ is the angle of turn of the drum corresponding to the displacement dl, ε is the angular acceleration of the drum.

$$\ddot{x}=\varepsilon r,\ d\varphi=\frac{dl}{r} \tag{3}$$

Substituting (3) into (2), we obtain

$$P\sin\alpha-\frac{P}{g}\,\varepsilon r-I_0\varepsilon\,\frac{1}{r}\,=0; \quad \varepsilon=\frac{P\sin\alpha}{\frac{Pr}{g}+\frac{I_0}{r}}. \tag{4}$$

The moment of inertia of the drum of radius r is

$$I_0=\frac{Qr^2}{2g},$$

and, hence, substituting this into (4), we get

$$\varepsilon=\frac{2\,Pg\sin\alpha}{r\,(2P+Q)}$$

554(37). If $\bar{w}$ is the acceleration of the centre of inertia of a material system, M is its mass and $\bar{F}_i$ are the external forces acting on the system, then

$$M\bar{w}=\sum_{i=1}^{n}\bar{F}_i. \tag{1}$$

Take now the axis x to the right, parallel to the foundation of the motor, and take the origin of the coordinates at the point of intersection of its axis of symmetry with the axis x. By projecting (1) onto the axis x we have

$$M\ddot{x}=\sum_{i=1}^{n} F_{ix}, \tag{2}$$

and

$$M=\frac{P+p+Q}{g}, \quad F_x=M\omega^2 r \sin \omega t, \tag{3}$$

where $r=OC$ is the modulus of the radius vector of the centre of mass of the system.

$$r=\frac{l(p+2Q)}{P+p+Q}$$

Using the values of r and M from (2) and (3), we obtain

$$\frac{P+p+Q}{g}\ddot{x}=\frac{\omega^2 l(p+2Q)}{g} \sin \omega t, \tag{4}$$

and integrating (4) we have $x=-\frac{l(p+2Q)}{P+p+Q}\sin \omega t+C_1 t+C_2$.

From the initial conditions at $t=0$, $x=0$, $\dot{x}=\frac{l(p+2Q)\omega}{P+p+Q}$, it follows that

$$C_2=0; \quad C_1=0.$$

Therefore, when the motor is fixed we have

$$x=-\frac{l(p+2Q)}{P+p+Q} \sin \omega t;$$

$$F_x=\frac{P+p+Q}{g}\ddot{x}=\frac{\omega^2 l(p+2Q)}{g} \sin \omega t$$

$$F_{x\,\max}=R=l\omega^2\frac{p+2Q}{g}.$$

568(38). Draw the axis x in the direction of the velocity $\bar{v}_1$. As far as the projections on this axis are concerned, the theorem on the change in the linear momentum of a system can be expressed in the equation

$$Mv_{1x}-Mv_{2x}=\Sigma S_x, \tag{1}$$

where M is the mass of the fluid falling on the blade at time dt, v_{2x} is the projection of the velocity $\bar{v}_2$ on the axis x, ΣS_x is the pro-

jection of the sum of all the impulses of external forces, acting on the blade, on the axis x

$$\Sigma S_x = N dt; \tag{2}$$

$$M = Q\frac{\gamma}{g}dt; \tag{3}$$

$$v_{2x} = -v_2 \cos \alpha. \tag{4}$$

From the equations (1), (2), (3) and (4), we obtain

$$N dt = Q\frac{\gamma}{g}(v_1 + v_2 \cos \alpha)\,dt; \tag{5}$$

and, finally, from the equation (5) we get

$$N = Q\frac{\gamma}{g}(v_1 + v_2 \cos \alpha).$$

572(39). The change in angular momentum of a system with respect to the axis z directed perpendicular to the plane of the platform through its centre may be expressed by the equation

$$\frac{dL_z}{dt} = \sum_{k=1}^{n} m_z(\overline{F}_k), \tag{1}$$

where L_z is the angular momentum of the system. $\sum_{k=1}^{n} m_z(\overline{F}_k)$ is the sum of all moments of the external forces of the system with respect to the axis z. In our case we have

$$\Sigma m_z(\overline{F}_k) = 0; \tag{2}$$

$$L_z = r\frac{P}{g}(u - r\omega) - \frac{PR^2\omega}{2g}, \tag{3}$$

where the first member is the projection of the angular momentum of the man on the axis z, and the second member is the projection of the angular momentum of the platform on the same axis. (The moment of inertia of the disk equals $\frac{PR^2}{2g}$.) It follows from (1) and (2), that $L_z = \text{const}$, and from the initial conditions we obtain $L_z = 0$. And hence, equating (3) to zero and solving this equation for ω, we can determine the modulus of the angular velocity of the platform.

651(40). The change in the kinetic energy of the system during any displacement is equal to the sum of the work done by all the external and internal forces acting on the system during that displacement. In our case the internal forces are not taken into consideration. Therefore, we have

$$T_2 - T_1 = \sum_{k=1}^{n} A(\bar{F}_k^e), \tag{1}$$

where T_2 is the kinetic energy of the system after the displacement of the wheel A at the distance S down the inclined plane ON, $\Sigma A(\bar{F}_k^e)$ is the sum of the work done by all external forces during this displacement. Therefore,

$$T_2 = \frac{I_A\omega_A^2}{2} + \frac{M_A v^2}{2} + \frac{I_C\omega_C^2}{2} + \frac{I_B\omega_B^2}{2} + \frac{M_B v^2}{2},$$

where ω_A, ω_B, ω_C and I_A, I_B, I_C are the angular velocities and moments of inertia of the wheels and the pulley with respect to their axes. Thus, according to the conditions of the problem (the wheels and the pulley are considered as identical homogeneous disks) $I_A = I_C = I_B = \frac{Mr^2}{2}$, where M is the mass, and r is the radius of the wheels or pulley. $M_A = M_B = M$. Whence, if the magnitude of the velocity of the axis of the wheel A is v, then its angular velocity is $\omega = \frac{v}{r}$ therefore, $\omega_A = \omega_B = \omega_C = \omega$. In this case the equation (2) takes the form

$$T_2 = \frac{7}{4} M v^2. \tag{3}$$

According to the conditions of the problem, $T_1 = 0$. Consequently, the work done by all forces in the displacement S will be equal to the change in potential energy during that displacement

$$\sum_{k=1}^{n} A(\bar{F}_k^e) = MgS(\sin\alpha - \sin\beta). \tag{4}$$

And now, taking into consideration (3) and (4) from (1), we finally obtain

$$v^2 = \frac{4}{7} gS(\sin\alpha - \sin\beta).$$

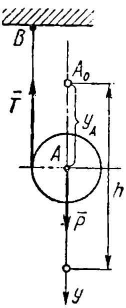


Fig. 541

664(41). The coordinate axes are shown in Fig. 541. The cylinder is under the action of the external forces: $\bar{P}$ is the weight of the cylinder, and $\bar{T}$ is the reaction of the cord. The differential equations of motion of the cylinder are

$$\frac{P}{g}\ddot{y}_A = P - T \tag{1}$$

$$I_A\ddot{\varphi} = TR, \tag{2}$$

where R is the radius of the cylinder, φ is the angle of

its rotation, and $I_A = \frac{P}{g}\frac{R^2}{2}$ is its moment of inertia about the axis of symmetry, passing through the point A, y_A is the coordinate of the point A. Whence,

$$y_A = R\varphi;\ \varphi = \frac{\ddot{y}_A}{R}. \tag{3}$$

Substituting (3) into (2), we obtain

$$y_A = \frac{2g}{P}T \tag{4}$$

Then, from the equations (1) and (4), excluding $\ddot{y}_A$, we can determine the reaction of the cord T

$T = \frac{1}{3}P = \frac{1}{3}mg$, where m is the mass of the cylinder Likewise, from (1) we have

$$y_A = \frac{2}{3}g;\quad \dot{y}_A = \frac{2}{3}gt + C_1;\quad y_A = \frac{1}{3}gt^2 + C_1t + C_2.$$

It follows from the initial conditions $t=0$, $y_A=0$, $y_A=0$, that $C_1=0$, $C_2=0$, when $y_A=h$, $t_0=\sqrt{\frac{3h}{g}}$.

$$y_A = \frac{2}{3}gt. \tag{5}$$

$$\dot{y}_A = \frac{1}{3}gt^2 \tag{6}$$

And, finally, substituting the value of t_0 in (5), we obtain the result given in the answer

702 (44). During the impact the impulse of the force $\overline{S}$ applied to the ball is directed in the way, shown in Fig. 542. The coordinate axes x and y are also drawn in this figure. If we apply the theorem on the change of the momentum of the ball during impact, we obtain for the y axis

$$mv_1 - mv = S_1.$$

The velocity of the ball after impact $\vec{v}_1 = 0$ and, therefore,

$$mv = -S_1. \tag{1}$$

Following the principle of the equality of action and reaction, we have $S = -S_1 = mv$, where S is the impulse of the force applied to the prism.

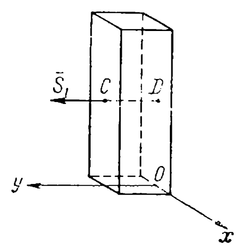


Fig. 542

And now, let us consider the ball and the prism as one unit. The change during the

impact of the principal angular momentum of a system of rigid bodies with respect to a fixed axis is equal to the sum of the moments taken with respect to the same axis of the impulses of the external instantaneous forces:

$$L_{x_2} - L_{x_1} = \Sigma m_x(\bar{S}), \tag{2}$$

where L_{x_2} and L_{x_1} are the projections of the angular momentum of the system before and after the impact, respectively, and $\Sigma m_x(\bar{S})$ is the sum of moments of the impact impulses about the axis x. In this particular case

$$L_{x_1} = 0, \ L_{x_2} = -I_x\omega, \tag{3}$$

where I_x is the moment of inertia of the prism and the ball with respect to the edge AB.

Substituting (3) into (2) and noting (1) we have

$$-I_x\omega = -\frac{3}{2}\,amv; \quad \omega = \frac{3}{2}\frac{amv}{I_x}. \tag{4}$$

The moment of inertia of the ball about the x axis equals $\frac{13}{4}\,ma^2$, and the moment of inertia of the prism about the x axis equals $10\,ma^2$.

Hence, $I_x = \frac{53}{4}\,ma^2$ and, substituting the value of I_x into (4), we obtain $\omega = \frac{6v}{53a}$.

Whence, the prism will tip over if the kinetic energy, concentrated at the impact, equals the work required to tip the system over.

$$\frac{I_x\omega^2}{2} = 4\,mgh, \tag{5}$$

where h is the height at which the centre of gravity of the system (prism — ball) should be raised to make it tip over.

Substituting the values of I_x, ω and h into (5), we, finally, obtain the velocity of the ball.

705, 707 (45). We may write the equation of motion of a system, whose mass changes continuously due to ejection of particles of negligibly small masses, in the following form:

$$m\frac{d\bar{v}}{dt} = \bar{F} + \frac{dm}{dt}(\bar{u} - \bar{v}); \tag{1}$$

(this equation states the differential equation of motion of a particle having variable mass, known as Meshchersky's equation), where m is the mass of the system at a given instant, and $\bar{v}$ and $\bar{u}$ are the velocities of the system and ejected particles, respectively.

Now, draw the x axis in the direction of motion of the rocket. According to the condition $m = m_0 f(t)$; $F_x = -R(x, \dot{x}) - mg$. The projection $\bar{u} - \bar{v}$ on the axis x will be v_r and

$$\frac{dm}{dt} = m_0 \dot{f}(t).$$

When we project the equation (1) on the x axis and substitute these values, we obtain

$$m_0 f(t) \ddot{x} = -R(x, \dot{x}) - m_0 \dot{f}(t) v_r - m_0 f(t) g,$$

or

$$\ddot{x} + \frac{R(x, \dot{x})}{m_0 f(t)} + \frac{\dot{f}(t) v_r}{f(t)} + g = 0. \tag{2}$$

Let $m = m_0 e^{-\alpha t}$ and $R(x, \dot{x}) = 0$. Then, from (2) we have

$$x = \alpha v_r - g. \tag{3}$$

Integrating (3), we obtain $x = (\alpha v_r - g) \frac{t^2}{2} + C_1 t + C_2$.

From the initial conditions, when $t = 0$; $x = 0$; $\dot{x} = 0$, we conclude that $C_2 = 0$, and $C_1 = 0$. Whence

$$x = (\alpha v_r - g) \frac{t^2}{g} \tag{4}$$

The altitude h of the rocket during the time t_0 equals

$$h = (\alpha v_r - g) \frac{t_0^2}{2}. \tag{5}$$

Starting from the instant of burning out of the fuel, i. e., at $t > t_0$ the rocket is acted on only by the force of gravity while the mass of the rocket remains constant. Taking the law of conservation of mechanical energy, we may write the following equation:

$$m_0 e^{-\alpha t_0} g \Delta h = \frac{m_0 e^{-\alpha t_0} \dot{x}^2(t_0)}{2}.$$

Denoting from (4) the value $\dot{x}(t_0)$, we obtain

$$\Delta h = \frac{t_0^2 (\alpha v_r - g)^2}{2}, \tag{6}$$

where Δh is an increment in the motion of the rocket, when the fuel is completely burned out, the time being $t > t_0$. Thus, the maximum altitude of the rocket will be

$$H = h + \Delta h = \frac{\alpha v_r}{2g} (\alpha v_r - g) t_0^2.$$

717(46). According to the text of the problem, both ends of a uniform rod of length l are free to slide without friction along

the curve, given by the equation $f(x, y)=0$. Therefore, the sum of the work done by the reaction forces at the points of contact of the ends of the rod in any displacement is zero. To provide the equilibrium for the rod it is quite sufficient for the latter to slide with its ends along the curve $f(x, y)=0$ so that the work done by its weight, applied at the centre of gravity of the rod, equals zero in any displacements of the rod along the curve

$$\bar{P}\delta\bar{r}=P\delta y=0, \tag{1}$$

where δy is the increment of the coordinate y of the centre of gravity of the rod, and P is its weight. Since the rod is a uniform body, its centre of gravity lies at the mid-point of the rod.

Let the coordinates of the ends of the rod, when in equilibrium, be x_1, y_1 and x_2, y_2, then the increment of the coordinate of the centre of gravity takes the form

$$\delta y=\frac{\delta y_1+\delta y_2}{2} \tag{2}$$

where δy_1 and δy_2 are the increments of the coordinates y_1 and y_2. Whence, from (1) and (2) we obtain

$$\delta y_1+\delta y_2=0. \tag{3}$$

It is evident that the coordinates of the ends of the rod located on the curve, will satisfy the equations

$$f(x_1, y_1)=0; \quad (4) \quad \text{and} \quad f(x_2, y_2)=0. \tag{5}$$

Besides, its length l is connected with the coordinates of the ends of the rod by the following expression

$$(x_2-x_1)^2+(y_2-y_1)^2=l^2, \tag{6}$$

as the rod and the straight lines drawn from the ends of the rod parallel to the coordinate axes form a right-angled triangle. Let us find total derivative of the function (6)

$$(x_2-x_1)(\delta x_2-\delta x_1)+(y_2-y_1)(\delta y_2-\delta y_1)=0. \tag{7}$$

Substituting the value δy_2 from (3) into (7), we obtain

$$(x_2-x_1)(\delta x_2-\delta x_1)=2(y_2-y_1)\delta y_1;$$

$$\delta x_2=\frac{dx_2}{dy_2}\frac{dy_2}{dx_1}\delta x_1. \tag{8}$$

And then, considering the equation (3), we have

$$\delta x_2=-\frac{dx_2}{dy_2}\frac{dy_1}{dx_1}\delta x_1=\frac{\frac{\partial f}{\partial y_2}\frac{\partial f}{\partial x_1}}{\frac{\partial f}{\partial y_1}\frac{\partial f}{\partial x_2}}\delta x_1, \tag{9}$$

where $\frac{\partial f}{\partial x_1}$, $\frac{\partial f}{\partial y_1}$, $\frac{\partial f}{\partial x_2}$, $\frac{\partial f}{\partial y_2}$ are the partial derivatives of the

function $f(x, y)$ at points x_1, y_1 and x_2, y_2, respectively. Substituting the value δx_2 from (9) into (8) and remembering that

$$\frac{\delta y_1}{\delta x_1} = \frac{-\dfrac{\partial f}{\partial x_1}}{\dfrac{\partial f}{\partial y_1}},$$

after some easy calculations we obtain

$$(x_2-x_1)\left(\frac{\partial f}{\partial y_2}\frac{\partial f}{\partial x_1}+\frac{\partial f}{\partial x_2}\frac{\partial f}{\partial y_1}\right)=2(y_2-y_1)\frac{\partial f}{\partial x_1}\frac{\partial f}{\partial x_2}. \qquad (10)$$

We conclude that the equations (4), (5), (6) and (10) determine the coordinates of the ends of the rod at equilibrium.

It is worth while noting that this solution is one of a rather limited class of functions $f(x, y)$.

721(46). Let us denote by y the distance between the centre of gravity of the rod, lying at its mid-point, and the straight line CO. Using the principle of virtual displacements we find, at equilibrium,

$$P\delta y=0, \qquad (1)$$

where P is the weight of the rod. $AC=2R\cos\varphi$

$$y=(AC-a)\sin\varphi=(2R\cos\varphi-a)\sin\varphi. \qquad (2)$$

By differentiating (2) we obtain

$$\delta y=(2R\cos 2\varphi-a\cos\varphi)\,\delta\varphi. \qquad (3)$$

Whence, the angle which corresponds to the position of equilibrium is determined by the equation

$$2R\cos 2\varphi-a\cos\varphi=\cos^2\varphi-\frac{a}{4R}\cos\varphi-\frac{1}{2}=0, \qquad (4)$$

which is obtained by substituting (3) into (1)

Hence, $\varphi_0<\frac{\pi}{2}$, therefore, $\cos\varphi_0>0$.

And, finally, taking only the positive root of the equation (4), we have

$$\cos\varphi_0=\frac{1}{8R}\left[a+\sqrt{a^2+32R^2}\right].$$

734(47). Let us take the distance from the intersection of the straight line AB with the z axis to the particle M as a generalized coordinate, as shown in Fig. 466.

The potential energy of the particle will be $V=mgr\sin\alpha$, where m is the mass of the particle.

The kinetic energy of the particle is given by the expression:

$$T=\frac{m}{2}\left(r^2\cos^2\alpha\,\omega^2+\frac{\dot r^2}{2}\right). \qquad (1)$$

Here the first part of the equation is the kinetic energy of the rotation of the particle about the z axis, and the second is the kinetic energy motion along the straight line AB.

Substituting $\frac{\partial L}{\partial \dot{r}}$ and $\frac{\partial L}{\partial r}$ into Lagrange's equation

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{r}}\right)-\frac{\partial L}{\partial r}=0,$$

where $L=T-V$, we obtain

$$\ddot{r}-r\omega^2\cos^2\alpha=-g\sin\alpha. \tag{2}$$

The quotient solution of this non-homogeneous equation will be

$$r_1=\frac{g\sin\alpha}{\omega^2\cos^2\alpha}.$$

In this particular case the auxiliary equation takes the form:

$$p^2+\omega^2\cos^2\alpha=0;\; p=\pm\omega\cos\alpha.$$

Hence, the solution of the homogeneous equation is

$$r_2=C_1e^{-\omega t\cos\alpha}+C_2e^{\omega t\cos\alpha}.$$

And the complete solution then will be

$$r=r_1+r_2=C_1e^{-\omega t\cos\alpha}+C_2e^{\omega t\cos\alpha}+\frac{g\sin\alpha}{\omega^2\cos^2\alpha}.$$

751(47). Take the variables φ and y as the generalized coordinates, as shown in Fig. 471.

The kinetic energy of the system is

$$T=\frac{m_1\dot{y}^2}{2}+\frac{m_2v_2^2}{2}, \tag{1}$$

where $\frac{m_1\dot{y}^2}{2}$ and $\frac{m_2v_2^2}{2}$ are the kinetic energies of the slider and the ball, respectively.

$\overline{v}_2=\overline{v}_A+\overline{v}_{BA}$, where $\overline{v}_A$ is the velocity of the slider, and $\overline{v}_{AB}$ is the velocity of the ball B with respect to the point A.

$$v_2^2=\dot{y}^2+l^2\dot{\varphi}^2+2l\dot{\varphi}\dot{y}\cos\varphi.$$

Then the potential energy V equals:

$$V=m_2gl(1-\cos\varphi). \tag{2}$$

And now make the following calculations:

$$\frac{\partial T}{\partial y}=0;\frac{\partial T}{\partial \dot{y}}=(m_1+m_2)\dot{y}+m_2l\dot{\varphi}\cos\varphi;\;\frac{\partial V}{\partial y}=0;$$

$$\frac{\partial T}{\partial \varphi}=-m_2l\dot{\varphi}\dot{y}\sin\varphi;\;\frac{\partial T}{\partial \dot{\varphi}}=m_2l(\dot{\varphi}l+\dot{y}\cos\varphi);\;\frac{\partial V}{\partial \varphi}=m_2lg\sin\varphi. \tag{3}$$

Substituting (3) into Lagrange's equation

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{q}_i}-\frac{\partial T}{\partial q_i}=\frac{\partial V}{\partial q_i}; \quad (i=1,2;\ q_1=y,\ q_2=\varphi),$$

we finally obtain the equation of motion of the elliptic pendulum.

795(48). Let us derive the equation of motion of the weight P at the period of time from 0 to τ.

$$\frac{P}{g}\ddot{x}=F-cx.$$

In this case we arrive at the auxiliary equation

$$p^2+\frac{cg}{P}=0;\ p_{1,2}=\pm i\sqrt{\frac{cg}{P}},$$

and the solution of the homogeneous equation is

$$A\cos\alpha t+B\sin\alpha t,$$

where $\alpha=\sqrt{\frac{cg}{P}}$.

The solution of the non-homogeneous equation will be $\frac{F}{c}$. Hence, the complete solution takes the form:

$$x=\frac{F}{c}+A\cos\alpha t+B\sin\alpha t.$$

From the initial conditions at $t=0$; $x=0$; $\dot{x}=0$, we may determine the constants A and B.

$$\frac{F}{c}+A=0;\ B=0.$$

And, finally,

$$x=\frac{F}{c}(1-\cos\alpha t);\ t[0,\ \tau], \tag{1}$$

If $t>\tau$, then the equation of motion of the weight will be

$$\frac{P}{g}x=-cx. \tag{2}$$

The solution of the equation (2) will become

$$x=A\sin\alpha t+B\cos\alpha t. \tag{3}$$

From the initial conditions $t=\tau$, $x=\frac{F}{c}(1-\cos\alpha\tau)$, $\dot{x}=\frac{\alpha F}{c}\sin\alpha\tau$, we obtain the system of equations for defining the constants A and B

$$\frac{F}{c}(1-\cos\alpha\tau)=A\sin\alpha\tau+B\cos\alpha\tau,$$

$$\alpha\frac{F}{c}\sin\alpha\tau=\alpha A\cos\alpha\tau-\alpha B\sin\alpha\tau.$$

Solving this system, we obtain

$$A=\frac{F}{c}\sin\alpha\tau,\ B=-\frac{F}{c}(1-\cos\alpha\tau). \qquad (4)$$

Substituting the values (4) into (3), we finally have

$$x=\frac{F}{c}[\sin\alpha(t-\tau)-\cos\alpha\tau].$$

814(49). The potential energy V and kinetic energy T of the system as a function of the coordinates φ and ϑ (Fig. 509) are given by the equations

$$V=Mgl(1-\cos\varphi)+mg(l+r-l\cos\varphi-r\cos\vartheta) \qquad (1)$$

$$T=\frac{Mv_A^2}{2}+\frac{I\omega^2}{2}+\frac{mv_m^2}{2} \qquad (2)$$

The expression $I=\frac{Mr^2}{2}$ in the formula (2) is the moment of inertia of the disk, and $v_A=l\dot{\varphi}$ is the velocity of its centre of mass; ω is angular velocity of the disk. $\bar{v}_m=\bar{v}_A+\bar{v}_\vartheta$, where v_ϑ is the velocity of the particle m with respect to the centre of the disk A.

$$v_m^2=l^2\dot{\varphi}^2+r^2\dot{\vartheta}^2+2\dot{\vartheta}\dot{\varphi}lr\cos(\vartheta-\varphi).$$

When small oscillations take place, $\cos(\vartheta-\varphi)=1$. Then

$$T=\frac{Ml^2\dot{\varphi}^2}{2}+\frac{Mr^2\dot{\vartheta}^2}{4}+\frac{m}{2}(l^2\dot{\varphi}^2+r^2\dot{\vartheta}^2+2\dot{\vartheta}\dot{\varphi}lr)=$$

$$=\frac{1}{2}(A_{11}\dot{\varphi}^2+2A_{12}\dot{\vartheta}\dot{\varphi}+A_{22}\dot{\vartheta}^2), \qquad (3)$$

where $A_{11}=(M+m)l^2$, $A_{12}=mlr$, $A_{22}=\left(\frac{M}{2}+m\right)r^2$.

In this particular case of equilibrium we have

$$V=0;\ \frac{\partial V}{\partial q_i}=0.$$

For small oscillations the potential energy is

$$V=\frac{1}{2}(a_{11}q_1^2+2a_{12}q_1q_2+a_{22}q_2^2)=\frac{1}{2}[\varphi^2gl(M+m)+\vartheta^2mgr], \qquad (4)$$

where

$$a_{11}=(M+m)gl;\quad a_{12}=0;\quad a_{22}=mgr;\quad a_{ik}=\left(\frac{\partial^2V(0)}{\partial q_i\partial q_k}\right),\ q_1=\varphi;\quad q_2=\vartheta;$$

Substituting (3) and (4) into Lagrange's equations,

$$\frac{d}{dt}\left(\frac{\partial T}{\partial\dot{q}_i}\right)-\frac{\partial T}{\partial q_i}=\frac{\partial V}{\partial q_i}\ (i=1,\ 2)$$

we find the equations of motion of the system:

$$\left.\begin{aligned}(M+m)\,l^2\ddot{\varphi}+mlr\ddot{\vartheta}+(M+m)\,gl\varphi=0\\ mlr\varphi+\left(\frac{M}{2}+m\right)r^2\ddot{\vartheta}+mgr\vartheta=0\end{aligned}\right\}\qquad(5)$$

The quotient solution of the system (5) is assumed to be in the form

$$\varphi=B\sin(kt+\alpha),\quad \vartheta=D\sin(kt+\alpha),\qquad(6)$$

where B, D, α are constants. Substituting (6) into (5), we obtain

$$\begin{aligned}B(a_{11}-k^2A_{11})-Dk^2A_{12}=0;\\ -Bk^2A_{12}+D(a_{22}-k^2A_{22})=0.\end{aligned}\qquad(7)$$

The system (7) has a solution different from zero, if its determinant Δ equals zero:

$$\Delta=\begin{vmatrix}a_{11}-k^2A_{11} & -k^2A_{12}\\ -k^2A_{12} & a_{22}-k^2A_{22}\end{vmatrix}=0.\qquad(8)$$

Substituting the values A_{ik} and a_{ik} from (3) and (4) and solving the determinant, we, finally, obtain the equation of the frequencies.

836 (50). According to the conditions of the problem the differential equations of the undisturbed motion of the system are

$$\left.\begin{aligned}m\ddot{x}+\beta\dot{x}+cx=A(\omega-\omega_0)\\ I\dot{\omega}=-Bx.\end{aligned}\right\}\qquad(1)$$

Substituting the values $x=x_0+\delta x$ and $\omega=\omega_0+\delta\omega$ into the equation (1), we obtain the equation of the disturbed motion.

$$m\delta\ddot{x}+\beta\delta\dot{x}+c\delta x+cx_0=A\delta\omega.\qquad(2)$$

$$I\delta\dot{\omega}=-B(x_0+\delta x).\qquad(3)$$

Differentiating (2) with respect to time, we obtain

$$m\delta\dddot{x}+\beta\delta x+c\delta\dot{x}=A\delta\dot{\omega}.\qquad(4)$$

Eliminating $\delta\dot{\omega}$ from (3) and (4), we have

$$m\delta\dddot{x}+\beta\delta\ddot{x}+c\delta\dot{x}+\frac{AB}{I}\,\delta x+\frac{AB}{I}\,x_0=0.$$

In this case we have the auxiliary equation in the form:

$$a_0p^3+a_1p^2+a_2p+a_3=0,\qquad(5)$$

where

$$a_0=m,\ a_1=\beta,\ a_2=c,\ a_3=\frac{AB}{I}.$$

Hence, the motion defined by the equations (1) is stable, if all the roots of the auxiliary equation (5) are negative, i. e., when:

$$a_1>0, \quad \begin{vmatrix} a_1 & a_0 \\ a_3 & a_2 \end{vmatrix} >0; \quad \begin{vmatrix} a_1 & a_0 & 0 \\ a_3 & a_2 & a_1 \\ 0 & 0 & a_3 \end{vmatrix} >0,$$

i. e.,

$$\beta>0; \quad c\beta-m\frac{AB}{I}>0; \ \frac{AB}{I}\left(\beta c-m\frac{AB}{I}\right)>0.$$

It is evident that these conditions are equivalent to the equation

$$\frac{Ic\beta}{m}>AB>0.$$

Иван Всеволодович Мещерский

Сборник задач
по теоретической механике
(на английском языке)

RRAT

	Reads	Should
	Ans. $\frac{P_r^2}{2g}\left(\frac{T_1}{T_2}\right) = 0.117$ kgfcmsec²	*An* $= \frac{P_r^2}{2g}\left(\frac{T_1}{T_2}\right)^2 = 0.117$ kgfcmsec²
bo	$T = \frac{P}{2\cos\beta} = \frac{lP}{2\sqrt{l^2-4r^2\sin^2\frac{\alpha}{2}}}$	$T = \frac{P}{2\cos\beta} = \frac{lP}{2\sqrt{l^2-4r^2}\frac{\alpha}{2}}$
	$w_1'^r = w_a^2 MK$	$w_1'^r = w_a^2 MK = 2\sqrt{5}R.$
rom	$dA = (F_{earth}$	$dA \cong (F_{earth}$
	at the auxiliary	at the auxiliary equation.
	where $A_{11} = (M+m)l_1^2$	$A_{11} = (M+m)l^2$

теоретической механике»